MEASURING REACTIVE POWER IN ENERGY METERS

The amount and complexity of household electrical equipment has increased tremendously over the last few years. Electronic ballast lighting, computer monitors and air conditioners are welcome additions to our homes but come with additional burdens. One of these is on the electricity grid, as these appliances generate more signal harmonics.

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This change in the end-consumer profile is a disadvantage for energy distributors which bill energy based only on active power. With the application of non-linear loads to power lines the active energy no longer represents the total energy delivered. As a response to improve billing, the measurement of reactive energy is gaining interest. For example, Italy's leading energy distributor has decided to install more than 20 million household energy meters with active and reactive power measurements.

This growing interest in measuring reactive energy leads to the question: What method should an energy meter designer implement to accurately measure the reactive energy? Although today's electronic digital signal processing (DSP) enables reactive energy measurements to be closer to the theoretical value, there is no consensus in the field of energy metering on the methods of measurement. This article aims to explain and compare the three main methods in use, namely the Power Triangle, the Time Delay and Low-pass Filter.

SYSTEM REQUIREMENTS
Electromechanical meters have set a precedent in reactive energy billing. Although they are bandwidth limited and cannot take into account harmonics of the line frequency, they are supported by the international standard for alternating current static var-hour meters for reactive energy (IEC-1268). The standard defines reactive energy measurements at the fundamental line frequency, which implies that it is not mandatory to include harmonics. It also specifies additional testing conditions to check the robustness of the measurements against the third harmonic, the dc offset in the current input, and the line frequency variation. The various reactive power measurement methods presented in this paper are evaluated against these critical tests of the IEC-1268 (Table 1).

REACTIVE POWER THEORY
The reactive power is defined in the IEEE Standard Dictionary 100-1996 under the energy "magner" as:

$$\text{Reactive } power = \sum_{n=1}^{\infty} V_n \cdot I_n \cdot \sin(\varphi_n) \qquad (1)$$

where V_n and I_n are respectively the voltage and current rms values of the nth harmonics of the line frequency, and φ_n is the phase difference between the voltage and the current n^{th} harmonics. A convention is also adopted stating that the reactive energy should be positive when the current is leading the voltage (inductive load).

In an electrical system containing purely sinusoidal voltage and current waveforms at a fixed frequency, the measurement of reactive power is easy and can be accomplished using several methods without errors. However, in the presence of non-sinusoidal waveforms, the energy contained in the harmonics causes measurement errors.

According to the Fourier theorem any periodic waveform can be written as a sum of sin and cosine waves. As energy meters deal with periodic signals at the line frequency both current and voltage inputs of a single phase meter can be described by:

$$v(t) = \sum_{n=1}^{\infty} V_n \cdot \sqrt{2} \cdot \sin(n\omega_0 t) \qquad (2)$$

$$i(t) = \sum_{n=1}^{\infty} I_n \cdot \sqrt{2} \cdot \sin(n\omega_0 t + \varphi_n) \qquad (3)$$

where V_n, I_n and φ_n are defined as in Equation 1.

Active power
The average active power is defined as:

$$\textit{Average Active Power} = \sum_{n=1}^{\infty} V_n \cdot I_n \cdot \cos(\varphi_n) \quad (4)$$

The implementation of the active power measurement is relatively easy and is done accurately in most energy meters in the field.
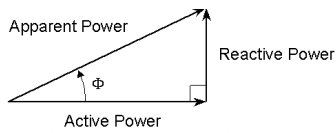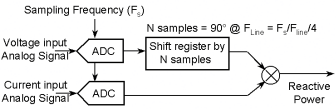
Figure 1 - Power triangle.

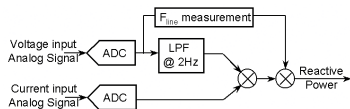

Figure 2 - Reactive power calculation with time delay.

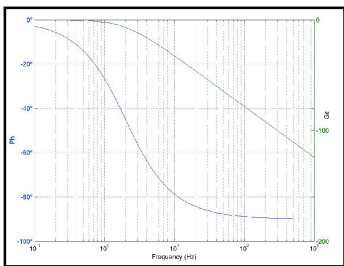*Figure 3 - Reactive power calculation with Analog Devices solution.*

Figure 4 - Low-pass filter for reactive power calculation.

Apparent power
The apparent power is the maximum real power that can be delivered to a load. As V_{rms} and I_{rms} are the effective voltage and current delivered to the load,

$$Apparent\ power = V_{rms} \cdot I_{rms} \qquad (5)$$

The correct implementation of the apparent energy measurement is bound by the accuracy of the rms measurements.

REACTIVE POWER CALCULATION
As explained above, different methods can be used to calculate the reactive power. The theoretical definition of the reactive power is difficult to implement in an electronic system at a reasonable cost. It requires a dedicated DSP to process the Hilbert transform necessary to get a constant phase shift of 90° at each frequency. Several solutions have been developed to overcome this limitation. They can be categorised in three groups:

Method 1: Power triangle
The Power triangle method is based on the assumption that the three energies, apparent, active and reactive, form a right-angle triangle as shown in Figure 1. The reactive power can

$$Reactive\ Power = \sqrt{Apparent\ Power^2 - Active\ Power^2} \qquad (6)$$

then be processed by estimating the active and apparent energies and applying:
Although this method gives excellent results with pure sinusoidal waveforms, noticeable errors appear in presence of harmonics (Table 1).

Method 2: Time delay
A time delay is introduced to shift one of the waveforms by 90° at the fundamental frequency and multiply the two waveforms:

$$Reactive\ energy = \frac{1}{T}\int_0^T v(t) \cdot i\left(t + \frac{T}{4}\right) dt \qquad (7)$$

where T is the period of the fundamental.

In an electronic DSP system, this method can be implemented by delaying the samples of one input by the number of samples representing a quarter-cycle of the fundamental frequency (F_{line}) (Figure 2)

This method presents drawbacks if the line frequency changes and the number of samples no longer represents a quarter-cycle of the fundamental frequency. Significant errors are then introduced to the results (Table 1).

Method 3: Low-pass filter
A constant 90° phase shift over frequency with an attenuation of 20 dB/decade is introduced. This solution, which has been implemented by Analog Devices, can be realised with a single pole low-pass filter on one channel input (Figure 3). If the cut-off frequency of the low-pass filter is much lower than the fundamental frequency, this solution provides a 90° phase shift at any frequency higher than the fundamental frequency. It also attenuates these frequencies by 20 dB/decade (Figure 4).

Similarly to method 2, this solution is susceptible to variations of the line frequency. However, a dynamic compensation of the gain attenuation with the line frequency can be achieved by evaluating the line period of the signal (Table 1).

CONCLUSION
With more and more non-linear loads in household appliances, measuring reactive energy accurately becomes a key issue for energy distributors. Traditional measurement methods like the Power triangle and the Time delay comply with international standards but show limitations in the presence of harmonics or line frequency variation. With the latest advancements in integrated circuit development, as proposed by Analog Devices, energy meter designers can now easily implement more accurate reactive energy measurements and thereby, satisfy emerging requirements from energy providers. mi

Test		Power triangle	Time delay	Low-pass filter
IEC1268 - Reference test	Voltage and current input @ F_{line} ; PF=0	Negligible	Negligible	Negligible
IEC1268 - Frequency variation test	Reference test @ F_{line} +/- 2% and PF=0.87	Negligible	5.4%	Negligible
IEC1268 - Harmonic test	Reference test + 10% of the third harmonics on the current signal	0.5%	Negligible	Negligible
IEC1268 - DC component test	Reference test with half way rectified sin wave on the current input	Negligible	Negligible	Negligible
Reference test + 10% of the third harmonics on voltage input and 20% of the third harmonics on current input ($\varphi_1 = \varphi_2 = 30°$)		1.9%	4%	1%

Table 1: Error benchmark of different reactive energy calculation methods.
(Note: The results from the three methods are compared to the results given by the true reactive energy measurement, equation 1)