How to Predict the Frequency and Magnitude of the Primary Phase Truncation Spur in the Output Spectrum of a Direct Digital Synthesizer (DDS)

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INTRODUCTION

Modern direct digital synthesizers (DDSs) typically employ an accumulator and a digital frequency tuning word (FTW) to produce a periodic, N-bit digital ramp at the accumulator output (see Figure 1). This digital ramp defines the output frequency (f_O) of the DDS according to Equation 1, where f_S is the DDS sample rate (or system clock frequency).

$$f_O = f_S \times \frac{FTW}{2^N} \qquad (1)$$

For a given DDS, the number of bits (N) that compose the FTW defines the smallest possible change in f_O, which occurs when the value of the FTW changes only by its least significant bit (LSB). That is, a 1 LSB change in the FTW defines the tuning resolution of the DDS. A DDS with N = 32, for example, has finer tuning resolution than a DDS with N = 24. To demonstrate the very fine tuning capability of a DDS, consider the AD9912,

for which N = 48 yields a tuning resolution of one part in 2^{48} (that is, one part in 281,474,976,710,656). In fact, with f_S = 1 GHz, the AD9912 yields a frequency tuning resolution of approximately 3.6 μHz (0.0000036 Hz).

Given a DDS with an FTW of N bits, close inspection of Figure 1 shows an apparent disparity between the number of bits at the output of the accumulator (N) and the number of bits at the input to the angle to amplitude block (P), namely, P ≤ N. This disparity leads to the presence of phase truncation spurs in the DDS output spectrum.

Knowing the value of P for a given DDS is essential for predicting phase truncation spurs. This application note describes a method for calculating the frequency and magnitude of a specific phase truncation spur, in particular, the primary phase truncation (PPT) spur for any given FTW.
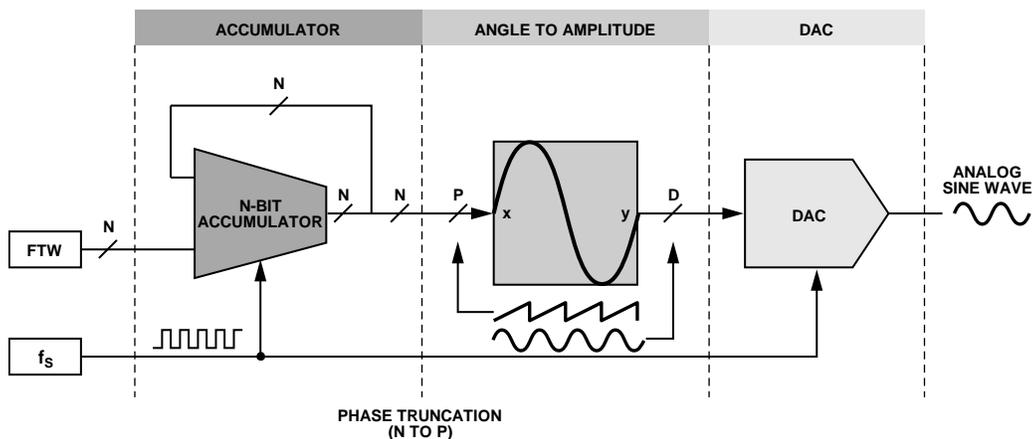

Figure 1. DDS Block Diagram

PHASE TRUNCATION

The accumulator and the FTW constitute the frequency control element of a DDS. However, in addition to the frequency control element, a DDS has an angle to amplitude block that converts the N-bit accumulator output from a phase value to an amplitude value. The angle to amplitude block accounts for the bulk of the digital circuitry in a DDS. Therefore, improving the tuning resolution of a DDS by increasing N significantly increases the amount of circuitry necessary for the angle to amplitude block. As such, it is not practical to convert all N bits of phase information to amplitude. Instead, as shown in Figure 1, a practical DDS uses a subset of the accumulator bits for phase to amplitude conversion, namely, the P most significant bits (MSBs). This truncation of bits significantly reduces the amount of circuitry needed for the angle to amplitude block. However, it comes at the cost of introducing possible spectral artifacts (specifically, phase truncation spurs) at the DDS output.

PHASE TRUNCATION SPURS

By definition, a DDS designed with P = N has no phase truncation. Therefore, no phase truncation spurs appear in its output spectrum. A practical DDS, however, has P < N, which implies phase truncation.

There are three categories of phase truncation spurs: first order, second order, and third order. These categories arise from the spectral characteristics of the cascaded combination of the phase to amplitude converter and the digital-to-analog converter (DAC) in a DDS. Figure 2 is a graphical representation of the spectral lines that result from applying Fourier transform techniques to the angle to amplitude block (with P bits of phase input) and a non-ideal DAC that exhibits harmonic distortion. In general, the spectrum consists of 2^P frequencies indexed from 0 to $2^P - 1$ and categorized as detailed in Table 1.
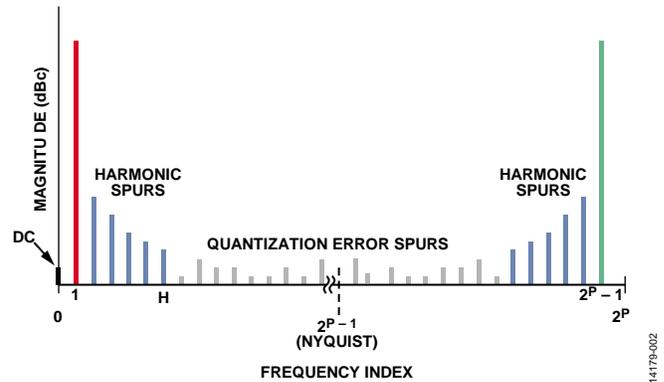

Figure 2. Spectral Characteristics of the Angle to Amplitude Block and the DAC

PRIMARY PHASE TRUNCATION (PPT) SPUR

Many phase truncation spurs of each order (first, second, and third) can be present in the output spectrum of a DDS depending on the specific value of the FTW. This application note focuses on the largest first order spur, which is the PPT spur.

Because the DDS output is the result of generating a waveform from phase samples (that is, the accumulator output), the DDS output spectrum follows the rules of Nyquist sampling theory. The output spectrum appears as two identical spectra, each spanning a frequency range of one half of the sampling frequency (f_S). These two spectra are mirror images of one another reflected about the Nyquist frequency ($f_S/2$). As such, the PPT spur expresses itself as two spurs. One PPT spur appears between 0 Hz and $f_S/2$, and the other appears as its mirror image between $f_S/2$ and f_S.

Note that although the two PPT spurs are the largest first order phase truncation spurs, these spurs may not be the largest phase truncation spurs overall. Because of the mechanism by which phase truncation spurs distribute in the DDS output spectrum, some second order phase truncation spurs can be of greater magnitude than the PPT spurs.

It is not possible to predict the magnitude of second order or third order phase truncation spurs. Second order phase truncation spur magnitude depends on the harmonic distortion characteristics of the DAC, which vary from device to device. Third order phase truncation spur magnitude relates to quantization errors, which are fundamentally random in nature.

Table 1. Frequency Categories Shown in Figure 2

Index	Color	Frequency Category
0	Black	DC.
1	Red	Fundamental or Fourier frequency.
$2^P - 1$	Green	First order phase truncation spur (Nyquist image of the fundamental).
2 to H, $2^P - H$ to $2^P - 2$	Blue	Second order phase truncation spurs. The first group of indices constitutes the DAC harmonic spurs, and the second group of indices constitutes their images.
H + 1 to $2^P - H - 1$	Gray	Third order phase truncation spurs. These spurs constitute the quantization spurs (and their images) associated with the angle to amplitude converter and the DAC.

THE RIGHTMOST NONZERO BIT

Calculating the magnitude and frequency location of the PPT spurs requires knowledge of the following:

- The DDS sample rate (f_S)
- The two DDS parameters, N and P
- The specific value of the FTW

For a given application, N and P are fixed, and f_S is generally a constant value. Conversely, the FTW is completely variable and controls the value of f_O as detailed in Equation 1. The value of the FTW controls not only the placement of f_O in the DDS output spectrum, but also the placement of the phase truncation spurs. In fact, the most important characteristic of a given FTW in terms of the DDS output spectrum is the number of trailing zeroes it has when expressed in binary form. The number of trailing zeroes defines an important parameter, L, which is the location of the rightmost nonzero bit of the FTW.

The position of Bit L in an FTW depends on the specific value of the FTW (recall that the value of the FTW varies according to the desired DDS output frequency, as detailed in Equation 1). This dependency is significant because the location of Bit L for any given FTW determines how phase truncation spurs distribute in the DDS output spectrum.

For any given FTW, Figure 3 demonstrates how to find the value of L. First, convert the FTW to binary. Next, assign index values to the FTW bits with the MSB having a starting index value of 1. Figure 3 is an example of a 32-bit FTW; therefore, the indices range from 1 to 32. The value of L is the index of the last bit having a value of 1 (reading from MSB toward LSB). The FTW in Figure 3 has a value of 0x0036e580 (hexadecimal), so using this protocol, the value of L is 25 for this particular FTW.

The value of L for any given FTW is of the utmost importance. Firstly, L determines whether any phase truncation spurs appear in the DDS output spectrum at all. If L ≤ P, there are no phase truncation spurs. For L > P, however, phase truncation spurs, including the PPT spurs, appear in the DDS output spectrum. Secondly, the values of L and P establish the magnitude and frequency of the PPT spurs (assuming L > P).

PPT MAGNITUDE

The two PPT spurs have identical magnitudes (see Equation 2). Given the value of a particular FTW and the value of P for a given DDS design, the magnitude of the two PPT spurs is:

$$PPT\ Magnitude = 20 \times \log_{10} \left| \frac{\sin\left(\dfrac{\pi}{2^L}\right)}{\sin\left(\dfrac{\pi(2^P - 1)}{2^L}\right)} \right| \qquad (2)$$

where the value of PPT Magnitude is in units of dBc: decibels relative to the magnitude of the main DDS output signal at the frequency, f_O.

Note that the arguments of the two sine functions in Equation 2 are in units of radians. For example, given a DDS with P = 19 and the FTW in Figure 3, the magnitude of the PPT spurs using Equation 2 is −114.38789 dBc.

PPT FREQUENCY

A multistep process is necessary to determine the two PPT frequencies. First, find K, the decimal value of the FTW truncated to L bits. To find K, given the FTW in binary form, eliminate the trailing 0s and convert the resulting L bit FTW to its equivalent decimal value. The FTW in Figure 3, for example, yields K = 28,107 (the number obtained by converting the binary number 0000000000110110111001011 to its decimal equivalent).

Use K to calculate the spectral index positions (R1 and R2) of the two PPT spurs with Equation 3 and Equation 4:

$$R1 = (K \times (2^P - 1))\ \text{modulo}\ 2^L \qquad (3)$$

$$R2 = (K \times (2^L - 2^P + 1))\ \text{modulo}\ 2^L \qquad (4)$$

For example, given a DDS with P = 19 and the FTW in Figure 3, the parenthesized quantity in Equation 3 is 14,736,134,709 and the parenthesized quantity in Equation 4 is 928,378,285,515. Applying the 2^{25} modulus to these values yields

$$R1 = 5,739,061$$

$$R2 = 27,815,371$$

Use R1 and R2 to determine the two PPT frequency locations (f_{PPT1} and f_{PPT2}) in the DDS output spectrum according to Equation 5 and Equation 6.

$$f_{PPT1} = f_S \times \frac{R1}{2^L} \qquad (5)$$

$$f_{PPT2} = f_S \times \frac{R2}{2^L} \qquad (6)$$

For example, given f_S = 250 MHz, the FTW in Figure 3, and the values of R1 and R2 calculated in the previous example, the two PPT spur frequencies are the following:

$$f_{PPT1} = 42.759336531162261962890625\ \text{MHz}$$

$$f_{PPT2} = 207.240663468837738037109375\ \text{MHz}$$

MSB																									L							LSB
INDEX	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
FTW	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	1	1	1	0	0	1	0	1	1	0	0	0	0	0	0	0
HEX		0				0				3				6				e				5				8				0		

Figure 3. Bit L of the FTW

CONCLUSION

Although this application note describes a relatively easy way to predict the two PPT spur frequencies and their magnitude, Equation 3 and Equation 4 can pose computational problems. In particular, the quantity in parentheses, which is by definition an integer, can be a very large integer.

To illustrate the problems posed by a large integer in these calculations, reconsider the AD9912, for which N = 48 and P = 19. Let the FTW be 0x400000000001 (hexadecimal), which is 70,368,744,177,665 (decimal). In this case, L = 48; therefore, K has the same value as the FTW.

Now consider the parenthesized quantity in Equation 4. The term $2^L – 2^P + 1$ yields the following value:

$$2^{48} – 2^{19} + 1 = 281,474,976,186,369$$

Therefore, the parenthesized quantity in Equation 4 yields

$K \times (2^L – 2^P + 1) =$
$70,368,744,177,665 \times 281,474,976,186,369 =$
$19,807,040,591,672,948,094,687,248,385$

It takes 94 bits to represent this integer in binary form. Most computational software tools cannot accurately represent an integer of this size. For example, MATLAB running on a 64-bit machine renders $K \times (2^L – 2^P + 1)$ as the floating-point number, $1.980704059167295 \times 10^{28}$. The corresponding integer is 19,807,040,591,672,950,000,000,000,000. This value deviates significantly from the true value and yields an incorrect result for R_2.

In fact, not only are most software tools incapable of handling very large integers, but also the processor chips inside most computers fail in this regard. For example, a computer with a 64-bit processor cannot handle a 94-bit integer. This problem is a key concern because calculating the values of R1 and R2 necessitates very large integer computations beyond the capability of most computers.

In addition to correctly rendering the parenthesized quantities in Equation 3 and Equation 4, it is also necessary to compute a 2^L modulus of the parenthesized quantity. In the previous example, L = 48, which implies a 48-bit modulus. Most computers cannot support modulus computations in excess of 32 bits.

In summary, the difficulties imposed by Equation 3 and Equation 4 necessitate great care when implementing the PPT spur calculations using software tools. A user must compute the parenthesized quantity correctly as an integer (with no truncation or rounding) and properly handle the modulus computation without loss of precision.

Even though most software tools and processors cannot handle very large integers directly, there are exceptions. For example, the software tool, Python, natively supports very large integer computations. Although MATLAB® cannot natively handle very large integers, a variable precision integer (VPI) toolbox supporting very large integer computations is available for free download on the MathWorks website at MATLAB Central.

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