On the Frequency Response of Resistors

Basic Resistor Equivalent Circuit

The basic lumped-parameter equivalent circuit for a two-terminal resistor is that shown here. For this circuit:

```
       R      L
   O+-/\/\/\-ωωωωω-+o
    !              !
    +------)(------+
           C
```

$$Z = \frac{R + j\omega[L(1-\omega^2LC) - R^2C]}{(1-\omega^2LC)^2 + (\omega RC)^2} = Rs + jXs \qquad [1]$$

$$Y = \frac{R + j\omega[R^2C - L(1-\omega^2LC)]}{R^2 + (\omega L)^2} = Gp + Bp \qquad [2]$$

From these we can get:

$$Rs = \frac{R}{(1-\omega^2LC)^2 + (\omega RC)^2} \quad \text{and} \quad Rp = R[1 + (\omega L/R)^2] \qquad [3]\&[4]$$

and also $$Q = \omega[(L/R)(1 - \omega^2LC) - RC] \qquad [5]$$

Note that Rs, the equivalent series value (real part of the impedance), is independent of the series inductance, except for the resonant LC term, and Rp, the equivalent parallel value (reciprocal of the real part of the admittance), is independent of the lumped parallel capacitance.

Dielectric Loss in the Lumped Shunt Capacitance

If the lumped parallel capacitance is lossy, the resistor is shunted by conductance of ωDC. This affects both the effective series and parallel values.

```
            R
  o-+-/\/\/\/\/\-+-o
    !      C      !
    +--+--)(--+----+
       !      !
       +-/\/\-+  G = ωDC
```

$$Rac = R/(1 + \omega DCR) \qquad [6]$$

This effect can greatly decrease the ac resistance, both Rp and Rs. For example if the lossy capacitance across the resistor is 10 pF and has a D of .01, then the decrease in a 100 kΩ resistor at 1 MHz is 6.3%.

Note that D is not constant, but varies with frequency. However, it generally varies slowly with frequency so that it causes an error in ac resistance that is not proportional to frequency squared as are the errors from simple lumped parameters.

Capacitance to Guard (Ground)

If a guarded (3-terminal) measurement is made and there is capacitance from the body of the resistor to guard, it produces an effective series inductance. If this capacitance is evenly distributed along the resistor, to the first order it can be approximated by a lumped capacitance of from the center of the resistor to guard that is 2/3 of the total value. If Cg is the total capacitance to guard, then the effective series inductance is $CgR^2/6$. This adds to the actual series inductance in equation [4]. If that real inductance is negligible, we have

```
      R/2    R/2
  o-/\/\/\+/\/\/\/-o
          !
         ---
         ---  Cg
          !
          O   Guard
```

$$Rp \cong R[1 + (\omega RCg)^2/36] \qquad [7]$$

$$Q \cong \omega CgR/6 \qquad [8]$$

Note the series resistance, Rs, is not affected if this capacitance is lumped. This is usually a small effect but can be important for physically-large, high-valued resistors.

Dielectric Loss in the Capacitance to Guard

If the capacitance to guard has losses, it effectively causes an increase in both Rp and Rs. If we let Dg be the dissipation factor of the stray capacitance Cg, we get a conductance $Gg = \omega DgCg$ from the middle of the resistor to guard (if we assume that simple model). This causes an increase in both Rp and Rs.

$$Rac \cong R[1 + \omega DgRCg/8] \qquad [9]$$

Distributed Capacitance Along Resistor

Resistors also have distributed capacitance between all points along its body to all other points. Modeling this can be difficult, but the first-order effect will be that of a single capacitance, Cy, shunting part of the resistor as shown. The value of Cy is hard to determine.

```
   R/2      R/2
 o-/\/\/\-+-/\/\/\-o
          !        !
          +--)(----+
                Cy
```

$$Rp \cong R/(1 + (\omega RCy)^2/16) \qquad [10]$$

$$Q \cong -\omega RCy/4 \qquad [11]$$

This is the classic cause of the decrease of Rp with frequency of high-valued resistors. This affects Rs as well but that would be more affected by the lumped shunt capacitance.

If there is loss in this distributed capacitance itwould reduce Rp slightly. Note also that, if equal capacitances are in series shunting equal parts of the resistance, then these capacitances are equivalent to a very small lumped capacitance across the whole resistance.

The effect of distributed capacitance is particularly important in shielded resistors when the shield is tied to one end of the resistor as in the case of the coaxial GR 1442 resistors when one end is shorted to make the resistor two-terminal.

These distributed capacitances can be internal to the resistor. High-valued, carbon-composition resistors are subject to the "Boella Effect", the error caused by internal capacitances between the granules of carbon.

Eddy Current Loss

If a wire-wound resistor is wound on a conducting form or if a conducting material is nearby, eddy currents will cause power loss that can be represented by resistance shunting the series inductance. This might be the case in some power resistors with metal heat sinks. An extreme case is the resistance of iron-cored coils or transformers which varies widely with frequency due to eddy-current and hysterisis losses.

"Skin Effect" is due to eddy currents in the resistor itself. If causes an increase in resistance and inductance in thick, low-resistance conductors at high frequencies. It is usually negligible compared to other errors except in resistors made of thick, low-resistance wire.

Combining Errors

The errors for each effect was considered separately in the above discussion. Generally speaking, these effects may simply be added. An expression for an equivalent circuit that represents all sources of error would have many more, second-order error terms that would be negligible compared to the largest terms.