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**RF & Microwave  
Measurement Symposium  
and Exhibition**

Electrical Characterization  
of Quartz Crystal Units  
Through High Performance  
Vector Network Analysis

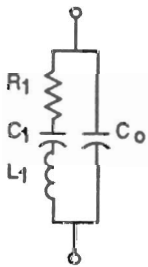
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OUTLINE:

- The Measurement Problem: Crystal Parameters
- The Measurement Solution: Network Analysis
- Practical Techniques

**The Measurement Problem**

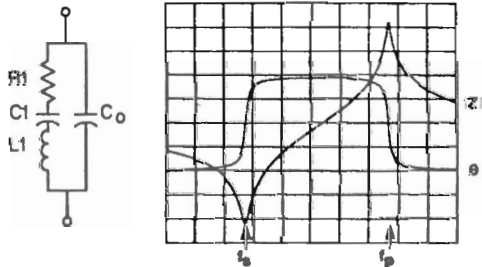


Crystal Parameters:

- $C_0$  Shunt Capacitance
- $C_1$  Motional Capacitance
- $R_1$  Motional Resistance
- $L_1$  Motional Inductance
- $f_s$  Series Resonant Frequency
- $f_p$  Parallel Resonant Frequency
- $Q$  Figure of Merit

The equivalent circuit diagram for a crystal includes a series resonant circuit ( $R_1$ ,  $C_1$  and  $L_1$  - the motional components) with  $C_0$  in shunt. Crystal characterization involves finding the lumped element values for these parts, as well as the associated resonant frequencies and figure of merit. These determinations must be made with accuracy and repeatability.

**Crystal Impedance**



These values will be derived from a plot of the frequency dependant vector impedance between the two ports (pins) of the crystal device. A review of the mathematical relationships between this plot and the desired measurements follows.

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## Series Resonant Frequency

$f_s$

- determined by L1, C1
- minimum impedance
- zero reactance (zero phase)
- $f_s = \frac{1}{2\pi\sqrt{L_1C_1}}$

The series resonant frequency  $f_s$  is determined by the motional reactances L1 and C1, as shown. It is recognizable on the impedance plot as the frequency of minimum impedance and zero phase angle.

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## Motional Resistance

$R_1$

- residual impedance at  $f_s$
- can be read directly from plot

With the reactances of L1 and C1 cancelling each other out at  $f_s$ , the remaining impedance is contributed entirely by R1, whose value can be read directly from the impedance plot.

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## Motional Inductance

$L_1$

- calculated from "slope" of reactance at  $f_s$
- $L_1 = \frac{1}{4\pi} \frac{dx}{df}$

The motional inductance is directly related to the rate of change of crystal reactance with frequency in the vicinity of  $f_s$ , as shown.

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## Motional Capacitance

$C_1$

Given the series resonant frequency ( $f_s$ ), and the motional inductance,  $C_1$  can then be calculated from the familiar series resonance equation.

- calculated from  $f_s, L_1$
- $C_1 = \frac{1}{L_1 (2\pi f_s)^2}$

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## Shunt Capacitance

$C_0$

To this point, shunt capacitance  $C_0$  has been ignored. While this may safely be done in some cases (particularly lower frequency crystals), it is important to understand its effects on the measurements.

- determines  $f_p$  (parallel, or anti-resonance)
- spreads  $f_m, f_r, f_s$

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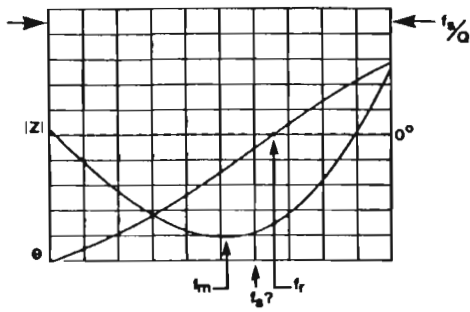
## Parallel Resonant Frequency

$f_p$

First, the shunt capacitance provides a second frequency of resonance, or zero phase, in this case an impedance maximum. This parallel resonance frequency ( $f_p$ ) will be related to  $L_1$  and  $C_1$  as well as  $C_0$ .

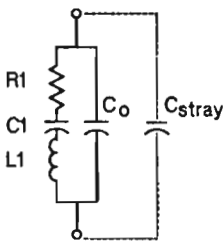
- determined by  $C_0, C_1, L_1$
- maximum impedance
- zero reactance (zero phase)
- $f_p = \frac{1}{2\pi} \sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_0}}$

### Spreading of $f_m$ , $f_r$ , $f_s$



The other effect can be seen on this plot of crystal impedance, taken over a very narrow frequency span centered on  $f_s$ . In the presence of  $C_0$ ,  $f_r$  (zero phase) and  $f_m$  (minimum impedance), no longer coincide, as previously indicated. Therefore,  $f_s$  (series resonance) cannot be correlated to any specific point on the plot.

### Influence of Stray Capacitance



- crystal behaves as if  $C_{stray}$  were part of  $C_0$
- large effect on  $f_p$
- lesser effect on  $f_s$

Further, any stray capacitance across the crystal (fixturing, etc.) will appear in parallel with, and indistinguishable from  $C_0$ . It will have a significant effect on parallel resonant frequency and can also lead to inaccurate values for the lumped motional elements. Means for overcoming these problems will be discussed later.

### Measurement of $C_0$

- find  $X_0$  at non-resonant frequency
- $$C_0 = \frac{1}{2\pi f X_0}$$
- or, find it analytically.....

$C_0$  can be easily found by measuring the crystal impedance a few percent away from series resonance. At such frequencies, the motional arm presents a high impedance in parallel with  $C_0$ , and the total device impedance is simply the capacitive reactance of  $C_0$  at the measurement frequency.  $C_0$  can also be found analytically using a technique to be presented later.

## Resonator Q

- Calculate from motional parameters

$$Q = 2\pi f_s L/R1$$

- calculate from phase slope at  $f_s$

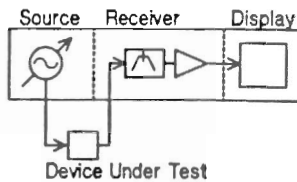
$$Q = -t_g \pi f_s, \text{ where}$$

$$t_g = -\frac{d\theta}{360 df}$$

The last crystal parameter to be derived is resonator Q. This can be calculated from the motional parameters already found, or from the rate of change of phase with frequency. The latter can often be measured directly as "group delay" ( $t_g$ ).

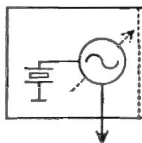
## The Measurement Solution

### Network Analysis



The crystal measurement problem is therefore one of very precise impedance measurements. Today's best answer is state-of-the-art network analysis. An appropriate network analyzer can be conceptualized as having three fairly simple functional blocks.

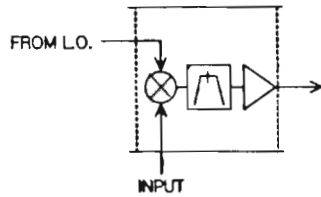
## Source



- furnishes stimulus, determines measurement frequency
- desirable characteristics:
  - frequency synthesis
  - frequency sweep
  - variable amplitude

The source provides sine wave energy to the device under test and determines the frequency at which the measurement is made. The hi Q nature of crystal devices will require that it have synthesizer accuracy and stability, and it should be able to sweep over a variety of frequencies.

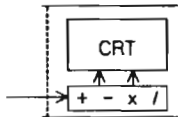
## Receiver



- quantifies test device response
- desirable characteristics:
  - vector measurements
  - narrowband

The receiver is the portion of the network analyzer that receives and quantifies the response from the device under test. It is desirable that it be able to respond to both the real and imaginary portions of the input signal, allowing both amplitude and phase measurements. Dynamic range requirements necessitate a narrowband detector, which in turn requires that the receiver and source share a common local oscillator, to provide tracking.

## Display

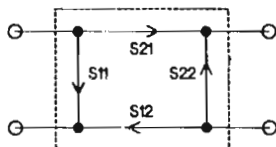


- presents measurement results
- desirable characteristics:
  - computational ability
  - graphical output

Finally, the display section of the analyzer calculates and displays the measurement results in a usable format, beginning from the raw data supplied by the receiver.

## Network Measurements

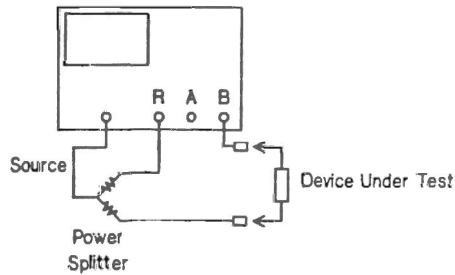
### S - Parameters



- S11 - input reflection coefficient
- S21 - forward gain
- S12 - reverse gain
- S22 - output reflection coefficient

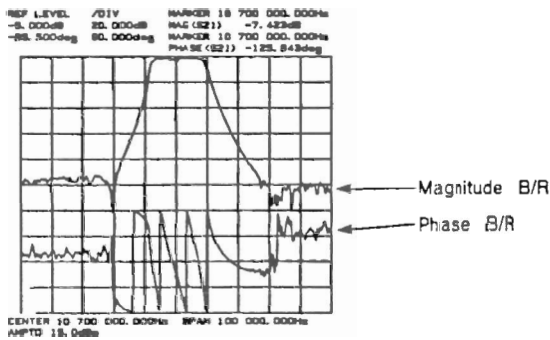
S-parameters are commonly used to describe the characteristics of single or multi-port RF devices. They are based on the (vector) ratios of power reflected from, or transferred between, the various ports under conditions of perfect source and/or load match. As these are the measurements most easily made with a network analyzer, they will be used as the basis for the following measurements.

### Test Setup - S21



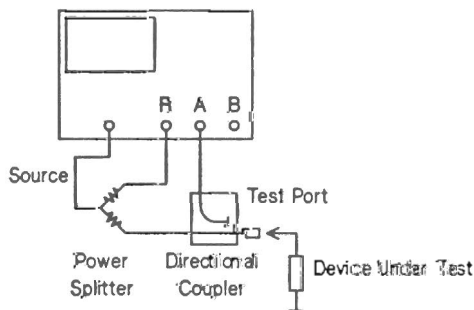
A typical S21 measurement (forward gain or frequency response) would be made in this manner. The internal source is split and applied to both the device under test (DUT) and one of the analyzer's receiver inputs. The output of the DUT is monitored by receiver input B.

### Typical S21 Measurement



A typical measurement result is shown as magnitude and phase versus frequency. Notice that by displaying the DUT output (B) in ratio with the actual input (R), any imperfections in source flatness automatically cancel out, because they appear in both terms. Results are in decibels or other relative units.

### Test Setup - S11



Crystal measurements focus on S11, the ratio of power reflected from a DUT relative to that applied to it. To the previous measurement setup has been added a directional coupler, capable of separating the reflected power from the incident and routing it to receiver A. S11 is therefore equal to the ratio A/R, and will take on a value between 0 (perfect impedance match, all power absorbed) and 1 (perfect reflection, no power absorbed by DUT) at any phase angle.



### Conversion of S11 to Impedance

$$Z = Z_0 \left[ \frac{1 + S_{11}}{1 - S_{11}} \right]$$

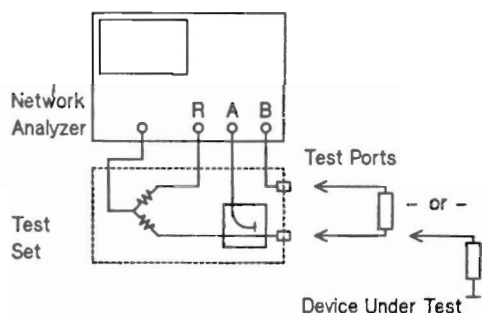
$$S_{11} = \frac{A}{R}$$

$Z_0$  = System characteristic impedance

Measurement range:  $Z_0 \pm 2$  decades

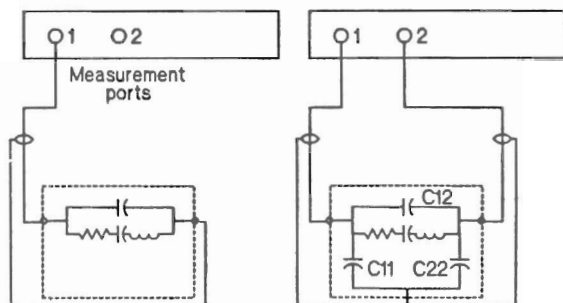
The familiar transmission line equation allows any value of S11 to be converted to a corresponding value of impedance Z. The practical limit of this technique is about two orders of magnitude around  $Z_0$ , the system characteristic impedance.

### S - Parameter Test Set



A measurement accessory available for many network analyzers is an S-parameter test set. It provides the interconnection of the power splitter, directional coupler, and switching functions in a single unit. The user need only connect his one or two port DUT to the appropriate measurement port(s) on the test set.

### Connecting Crystal to Test Set



Crystals can be measured using either one or two port techniques. The latter offers the additional information of case-to-pin capacitances C11 and C22, at the expense of somewhat more complex math.

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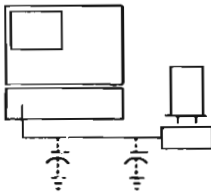
### Fixturing

- repeatability
- compatibility

Fixturing for a crystal measurement will involve problems that are often more mechanical than they are electrical. It is generally more important that stray impedances be highly repeatable than that they be held to an absolute minimum. In addition, the calibration standards to be used will have to be adapted for use directly at the crystal pin socket.

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### Measurement Calibration



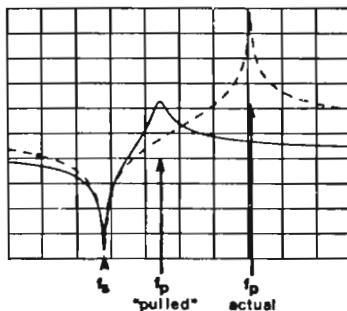
Required because of:

- cabling losses,  $\theta$  shift
- stray L, C
- instrument errors

Most of the sources of measurement inaccuracy are actually external to the network analyzer. Stray fixture impedances will appear in parallel with the crystal and change its electrical characteristics. Even the phase shift and losses of the interconnect cables will appear superimposed on the device response.

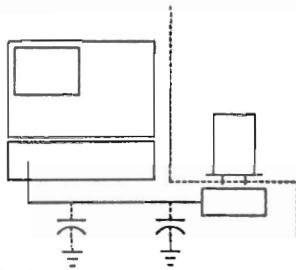
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### Uncalibrated Measurement



In this example, the shunt capacitance of the test fixture has "pulled", or lowered the crystal's parallel resonant frequency. The solid line shows the actual, uncalibrated measurement that has resulted, as compared to the actual value (dashed line).

## Calibration "Reference Plane"

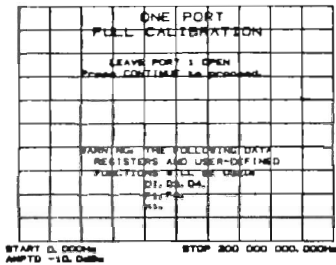


### Definition:

the point in space to which all amplitude and phase data are referenced

Calibration creates a measurement "reference plane", the physical point from which all measured values are referenced. Impedances within the reference plane are effectively removed, allowing those external to it (i.e. the DUT) to be measured independently.

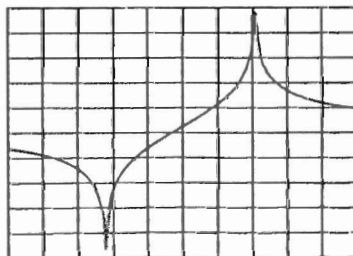
## Internal Calibration Firmware



Calibration firmware within the HP 3577A network analyzer provides step by step operator instructions whereby standard open, short and fifty ohm terminations are attached to the reference plane and measured. Data from these known devices are then used to correct future measurements.

## Calibrated Measurement

REF LEVEL /DIV MARKER 8 868 837.800Hz  
180.000dB 18.000dB MAG (LOP) 21.273dB



Shown is a measurement following correction for all of the error-causing effects previously mentioned. The calibration procedure has allowed the crystal to be measured as if it was in complete isolation from the external world.

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## General Measurement Sequence

The following slides will provide a practical sequence of steps whereby these measurements might be made on an HP 3577A Network Analyzer.

### Step 1: Instrument Setup

- one-port connections
- analyzer settings

The first step is to set-up the instrument with the proper connections and front panel settings.

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## Network Analyzer Initial Settings

INPUT	$\frac{1+S_{11}}{1-S_{11}}$ or UDF "F4"
DISPLAY FUNCTION	trace 1 - linear magnitude trace 2 - phase
FREQUENCY	center = $f_s$ span = $5 f_s / Q$
AMPLITUDE	as required

### INPUT

UDF "F4" identifies user defined function F4, which is pre-defined to be impedance, as calculated from S11.

### DISPLAY FUNCTION

Trace 1 - linear magnitude (in ohms)  
Trace 2 - phase

### FREQUENCY

Center frequency - as appropriate  
Frequency span - approx. 5 times the Q bandwidth

### AMPLITUDE

As required

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### Step 2: Calibrate

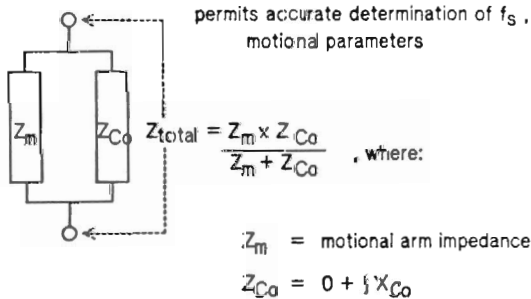
Don't forget to calibrate!

**Step 3:** Refine Instrument Settings

- frequency
- scaling
- bandwidth
- averaging
- sweep time

Refinement of the initial instrument settings may be desirable for optimum measurement performance. Very high performance devices may, for example, require use of signal averaging or a narrower bandwidth in order to maximize dynamic range. Re-calibration is a must after any of these steps, or after any change in frequency span.

**Step 4:** Removal of  $C_0$



Finally, the effects of  $C_0$  can be mathematically removed from the impedance plot, allowing the motional parameters ( $R_1$ ,  $C_1$ ,  $L_1$ ) and series resonant frequency to be conveniently found. By modelling the crystal as two parallel impedances, one of which is the reactance of  $C_0$ , it is possible to solve for the motional impedance given any assumed value for  $C_0$ .

- define display as  $Z_m$

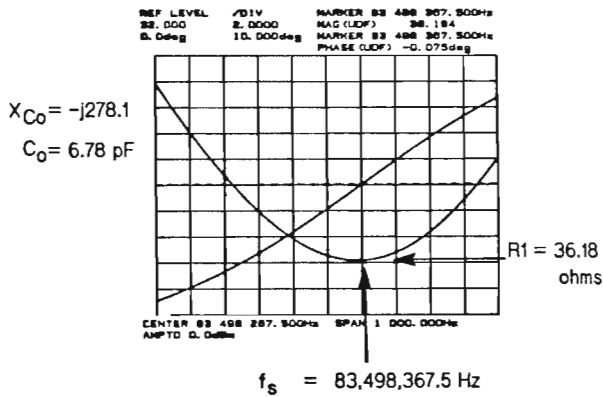
$$= \frac{Z_{Co} \times Z_{tot}}{Z_{Co} - Z_{tot}}$$

- adjust  $Z_{Co}$  until  $f_m = f_r = f_s$

- $C_0 = \frac{1}{2 \pi f X_{Co}}$

Starting from an arbitrarily chosen value,  $Z_{Co}$  (or  $X_{Co}$ ) is adjusted until the resultant plot shows the characteristics of a simple series resonant circuit (i.e.  $f_m = f_r$ ). This identifies both  $f_s$  and  $C_0$ . Alternatively, if  $C_0$  has been separately measured, it's reactance can be calculated and "plugged into" the equation at this point.

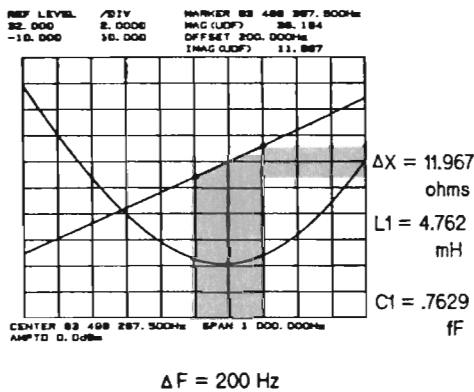
### Measurement Results: $f_s, R1$



This is the resulting impedance plot of a typical overtone crystal unit.  $f_m$  and  $f_r$  coincide because 278.1 ohms of capacitive reactance have been removed from in parallel with the network. This corresponds to a shunt  $C_0$  of about 6.8 pF.

$f_s$  is read directly as about 83.498 MHz. Motional resistance  $R1$  is read directly as 36.18 ohms.

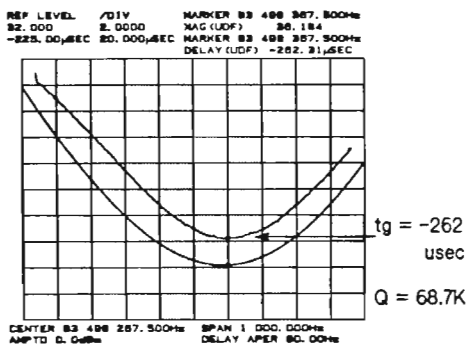
### Measurement Results: $L1, C1$



$L1$  is found by displaying the imaginary (reactive) portion of the device impedance, and using the display markers to determine its slope. In this case, with a 12 ohm change over a 200 Hz span,  $L1$  is about 4.8 millihenrys.

$C1$  is then calculated from the values for  $f_s$  and  $L1$ , and is equal to about .76 fF.

### Measurement Results: $Q$



The  $Q$  of the device can be determined without taking a separate measurement. After selecting the group delay display function, the value at series resonance is found to be -262 microseconds.  $Q$  is therefore about 68.7K.

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## Conclusions

- Quartz crystal electrical parameters are readily obtained from common network measurements.
- Today's network analyzers provide a more complete solution than ever before by:
  - simplifying measurement setup
  - self-calibrating
  - performing complex data manipulation
  - displaying results in more usable forms

Network analysis is gaining rapid acceptance as the preferred method for crystal device characterization. Excellent measurement performance and extensive mathematical capabilities makes the HP 3577A an ideal solution in either stand-alone or ATE applications.