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A METHOD OF CALIBRATING STANDARD VOLTAGE DIVIDERS UP TO 1000 V

A. S. Katkov

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A method of calibrating a resistive voltage divider for voltage ratios of 1000 V/10 V and 100 V/10 V with an uncertainty at the level of 10^{-7} is developed. The method enables a calibration to be carried out at the working voltage of the divider and can be used to calibrate a transportable voltage ratio standard when carrying out CCEM-K8 key comparisons.

Key words: *voltage divider, calibration.*

Precision voltmeters and calibrators with voltage ranges up to 1000 V are calibrated using accurate voltage dividers, which should have an error of the voltage ratio of less than 10^{-6}. The provision of dc precision voltmeters and calibrators to measure voltages up to 1000 V is based on the use of resistive voltage dividers. Various methods are used to determine the voltage ratios of a divider including bridge methods, a comparison of the resistances and voltages of individual sections of a resistive divider, and the Hammon method. The accuracy with which a voltage ratio can be measured depends on the error in measuring the voltage difference of the sections compared, the change in the resistances of the sections of the divider, the difference in the voltages of the calibrator and the operating divider, and the effect of leakage resistances, the connecting wires and the contact resistances of switches. The accuracy that can be achieved when measuring ratios for coefficients of 1000 V/10 V and 100 V/10 V when using R35 precision voltage dividers is estimated to have an error of 0.005–0.001%. The Kelvin–Varley 720A precision voltage divider made by the Fluke Company guarantees an absolute accuracy of the order of 10^{-7} with respect to the input voltage, which, on a scale with ratios of 1000 V/10 V and 100 V/10 V gives errors of $4.5\cdot10^{-6}$ and 10^{-6} respectively. The method developed is intended for calibrating a divider consisting of similar nominal sets of resistances using a 1:1 ratio.

The voltage ratio of a resistive divider can be expressed in general form in terms of the ratio of the voltage drops across two different groups of resistors.

It was suggested in [1, 2] that the ratios of the voltages of the divider resistors should be measured with respect to a reference resistor using additional voltage sources. However, this method imposes high requirements on the stability of the divider power supply and the reference power supplies at 10 V and 100 V, and there is also the possibility of leakages both in the null-indicator circuit and in the voltage supply circuits.

When the resistors are connected in series, the same current flows through both of them and this enables one to change from the ratio of the voltage drops to the ratio of the resistances of the groups of resistors, and one can therefore measure the 1:1 ratio most accurately. This method is employed in bridge circuits, used to calibrate dividers [3]. The drawbacks of the methods employed include the difference between the power applied to the divider when carrying out the calibration and the power applied to the divider in the working state, which gives rise to additional errors, connected with the temperature conditions in the voltage divider, and, in addition, the resistances of the connecting wires and switches may limit the accuracy attainable using bridge circuits.

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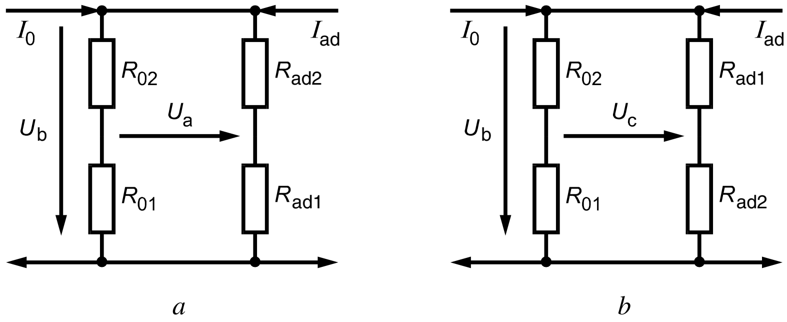

Fig. 1. Circuit for measuring the ratio 1:1 with the branches of the bridge interchanged.

The idea of the method proposed here will be considered using the example of calibrating the ratios 3:1 and 10:1. For a circuit consisting of three similar nominal resistors R_1, R_2 and R_3, the 3:1 ratio can be expressed in the form

$$r_3 = \frac{1}{R_1}\sum_{m=1}^{3} R_m = 1 + \frac{R_2}{R_1}\left(1 + \frac{R_3}{R_2}\right), \tag{1}$$

where R_2/R_1 and R_3/R_2 represent the ratio 1:1. Hence, a measurement of the ratio r_3 = 3:1 can be replaced by a measurement of the ratio 1:1 for two groups of this series of resistors.

Similarly, for a chain of ten similar nominal resistors R_1, ..., R_{10} the ratio 10:1 can be written as

$$r_{10} = \frac{1}{R_1}\sum_{m=1}^{10} R_m = 1 + \frac{R_2}{R_1} + \left\{\frac{R_2}{R_1}\frac{R_3}{R_2} + \frac{R_4+R_5+R_6}{R_1+R_2+R_3}\left[1+\frac{R_2}{R_1}\left(1+\frac{R_3}{R_2}\right)\right]\right\}\left(1+\frac{R_7+R_8+R_9+R_{10}}{R_3+R_4+R_5+R_6}\right), \tag{2}$$

where R_2/R_1, R_3/R_2, $(R_4 + R_5 + R_6)/(R_1 + R_2 + R_3)$, and $(R_7 + R_8 + R_9 + R_{10})/(R_3 + R_4 + R_5 + R_6)$ represent the ratio 1:1. Hence, a measurement of the ratio r_{10} = 10:1 can be replaced by a measurement of the ratio 1:1 for four groups of this series of resistors.

When two series of resistors are connected in series, the sum of the resistances of one of which is equivalent to the resistor of the other group, the divider ratio can be obtained as the product of the ratios of the first and second series. For example, the first series consists of ten resistors:

$$R_1^f \approx R_2^f \approx R_3^f \approx R_4^f \approx R_5^f \approx R_6^f \approx R_7^f \approx R_8^f \approx R_9^f \approx R_{10}^f.$$

In the second series, the resistance of the resistor R_1^s is represented by the sum of the resistances of the resistors of the first series:

$$R_1^s = \sum_{i=1}^{10} R_i^f \quad \text{and} \quad R_1^s \approx R_2^s \approx R_3^s \approx R_4^s \approx R_5^s \approx R_6^s \approx R_7^s \approx R_8^s \approx R_9^s \approx R_{10}^s.$$

In this case, the ratio 30:1 can be obtained in the form

$$r_{30} = r_3^s r_{10}^f \tag{3}$$

when measuring two 1:1 ratios in the second series of resistors and four 1:1 ratios in the first series of resistors, while the ratio 100:1

$$r_{100} = r_{10}^s r_{10}^f \tag{4}$$

when measuring eight 1:1 ratios.

It is proposed measuring the ratio 1:1 using a bridge circuit, consisting of the two resistors (or groups of resistors) of the main divider to be compared R_{01} and R_{02}, and two additional resistors R_{ad1} and R_{ad2} (Fig. 1), where the branches of the bridge obtain their bias from independent dc sources I_0 and I_{ad}. This circuit enables ratios to be measured at the specified (working) voltage of the resistors, which reduces the error connected with the power, while the introduction of independent bias sources enables the error due to the resistance of the connecting wires and the contact resistances of the switches to be reduced. In order to obtain high accuracy when measuring the ratio 1:1, in the bridge circuit the arms R_{ad1} and R_{ad2} are interchanged, which reduces the requirements regarding the accuracy and stability of the additional resistors.

The equation for calculating the ratio $K = R_{02}/R_{01}$ in this measurement circuit can be written in the form

$$K = 1 + 2(U_c + U_a)/U_b = 1 + k, \tag{5}$$

where U_b is the bridge supply voltage, U_a and U_c are the voltages in the diagonals of the bridge for direct connection (see Fig. 1*a*) and inverse connection (see Fig. 1*b*) of R_{ad1} and R_{ad2}, and k is the difference of the ratio from unity when the resistances R_{01} and R_{02} are unequal.

If we write the ratios $R_2/R_1 = 1 + k1$; $R_3/R_2 = 1 + k2$; $(R_4 + R_5 + R_6)/(R_1 + R_2 + R_3) = 1 + k3$; and $(R_7 + R_8 + R_9 + R_{10})/(R_3 + R_4 + R_5 + R_6) = 1 + k4$, we can express (1) and (2) using the results of measurements obtained with the bridge:

$$r_{3b} = 3(1 + (2/3)k1 + (1/3)k2), \tag{6}$$

$$r_{10b} = 10(1 + 0.7k1 + 0.4k2 + 0.6k3 + 0.4k4). \tag{7}$$

The relative systematic error of Eqs. (8) and (9), obtained after mathematical reduction of the initial equations (1) and (2) and dropping terms of the second order of smallness, does not exceed 10^{-8} when the resistances of resistors $R_1, ..., R_{10}$ differ from their mean value by less than 10^{-4}.

Apparatus and Method of Calibration. The method we have developed was used to calibrate a Datron 4902S resistive voltage divider, which is employed as a comparison standard in the CCEM-K8 key comparisons [4] and as a voltage standard in the 100–1000 V range, which comprises a divider with similar characteristics.

The divider to be calibrated consists of ten series-connected resistors with a nominal resistance of R_{nom} = 10 kΩ and nine with R_{nom} = 100 kΩ. The divider is designed to operate at a voltage of up to 1000 V, in which case the output voltage on a resistance of 100 kΩ will be 100 V, and on a resistance of 10 kΩ it will correspondingly be 10 V. To reduce the effect of leakage, the divider includes a chain of resistors, similar to the main divider, which enables protective potentials to be produced in the measuring circuit, which do not affect the main divider.

Ratios of 1000V/100 V, 100 V/10 V, 300 V/100 V, 30 V/10 V and also 1000 V/10 V and 300 V/10 V were calibrated. As was shown in (3) and (4), the last two ratios can be obtained from the results of a calibration of the first three ratios. The relative spread in the resistances of the resistors did not exceed 10^{-5}. The divider contained resistors with a low temperature coefficient and high long-term stability.

To carry out the calibration, we used MRKh-type commercial resistors as the additional resistors, from which the measuring circuit was assembled (Fig. 2) using a construction similar to the divider being calibrated. We used P309-type switches as the switches *S1* and *S2*. The voltage across the bridge diagonal was measured by a voltmeter with a resolving power of 1 μV, and voltage calibrators with an instability of 0.001% served as bias sources. A feature of the construction is the use of protective circuits to produce a zero potential relative to ground in the measuring diagonal of the bridge. In this circuit, the effect of leakage due to the calibrator, voltmeter, and connecting wires was reduced considerably. When using two bias sources in combination with the bridge circuit, the requirements regarding the stability of the sources were reduced by a factor of approximately R_{ba}/r_{cw} (R_{ba} is the resistance of the bridge arm and r_{cw} is the resistance of the connecting wires of the bridge), i.e., by not less than a factor of 10^4 for R_{ba} = 10 kΩ and r_{cw} = 1 Ω. As a result of these two bias sources, the effect of the resistances of the connecting wires between the main and the supplementary dividers is also reduced by a factor of approximately R_{ba}/r_{cw}.

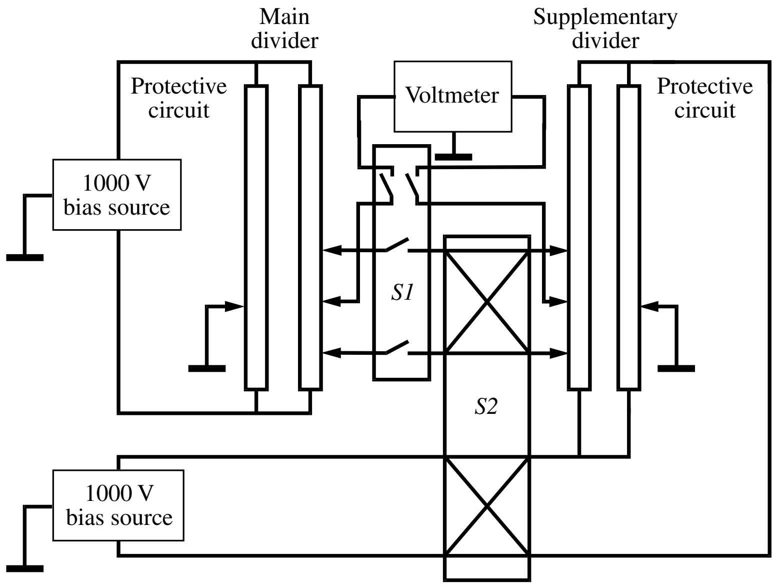

Fig. 2. Circuit for calibrating the divider.

The divider calibration cycle consists of the following:

- a bias voltage is applied to the main and supplementary dividers;
- the potential leads of resistors R_2 and R_1 are connected to the corresponding leads of the additional resistors R_{ad2} and R_{ad1};
- the zero potential is connected to the points of the protective circuits of the corresponding bridge diagonals;
- the voltmeter is connected to the bridge diagonal formed by resistors R_2, R_1, R_{ad2} and R_{ad1} and voltage difference U_{a1} is measured;
- the resistors R_{ad2} and R_{ad1} are interchanged;
- the voltage difference U_{c1} is measured.

After the first cycle of calibration is completed, the bridge circuit is connected to resistors R_3 and R_2 and to the corresponding resistors of the supplementary divider, the points of the protective circuits and the voltmeter are simultaneously switched and the voltages U_{a2} and U_{c2} are measured. Measurements in all eight calibration cycles are made similarly. The data obtained from the voltage measurements are used to calculate the ratios using formulas (3)–(7). When there are large thermoelectric emfs present in the measurement circuit (greater than 1 μV), the calibration is carried out for two polarities of the bias sources and the results obtained are averaged.

Calibration Uncertainty. Since this method was used when making the CCEM-K8 international key comparisons, the accuracy of the voltage ratio was estimated using the Guide to the Expression of Uncertainty in Measurement [5]. In these calculations, the numerical values of the relative standard uncertainty are practically identical with the values of the relative error of the voltage ratio, expressed in the form of the root mean square deviation. The uncertainty of the calibration of the voltage ratio of the divider depends on the chosen ratio and, in accordance with the calibration method, is defined in terms of the uncertainty of the 1:1 ratio.

According to (5), the uncertainty of the ratio $K \approx 1$ is defined as

$$u(K) = 2\sqrt{\frac{u^2(U_a)+u^2(U_c)}{U_b^2} + \frac{u^2(U_b)(U_a+U_c)^2}{U_b^4}} \approx 2\sqrt{\frac{u^2(U_a)+u^2(U_c)}{U_b^2}}. \quad (8)$$

The effect of uncertainty in measuring the bridge voltage $u(U_b)$ in (8) can be neglected, since a typical value of U_a/U_b and U_c/U_b does not exceed 10^{-5}, while $u(U_b)/U_b \le 10^{-4}$.

When carrying out the calibration, various factors have an effect on the value of the voltages U_a and U_c:

• the difference in the voltages of the bias sources, measured when the output voltage of one of the sources changes by 10%, which is converted to a level of 0.01%. The effect of unbalance of the source voltages was determined from the change in the readings of the voltmeter in the bridge diagonal. Measurements showed that the error due to an unbalance of the source voltages of 0.01% does not exceed 10^{-9} for all the calibration cycles;

• the resistance of the insulation between the wires connecting the resistors of the main divider, which is measured directly when the dividers, the bias sources, and the voltmeter are disconnected. The resistance of the insulation between the wires was $(0.5–1)\cdot 10^{13}$ Ω for the 10 kΩ sections for a test voltage of 100 V, and $(1–2)\cdot 10^{13}$ Ω for the 100 kΩ sections at a test voltage of 500 V. The resistance of the insulation of one of the sections of the switch, connected to the group of resistors with an overall resistance of 400 kΩ, was 5 ± 0.6 TΩ; the error from this leakage does not exceed $(8 \pm 1)\cdot 10^{-8}$, and it is taken into account in the form of a correction. The estimate of the effect of leakages between the wires, connected in parallel with the calibrated resistors, depends on the nominal value of the resistance of the resistor and does not exceed $4\cdot 10^{-8}$ for all the calibration cycles;

• leakages of the measurement circuits to ground (the output circuits of the bias sources, the switches, and the input circuit of the voltmeter), which are measured in the circuit with the connected circuits of the protective dividers with the possible leakage resistances shunted by a resistor of 10 MΩ. The effect of the shunting was recorded from the change in the readings of the voltmeter in the bridge diagonal. The results of measurements showed that the action of the protective dividers reduces the effect of the leakage resistances by a factor of 10^4. Since the leakage resistances to ground are estimated to be 10^9–10^{12} Ω, the estimate of the effect of these leakages, due to the action of the protective circuits, does not exceed $4\cdot 10^{-9}$ for all the calibration cycles;

• the voltmeter drift, which is found from the change in the voltmeter readings over a period of one minute after it is connected in the measuring circuit and after a two-minute pause. This phenomenon is due to the inherent instability of the voltmeter readings and the establishment of the polarization current in the insulation of the measuring circuit. An estimate of the effect of the drift depends on the nominal value of the resistance of the resistor and does not exceed $3\cdot 10^{-8}$ for all the calibration cycles;

• calibration of the voltmeter, the effect of which is estimated to be at the level of $2\cdot 10^{-9}$, since the measured difference of the readings does not exceed 10^{-5}, while the uncertainty in the gain and linearity of the voltmeter is not greater than $2\cdot 10^{-4}$;

• the uneliminated thermoelectric emf, which was measured in the switching circuit for multiple switchings and was estimated as $4\cdot 10^{-9}$ for a bridge supply voltage of 20 V.

The results of investigations of the uncertainties of measurements of type B for ratios of 1:1 are presented in Table 1 for sections with resistances of 10 kΩ and 100 kΩ. The estimate of the uncertainty is expressed in arbitrary units without the factor of 10^{-6} with a corresponding degree of freedom of ν_i.

The standard uncertainty of type B for the ratios 30 V/10 V, 100 V/10 V, 300 V/100 V, and 1000 V/100 V was estimated using estimates of the unit ratios (see Table 1) from the following formulas, obtained from Eqs. (6) and (7):

$$u(r_{3b}) = [(2/3u(k1))^2 + (1/3u(k2))^2]^{1/2}; \tag{9}$$

$$u(r_{10b}) = [(0.7u(k1))^2 + (0.4u(k2))^2 + (0.6u(k3))^2 + (0.4u(k4))^2]^{1/2}. \tag{10}$$

The standard uncertainty of type B for the ratios 300 V/10 V and 1000 V/10 V was estimated using estimates (9) and (10) from the formulas obtained from (3) and (4):

$$u(r_{3b}^{s} r_{10b}^{f}) = [u(r_{3b}^{s})^2 + u(r_{10b}^{f})^2]^{1/2};$$

$$u(r_{10b}^{s} r_{10b}^{f}) = [u(r_{10b}^{s})^2 + u(r_{10b}^{f})^2]^{1/2},$$

where r_{3b}^{s} denotes the ratio 3:1 for the 100 kΩ section of resistors, r_{10b}^{s} denotes the ratio 10:1 for the 100 kΩ section of resistors, r_{3b}^{f} denotes the ratio 3:1 for the 10 kΩ section of resistors, and r_{10b}^{f} denotes the ratio 10:1 for the 10 kΩ section of resistors.

TABLE 1. Uncertainties of Measurements of Type B for a Divider with Section Resistances of 10 kΩ and 100 kΩ

Source of uncertainty	Section 10 kΩ						Section 100 kΩ					
	10 V/10 V		30 V/30 V		40 V/40 V		100 V/100 V		300 V/300 V		400 V/400 V	
	$u(k1)$, $u(k2)$	ν_i	$u(k3)$	ν_i	$u(k4)$	ν_i	$u(k1)$, $u(k2)$	ν_i	$u(k3)$	ν_i	$u(k4)$	ν_i
Difference of the bias source voltages	0.001	5	0.001	5	0.001	5	0.001	5	0.001	5	0.001	5
Leakages in the measurement circuit	0.001	5	0.003	5	0.004	5	0.010	5	0.030	5	0.040	5
Leakages to ground	0.001	5	0.001	5	0.001	5	0.002	5	0.004	5	0.004	5
Drift of the voltmeter readings	0.030	10	0.015	10	0.012	10	0.020	10	0.014	10	0.011	10
Voltmeter gain and linearity	0.002	5	0.002	5	0.002	5	0.002	5	0.002	5	0.002	5
Thermo emf of the switching circuit	0.004	10	0.001	10	0.001	10	0.001	10	0.001	10	0.001	10
Standard uncertainty	0.030	11	0.016	11	0.013	13	0.023	14	0.033	8	0.042	6

TABLE 2. Results of Investigations of the Divider for Different Voltage Ratios

Voltage ratio of the divider	Difference from the nominal ratio, $\times10^{-6}$	Number of measurements	Standard uncertainty of type A, $\times10^{-6}$	Standard uncertainty of type B, $\times10^{-6}$	Number of effective degrees of freedom
1000 V/10 V	–4.162	31	0.007	0.034	47
100 V/10 V	–4.485	31	0.022	0.027	23
300 V/10 V	–4.242	31	0.022	0.042	56
30 V/10 V	–5.082	31	0.005	0.031	28

The results of investigations of the divider in the form of the differences from the nominal ratios and the corresponding relative standard uncertainties of type A and type B are presented in Table 2.

An estimate of the standard uncertainty of type A was obtained from the results of measurements of the voltage ratios using the calibration method described. The measurements were made over a period of 10 days. The time dependence of the results of the measurements for two voltage ratios are shown in Fig. 3.

Since the proposed method enables one to calibrate a voltage ratio for any voltage applied to the divider, it is possible to use this method to measure how the division factor depends on the applied voltage or power. In Fig. 4 we show a graph of the uncertainty of the results of a measurement of one of the division factors as a function of the applied voltage in the 10–1000 V range.

In conclusion, it should be noted that the proposed method of calibrating a voltage divider consisting of groups of similar nominal resistors connected in series has the following advantages:

• the calibration is carried out at the working voltage of the divider, which reduces the effect of changes in the power on the uncertainty of the results of measurements due to the different thermal operating conditions of the divider or when the calibration is carried out with a voltage which differs from the working voltage, or for a calibration which takes place at the voltage of individual divider resistors;

• the effect of contact connections and the wires is eliminated when carrying out the calibration;

• the effect of leakages to ground of the measurement circuits and of the instruments used is eliminated;

• the input circuits of the voltmeter are at zero potential with respect to ground;

• instability of the bias sources has no effect;

• the requirements on the stability and accuracy of the additional resistors used for the calibration are reduced.

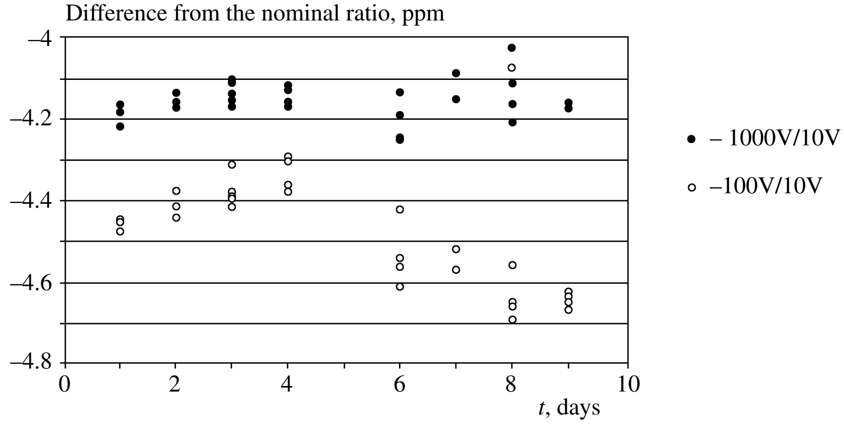

Fig. 3. Results of measurements of the ratios 1000 V/10 V and 100 V/10 V of the Datron 4902S divider.

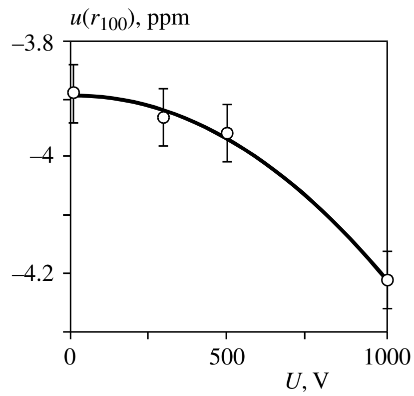

Fig. 4. 100:1 ratio as a function of the applied voltage.

These advantages of the proposed method enables a high accuracy (10^7) to be achieved when calibrating a voltage ratio, and also enable one to measure how the ratio depends on the voltage applied to the divider.

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