HEWLETT PACKARD

TECHNIQUE FOR MEASURING PHASE ACCURACY AND AMPLITUDE PROFILE OF CONTINUOUS-PHASE-MODULATED SIGNALS

Application to GMSK.3

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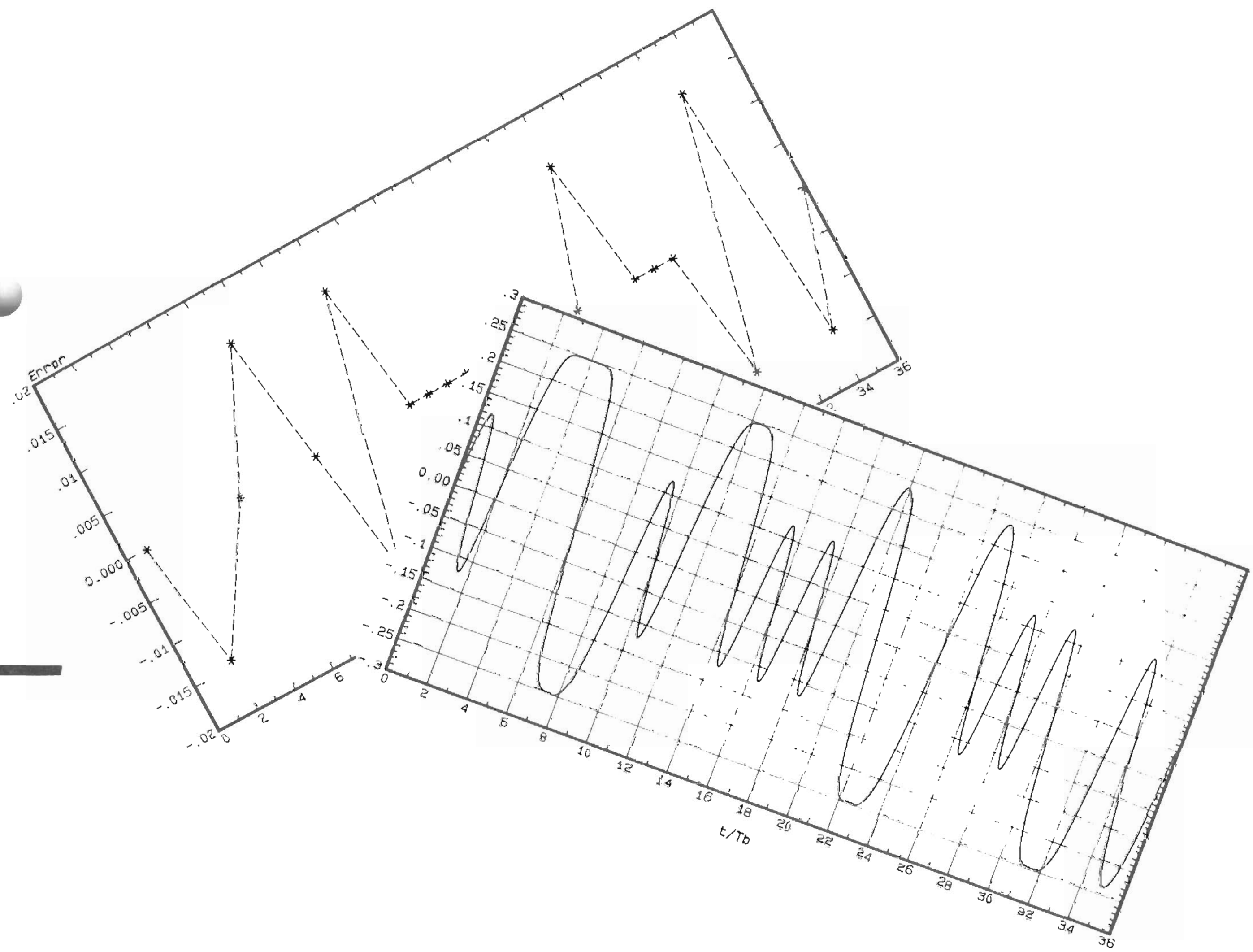


ABSTRACT

A method and apparatus for determining the phase and amplitude accuracy of continuous-phase-modulated signals is described. The modulated RF signal generated by the transmitter unit under test is down-converted to a relatively low intermediate frequency which is filtered and sampled by a high sampling rate analog-to-digital converter. A digital signal processor processes the digital signals to produce measured amplitude and phase functions of the modulated transmitter signal. From the measured values of amplitude and phase, ideal amplitude and phase functions of the transmitter signal are synthesized mathematically. The synthesized functions are compared to the measured functions to determine instantaneous amplitude and phase errors as a function of time.

I. Introduction

A number of manufacturers manufacture and market radios for use in communications, such as digital cellular radios and the like. Typically each manufacturer provides its own specifications for its products. Traditionally the accuracy of these specifications has been measured using many separate, possibly indirect methods. Phase accuracy of the transmitted signal, for example, typically is indirectly determined by measuring spurious signals, phase-noise, modulation index, frequency settling time, carrier frequency and data clock frequency. Furthermore, amplitude measurements present special problems because the amplitude versus time profile must be synchronized to the data typically utilizing external equipment.

It has been proposed that a standardized mobile digital radio system be implemented throughout Europe. Such a radio system would require that all components such as transmitter and receiver, for example, be manufactured to standard specifications measured by a common method. The Groupe Special Mobile (GSM) has proposed a measurement technique to measure the accuracy of the modulation of the transmitted signal. In the measurement technique described in this paper, a sampled measurement of the phase trajectory of the transmitted signal is obtained. This measurement is compared with the mathematically computed ideal phase trajectory to determine the phase difference between the transmitted signal and the ideal signal. The phase difference is fitted to a linear regression line. The slope of the regression line provides an estimate of the frequency error of the transmitter, and the regression line subtracted from the phase difference provides an estimate of the instantaneous phase error. The utilization of the standard method such as this would simplify the testing and manufacture of radios. An individual manufacturer would then only need to ensure that the standardized overall phase error specification be met rather than several interrelated specifications.

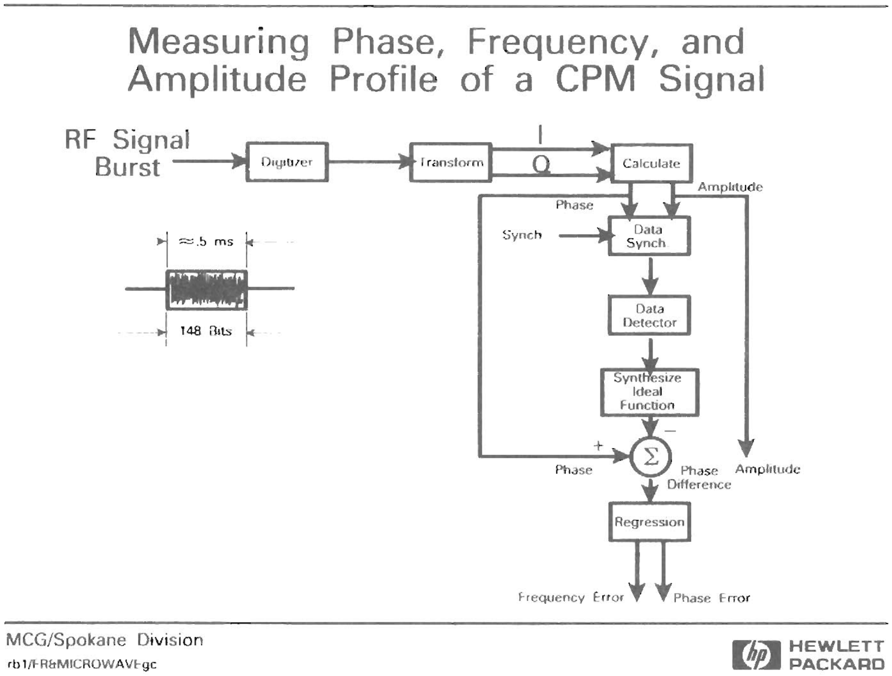
In section II of this paper, a general mathematical description of the phase and amplitude measurement method is presented. Included are considerations of quantization effects, estimation of transmitter clock phase and frequency, and detection of data. Then in section III an implementation of the method utilizing an HP 70700A digitizer and an HP 9000-350 computer is described. In this section, measurements of the phase and amplitude errors of a prototype continuous-phase-modulated transmitter obtained by utilization of the method described in this paper are presented. Finally in section IV, the summary and conclusions of the paper are presented.

II. Mathematical Description of the Method

The Method

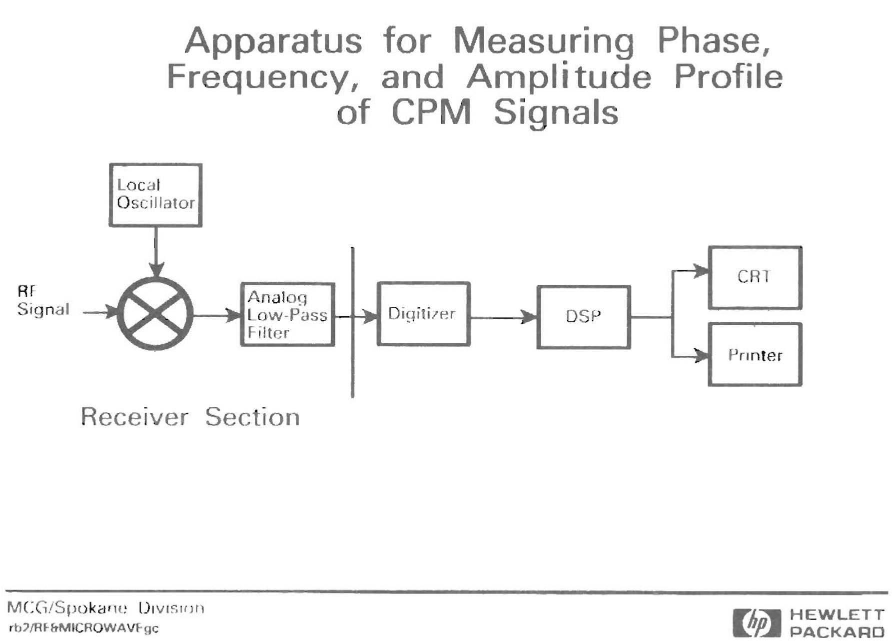
Referring to Figure 1, a flow chart illustrating a method for measuring the phase and frequency errors and amplitude profile of a continuous-phase-modulated RF signal is shown. A modulated RF signal generated by a transmitter is received and converted to digital form by a digitizer. The digitized signal is then converted or transformed into its component in-phase and quadrature-phase signals by a transformation circuit, and the transmitted signal amplitude and phase functions are computed from the component signals. Utilizing a known synchronization signal which may comprise a known sequence of data bits in a preamble or midamble, the bit sequence representing the transmitted data is synchronized to provide the transmitter data clock and a test interval. A data detector detects the data bit sequence and provides the transmitter data clock, test data interval and the data bit sequence to a synthesizer to synthesize or mathematically calculate an ideal phase function corresponding to the transmitted signal. The data detector may be implemented as a maximum likelihood sequence estimator utilizing the Viterbi algorithm. The ideal phase function thus synthesized is subtracted from the

Figure 1



measured phase function (i.e., the transmitted signal phase) to provide a phase difference. A linear regression of the phase difference then provides the frequency error and the instaneous phase error (as shown in Figure 16).

Figure 2



Phase and Amplitude Measurement

Referring now to Figure 2, a conceptual block diagram of an apparatus for measuring the phase and frequency errors and the amplitude profile of a continuous-phase-modulated RF signal is shown. The modulated RF signal is coupled to a down conversion mixer to provide an intermediate frequency (IF) signal. The IF signal is filtered in an analog anti-aliasing filter and coupled to a digitizer to convert the analog IF signal to a discrete-time data sequence. An HP 70700A digitizer manufactured by Hewlett-Packard Company may be used for this purpose, or the digitizer may be implemented by an ADC sampling at a high rate. After conversion to an IF the signal may be represented as:

$$y(t) = \tilde{A}(t)\cos[(\omega_o+\Delta\omega)t + \tilde{\phi}(t;\underline{a}) + \phi_o] \qquad (1)$$

where:

$\tilde{A}(t)$ is the received signal amplitude;

ω_o is the nominal IF signal frequency;

$\Delta\omega$ is the frequency uncertainty;

$\tilde{\phi}(t;\underline{a})$ is the received signal phase modulation function;

and ϕ_o is an unknown offset phase.

As given here only $\tilde{\phi}(t;\underline{a})$ is a function of the data sequence $\underline{a}$; however, in general $\tilde{A}(t)$ may also be a function of $\underline{a}$.

A transmitted RF signal or the IF signal down converted from the RF transmitted signal defined by equation (1) typically will be received in bursts having a duty cycle of .125 and being approximately 0.5 milliseconds (ms) in duration.

$\tilde{A}(t)$ and $\tilde{\phi}(t;\underline{a})$ are, respectively, the amplitude modulation and phase modulation of the received signal (i.e., the transmitted signal) which will be different than the ideal modulation of the transmitted signal. The present method determines the difference between the values of the received signal functions $\tilde{A}(t)$ and $\tilde{\phi}(t;\underline{a})$ and the ideal values of these functions.

The digitizer converts the IF signal defined by equation (1) to a sequence of discrete time samples. If the sampling points are given as $t=kT_s$, $k=0, 1, 2, \ldots$ where T_s is the time period between samples, and if we define $\Omega_o=\omega_o T_s$ and $\Delta\Omega=\Delta\omega T_s$, then the sequence of samples can be written as

$$y[k]=\tilde{A}[k]\cos[(\Omega_o+\Delta\Omega)k+\tilde{\phi}(k;\underline{a})+\phi_o] \qquad (2)$$

$$k=0, 1, 2, \ldots$$

Quantized values of equation (2) provide the sequence of binary numbers coupled to the digital signal processor.

The resulting phase error, frequency error and amplitude profile, as computed by the digital signal processor, are output to various display menus such as a cathode ray tube (CRT) and a printer. Typically, the phase, frequency and amplitude information are plotted versus time with the time interval defined by the number of data bits contained in a transmitted signal burst. Figures 16 and 17 are examples of phase difference and phase error plots while Figures 18a, 18b, and 18c are plots of the transmitted signal amplitude profile.

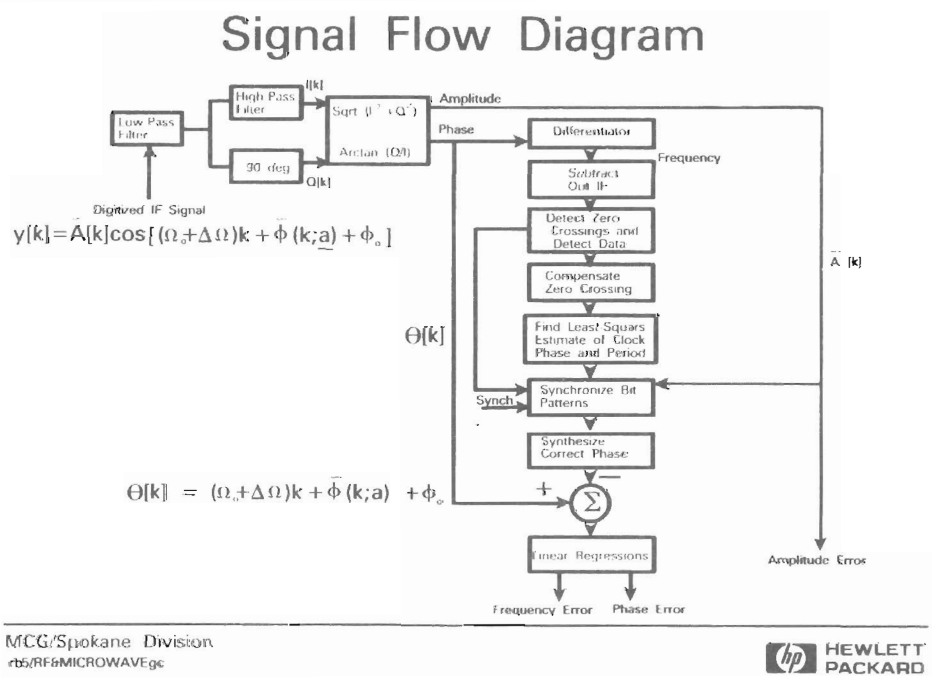
Figure 3 is a flow chart illustrating the method for determining the received RF signal amplitude, $\tilde{A}[k]$, and the difference between the measured phase modulation, $\tilde{\phi}(k;\underline{a})$, of the received RF signal and the ideal phase modulation, $\phi(k;\underline{a})$. The modulation functions have been discretized by replacing "t" with kT_s, k=0, 1, 2, ...

The first step in the flow diagram is to pass the digital IF samples through a low-pass digital filter. The low-pass digital filter would preferably be a finite impulse response (FIR) filter that would have a linear phase response to avoid distortion of the phase modulation of the signal passed by the filter. The purpose of the low-pass filter is to eliminate the harmonics of the IF signal. An FIR digital filter can perform this job with relative ease and with less cost than an analog filter which otherwise would be required.

After the initial low-pass filtering, the signal is converted to two component signals that are in phase quadrature with each other.

One method of conversion to in-phase, I[k], and quadrature-phase, Q[k], (I-Q conversion) signals utilizes a Hilbert transformer. The Hilbert transformer comprises a filter with a constant magnitude response and a phase response of −90 degrees for positive frequencies and +90 degrees for negative frequencies. An approximation to the Hilbert transformer can be realized with an anti-symmetric FIR filter that has an ideal phase response and an amplitude response that is nearly ideal over the range of frequencies of the signal. In parallel with the Hilbert transformer is a high-pass filter that provides the output, I[k]. The high-pass filter provides the block necessary to remove the D-C component from the signal burst. The outputs from the high-pass filter and Hilbert transformer are given as

Figure 3



$$I[k]=\tilde{A}[k]\cos[(\Omega_o+\Delta\Omega)k+\tilde{\phi}(k;\underline{a})+\phi_I]$$

and (3)

$$Q[k]=\tilde{A}[k]\sin[(\Omega_o+\Delta\Omega)k+\tilde{\phi}(k;\underline{a})+\phi_I];$$

$$k = 0, 1, 2, \ldots.$$

After I[k] and Q[k] are produced, amplitude and phase functions are computed. The amplitude function is given as

$$\tilde{A}[k]=\sqrt{I^2[k]+Q^2[k]}$$

k =0, 1, 2, . . . , K (4)

and the phase function is given as

$\theta[k]=\tan^{-1}(Q[k]/I[k])$

$k = 0, 1, 2, \ldots, K$ (5)

K+1 is the number of samples in a burst. For example, if the duration of a burst is 0.5 milliseconds and the sampling rate is 2500 Ksps, then K=1250.

The phase samples given by equation (5) are passed through a differentiator to produce samples of the frequency versus time function. The differentiator would preferably be an anti-symmetric FIR digital filter that has a linear magnitude response and a 90° phase shift over the range of frequencies of the test signal. Like the Hilbert transformer, the differentiator is a well-known digital filter that is easily and accurately implemented in digital hardware.

Data Detection and Estimation of Clock Phase

Referring now also to Figures 4 and 5, a typical frequency deviation function for GMSK.3 modulation which is a modulation scheme proposed in Europe for digital mobile radios is shown. In Figure 4, $(f-f_c)T_b$ is the frequency deviation from the signal carrier (IF) frequency, f_c, normalized by the bit rate $f_b = 1/T_b$ where T_b is the bit interval. The frequency deviation is shown for 36 bits in Figure 4. A positive value of frequency deviation over a bit interval represents one binary state and a negative value the other binary state. The frequency function shown in Figure 4 represents the bit sequence

101111000101110101011000110101000100 (6)

or the complement of this sequence.

From Figure 4, it can be seen that the frequency deviation passes through zero approximately at integer multiples of T_b as shown in Figure 5. From Figures 4 and 5, it can be seen that if the bit pattern is known, then differences in the zero-crossings from integer multiples of T_b are predictable. For example, if bit pair "10" is followed by bit pair "11", the zero crossing of the frequency deviation will occur $-0.0142\ T_b$ from the zero crossings that would occur if bit pair "00" were followed by bit pair "11". Likewise, bit pair "00" followed by bit pair "10" will cause a positive shift in the zero crossing of $0.0142\ T_b$.

Figure 4

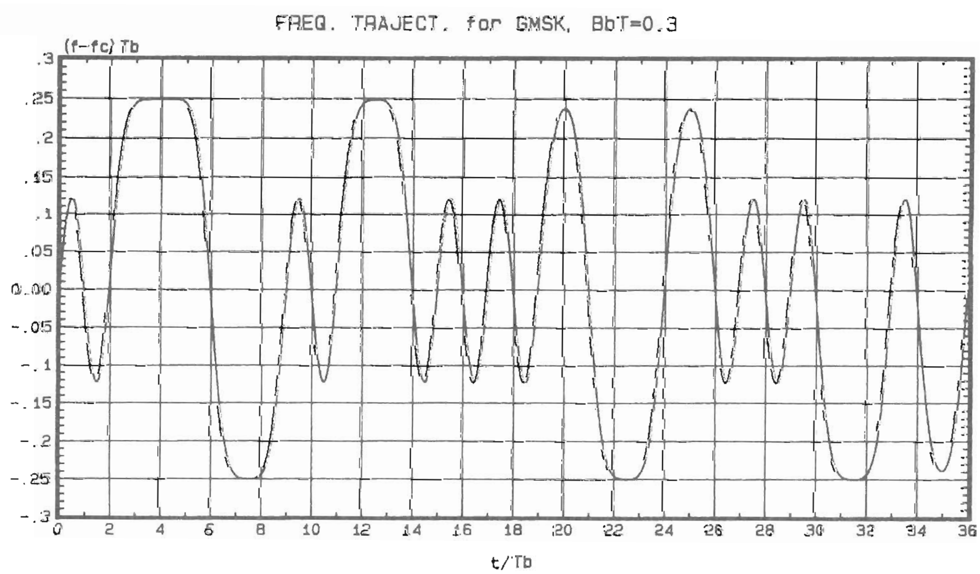
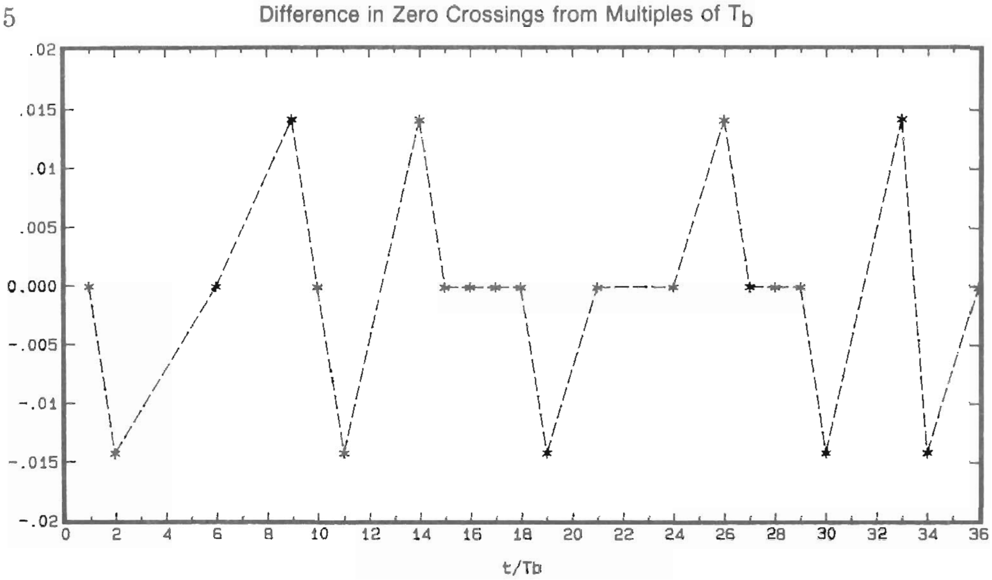


Figure 5



The output of the differentiator is not a continuous time function as shown in Figure 4 but is actual samples of the frequency function. For example, if the bit rate is 270.83 kbps and the sampling rate is 2.5 Msps, then there would be 9.23 samples per bit.

Referring again to Figure 3, following the differentiator, the IF frequency is subtracted from the frequency function to produce the frequency deviation function as presented in Figure 4. The next step is to detect the zero-crossing from which the received data sequence is detected as illustrated by bit sequence (6). Since only discrete time samples of frequency deviation are available, the zero-crossings are detected using an interpolation algorithm. From the detected data sequence, a correction is made to compensate for the difference in zero-crossings from multiples of T_b. These compensated zero-crossings provide the data used to establish a data clock synchronized to the transmitter (not shown) data clock.

The period and phase of the transmitter data clock must be estimated very accurately to minimize errors in the measured phase error. For example, an error of 1 percent in the data clock phase will result in a phase measurement error as large as 0.9 degrees, which may not be acceptable. Even though measured zero-crossings are compensated, measurement noise may result in an unreliable data clock unless the data clock is estimated in an optimal manner. The transmitter data clock may be represented as

$$t_K = kT + b, \; k = 0, 1, 2, \ldots \tag{7}$$

where T is the transmitter data clock period and b is the unknown data clock phase. The *a priori* clock period $\tilde{T}$ is known within a specified tolerance of T. The objective is to obtain estimates $\hat{T}$ and $\hat{b}$ of T and b from the measured zero-crossings.

Suppose s_i, i=1, 2, . . . , N are the measured and compensated zero-crossings of the frequency deviation function. An estimate of the zero-crossings spaced by multiples of T can be written as

$$\hat{s}_i = k_i \hat{T} + \hat{b} \tag{8}$$

where

$$k_i = \mathrm{INT}[(s_i - \epsilon_1)/\tilde{T} + .5) \tag{9}$$

and ϵ_1 is a time reference which may be a zero-crossing near the center of the signal burst. Values of $\hat{T}$ and $\hat{b}$ are obtained such that the mean-square error between the sets s_i and $\hat{s}_i$, i=1, 2, . . . , N given by

$$\overline{\epsilon^2} = \frac{1}{N} \sum_{i=1}^{N} (s_i - k_i \hat{T} - \hat{b})^2 \tag{10}$$

is minimized. The resulting estimates are

$$\hat{T} = \frac{\sum_{i=1}^{N} s_i k_i - \frac{1}{N} \left(\sum_{i=1}^{N} s_i \right) \left(\sum_{i=1}^{N} k_i \right)}{\sum_{i=1}^{N} k_i^2 - \frac{1}{N} \left(\sum_{i=1}^{N} k_i \right)^2} \tag{11}$$

and

$$\hat{b} = \frac{1}{N} \left[\sum_{i=1}^{N} s_i - \hat{T} \sum_{i=1}^{N} k_i \right] \tag{12}$$

The receiver data clock synchronized to the transmitter data clock is given as

$$\hat{t}_k = k\hat{T} + \hat{b}; \quad k = 0, 1, 2, \ldots \tag{13}$$

If the clock period T is known *a priori* with sufficient accuracy for the required measurement, or it is required that the measurement include the measurement of phase errors attributable to inaccuracies in T, T would not be estimated. In this case $\hat{T} = \tilde{T}$ in equations (12) and (13) and only the data clock phase is estimated as given by equation (12).

The next step is to synchronize bit patterns to establish the active time interval of a signal burst over which the phase and amplitude errors are determined and displayed. If a synchronizing pattern such as a preamble or midamble is available, i.e., included in the transmitted signal burst, then the leading and trailing edges of the envelope of the burst obtained from $\tilde{A}[k]$, as given by equation (4), are used to establish the range over which the preamble or midamble must exist. A discrete-time cross-correlation of the detected bit pattern with the known synchronizing pattern is performed to align the two patterns and establish the active interval. If a synchronizing pattern does not exist, then the active interval of the test is centered between the leading and trailing edges of the envelope of the burst.

Synthesis of Ideal Modulated Signal

Knowledge of the clock phase and period, the data sequence and the time interval of interest provide the information needed to mathematically compute the ideal phase modulating function $\phi[k;\underline{a}]$ and amplitude profile A[k]. These computed functions are then compared with the corresponding measured values of amplitude and phase to obtain measurements of amplitude and phase errors.

By way of example, synthesis of the phase function for continuous-phase-modulated signals (CPM) will be considered here.

The phase function for CPM can be written as

$$\phi(k;\underline{a}) = 2\pi \sum_{i=-\infty}^{\infty} h_i\, a_i\, q_i\, [(k-i)T_b] \qquad 14)$$

where

$$\underline{a} = (\ldots, a_{-1}, a_0, a_1, a_2, \ldots)$$
$$a_i = \pm 1, \pm 3, \ldots, \pm M-1$$

is the data sequence. For binary modulation M=2 and $a_i = \pm 1$.

h_i is the modulation index which in general may be a cyclic function of time. For many common modulations such as minimum shift key (MSK) and Gaussian minimum shift key (GMSK), h_i=1/2 (constant). q(t) is the phase pulse-shape function which has the property that

$$q(t) = 0,\ t<0 \qquad (15)$$
$$= \frac{1}{2},\ t>LT_b$$

where L is a positive integer. The type of modulation is determined by q(t). Phase pulse response curves for MSK and GMSK.3, L=5, are plotted in Figures 6a and 6b, respectively.

Figure 6a

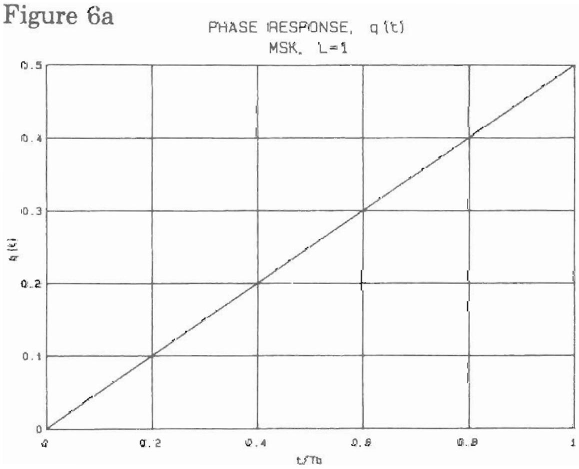
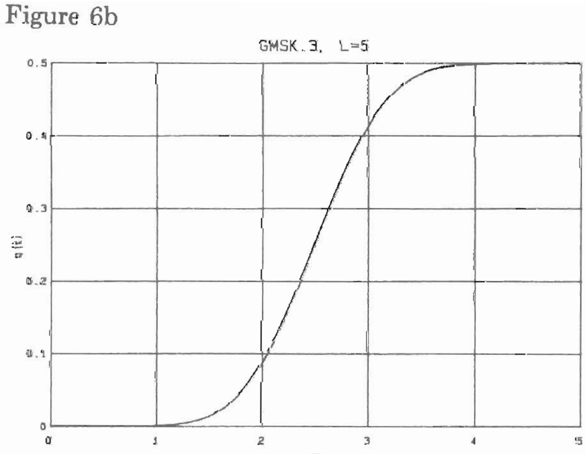


Figure 6b



Comparison of Measured Values of Amplitude and Phase With the Ideal

After the ideal phase function $\phi[k;a]$ is synthesized, it is subtracted from the measurement phase function

$$\Theta[k] = (\Omega_o + \Delta\Omega)k + \tilde{\phi}[k;\underline{a}] + \phi_1 \qquad (16)$$

to produce the phase difference given as

$$\Delta\Theta[k] = \Theta[k] - \Omega_o k - \phi[k;\underline{a}] = \Delta\Omega k + \tilde{\phi}[k;\underline{a}] - \phi[k;\underline{a}] + \phi_1 \tag{17}$$

The phase error is defined as

$$\epsilon_\phi[k] = \tilde{\phi}[k;\underline{a}] - \phi[k;\underline{a}] \tag{18}$$

i.e., the difference between the received and synthesized ideal phase functions, so that the phase difference is

$$\Delta\Theta[k] = \Delta\Omega k + \epsilon_\phi[k] + \phi_1 \qquad k = 1, 2, 3, \ldots, K \tag{19}$$

where

$\Delta\Omega$ is the frequency error and ϕ_1 is the unknown offset phase.

The phase difference, $\Delta\theta[k]$ has a linear term $\Delta\Omega k$ with slope $\Delta\Omega$ and a constant term ϕ_1, that can be estimated by fitting the K values given by equation (19) to a linear regression curve

$$\widehat{\Delta\Theta}[k] = \widehat{\Delta\Omega} k + \hat{\phi}_1 \tag{20}$$

The difference between equations (19) and (20) given as

$$\hat{\epsilon}_\phi[k] = \epsilon_\phi[k] + (\Delta\Omega - \widehat{\Delta\Omega})k + (\phi_1 - \hat{\phi}_1) \qquad k = 1, 2, 3, \ldots, K \tag{21}$$

along with statistics of $\hat{\epsilon}_\phi[k]$ are the desired outputs of the method.

Quantization Effects to Phase Measurement Accuracy

The digitized quadrature signals can be written as

$$I[k] = A\cos[(\Omega_o + \Delta\Omega)k + \phi(k;\underline{a}) + \phi_1] + N_{qI}[k]$$

and (22)

$$Q[k] = A\sin[(\Omega_o + \Delta\Omega)k + \phi(k;\underline{a}) + \phi_1] + N_{qQ}[k]$$

where $N_{qI}[k]$ and $N_{qQ}[k]$ are independent errors introduced by the analog-to-digital converter (ADC) and the digital filters. In general, if an n-bit ADC converter is used with saturation set at A, then quantization noise power is

$$E\{N_{qI}^2[k]\} = E\{N_{qQ}^2[k]\} = \frac{A^2}{3 \cdot 2^{2n}} \cdot k_N^2 \tag{23}$$

where k_N^2 is the ratio of the noise bandwidth of the digital filters following the ADC to the folding frequency, $f_s/2$.

The phase error is given approximately as

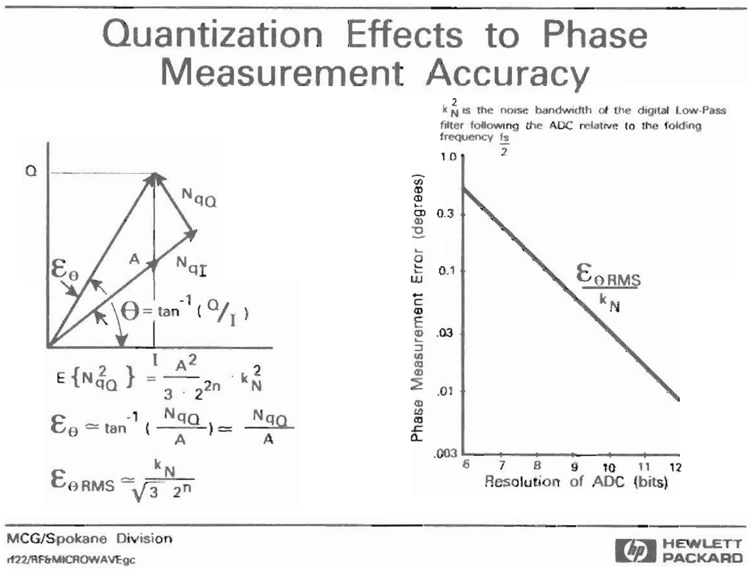
$$\epsilon_\Theta \simeq \tan^{-1}\left(\frac{N_{qQ}}{A}\right) \simeq \frac{N_{qQ}}{A}$$

from which (24)

$$\epsilon_{\Theta RMS} \simeq \sqrt{\frac{E\{N_{qQ}^2\}}{A^2}} \simeq \frac{k_N}{\sqrt{3 \cdot 2^n}}$$

$\dfrac{\epsilon_{\Theta RMS}}{k_N}$ is shown plotted as a function of the number of bits, n, of the ADC in Figure 7.

Figure 7



III. An Implementation of the Method

In this section, an implementation of the phase and amplitude measurement method utilizing an HP 70700A digitizer and an HP 9000-350 computer is described. A picture of the hardware is presented in Figure 8 and the block diagram of the implementation is presented in Figure 9.

As shown in Figure 9, the device under test feeds a signal at a carrier frequency of 900 MHz to a down-converter. A local oscillator at 899.3 MHz is utilized to produce an IF frequency of 700 kHz which passes through a low-pass filter with a cut-off frequency of approximately 6 MHz. The schematic of the down-converter and low-pass filter, and the frequency response of the low-pass filter are shown in Figure 10. The purpose of the low-pass filter is to reject the double frequency term and feed-through from the local oscillator. The cut-off frequency is selected to provide a flat magnitude and linear phase response over the bandwidth of the IF signal and to allow operating the local oscillator at a frequency as low as 10 MHz.

Figure 8

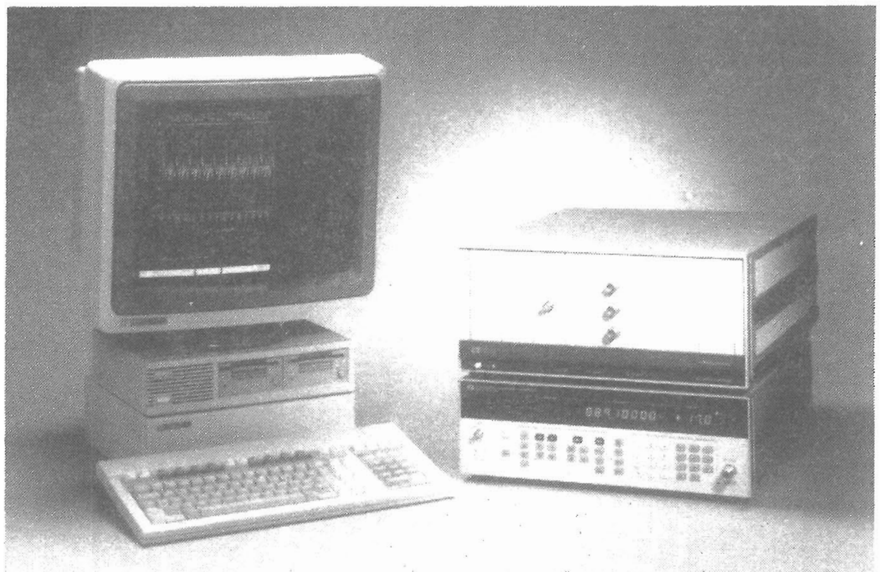


Figure 9

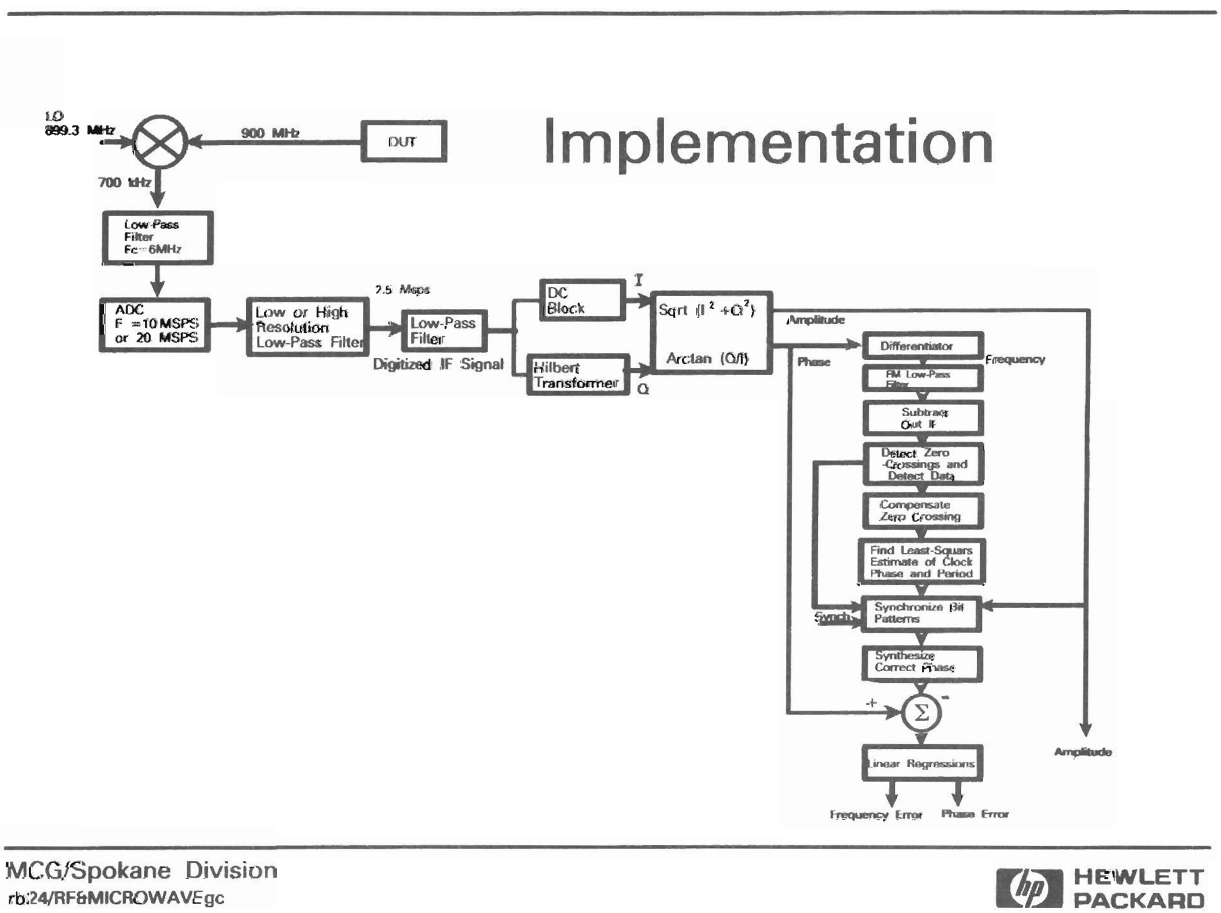


Figure 10a

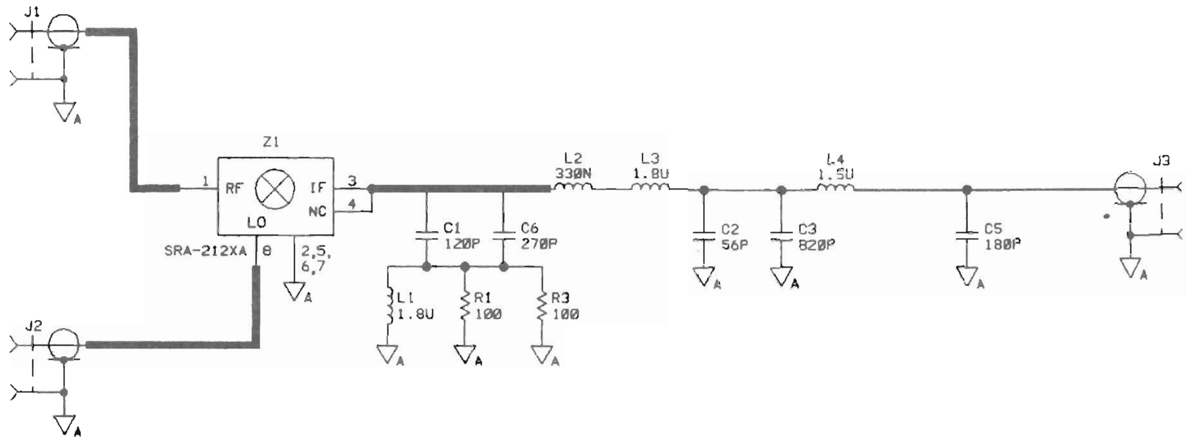
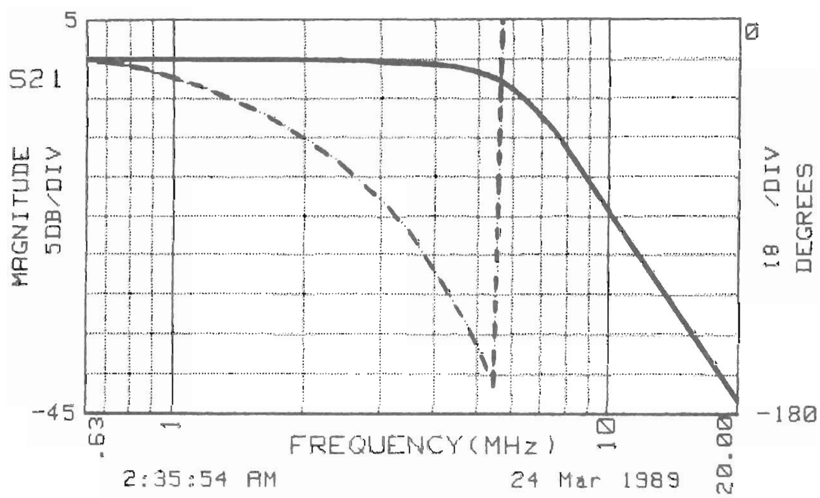


Figure 10b



The output of the low-pass filter is fed to the HP 70700A digitizer. The HP 70700A digitizer free runs at 20 MSps with a tolerance of ±0.01% which will produce a ±70 Hz frequency measurement error. To improve the accuracy of the sampling rate, the digitizer can be triggered externally from a 10 MHz or 20 MHz clock.

An analog envelope detector (not shown) is utilized to detect the leading edge of a burst and trigger the digitizer. The digitizer then samples at either the 10 MSps or 20 MSps rate and stores the 10 bit sampled values in a 262K word memory. At the 20 MSps rate, the capacity of the memory would allow sampling for a 13.1 msec interval which is much longer than the duration of a burst. After the samples of a burst have been captured, the remaining signal processing operations shown in Figure 9 are performed within the HP computer.

Referring to Figure 9, the digitizer is followed by a cascaded pair of low-pass filters. The first filter is either a low-resolution or high-resolution, linear phase, Chebyshev low-pass filter with frequency response shown in Figures 11a or 11b. The low-resolution filter is used with the 10 MSps sampling rate and the high-resolution filter is used with the 20 MSps sampling rate. The purpose of these filters is the elimination of the third harmonic at a carrier frequency of 2.1 MHz generated by the down-converter. The output of the third harmonic filter is resampled at a 2.5 MSps rate for both the low- and high-resolution filters. The ratio of noise bandwidth to folding frequency is given approximately as $k_N^2 \simeq 0.3$ for the low resolution filter and $k_N^2 \simeq 0.15$ for the high resolution filter.

From Figure 7 for a 10-bit ADC and $k_N^2 = 0.3$, the phase measurement error introduced by the ADC will be less than 0.02 degrees RMS and, therefore, will be negligible.

After the third harmonic filter, the samples at a rate of 2.5 MSps are fed to the second harmonic filter to eliminate the signal at 1400 kHz generated by the down-converter. Since the folding frequency for this filter is 1250 kHz, the 1400 kHz harmonic will appear at 1100 kHz or at the normalized frequency of 0.44. From the frequency response of the second harmonic filter presented in Figure 11c, it is seen that the second harmonic is suppressed by 20 dB.

Figure 11a

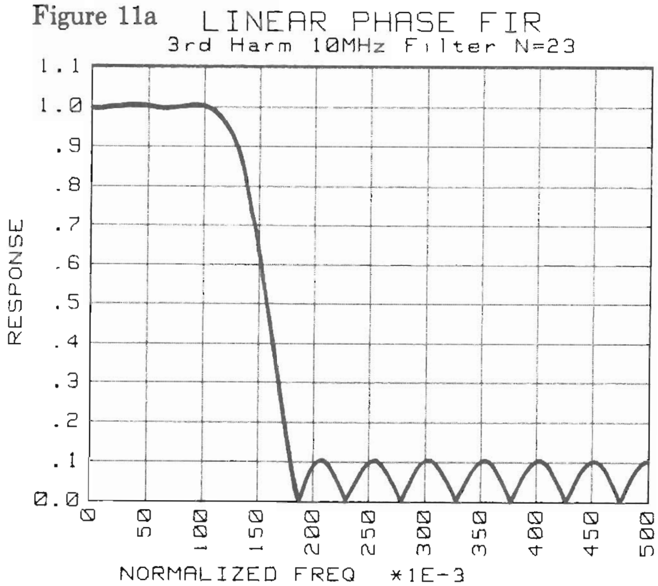


Figure 11b

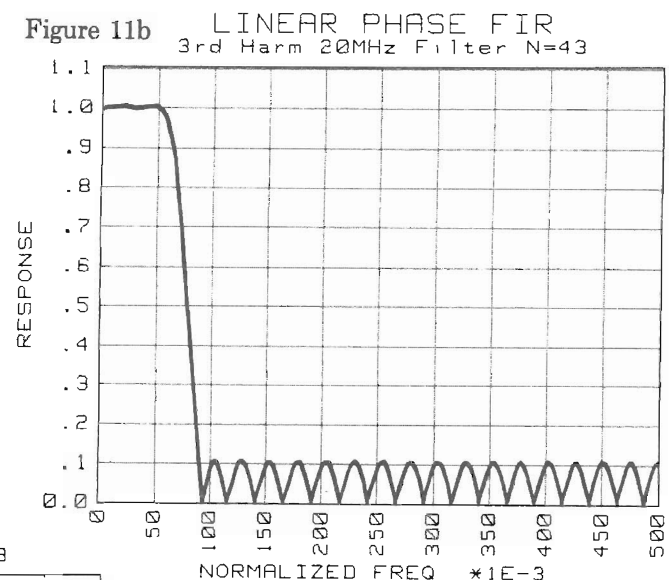
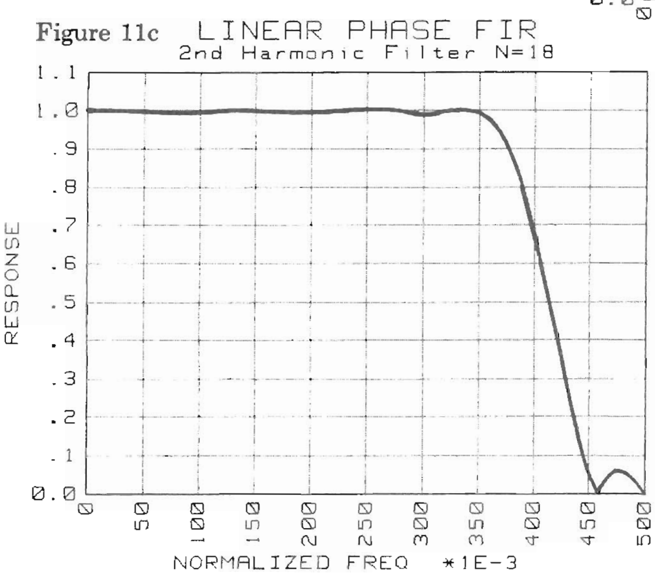


Figure 11c



From the output of the second harmonic filter, the signal is fed to the Hilbert transformer and dc block in parallel to provide the two quadrature signals, I and Q. The Hilbert transformer is an anti-symmetric, linear-phase, FIR filter that provides precisely 90 degrees of phase shift for frequencies from 0 to the folding frequency and −90 degrees phase shift from the folding frequency to the sampling frequency. The magnitude frequency response for the Hilbert transformer is presented in Figure 12. It should be pointed out that for the fifteenth order FIR Hilbert transformer, the even indexed filter coefficients are equal to zero so that only eight multiplications per sample are required to implement this filter.

The dc block is a fifteenth order high-pass FIR filter with magnitude frequency response presented in Figure 13. The dc block is necessary to remove the dc component from the IF burst fed from the down converter through the low-pass filters. A dc signal component present in either the I or Q signal will cause gross errors when calculating the phase given as $\tan^{-1}(Q/I)$.

Figure 12

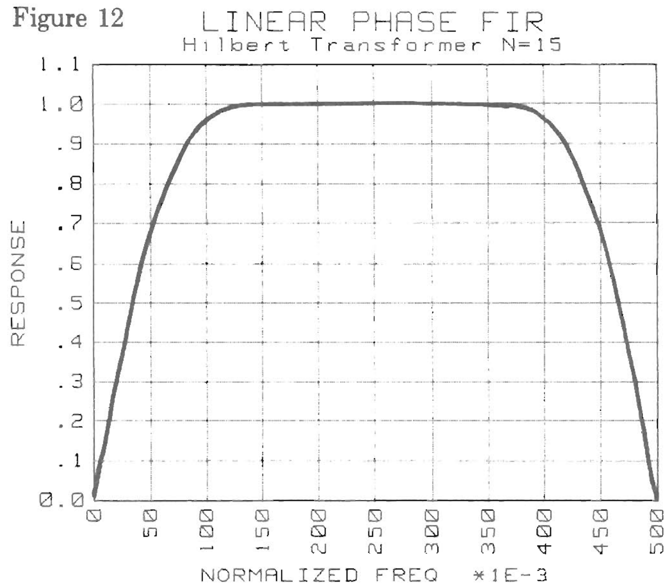
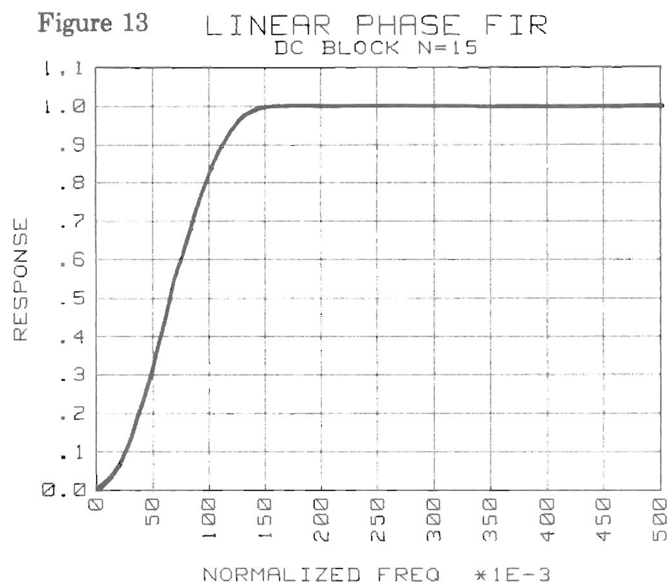


Figure 13



After the two quadrature signals are obtained, the amplitude and phase are computed as

$$\text{Amplitude} = \sqrt{I^2[k]+Q^2[k]}$$

and

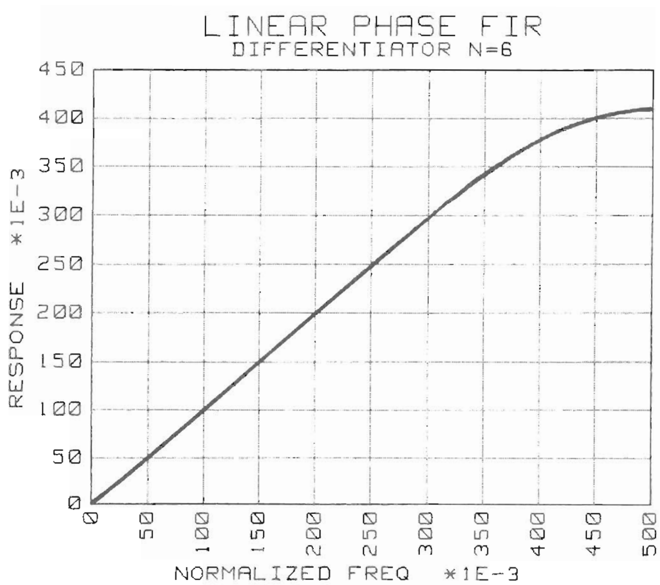
$$\text{Phase} = \tan^{-1}(Q/I)$$

and are stored in arrays over the time interval of the burst.

The amplitude is fed to a display where it is compared with a mask for the acceptable amplitude profile and is later used to give a bound on the range of the active interval of the burst used to synchronize the bit patterns. The phase is fed through a differentiator to provide samples of instantaneous frequency deviation and is used later to compare with the ideal synthesized phase.

The differentiator is an anti-symmetric, sixth order, linear phase, FIR filter with a phase shift of precisely 90 degrees over the range of the signal, and magnitude frequency response shown in Figure 14.

Figure 14

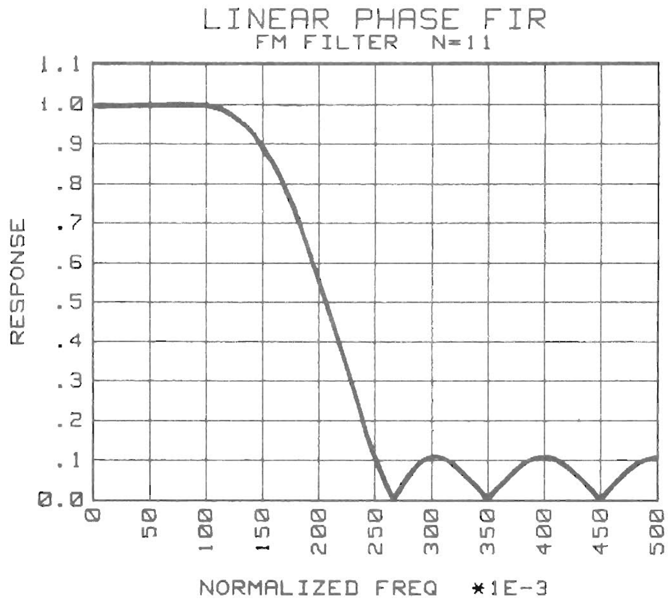


The output of the differentiator is fed through the eleventh order, linear-phase, low-pass filter with magnitude frequency response shown in Figure 15. The purpose of this filter is to reduce the high frequency components of noise introduced by the differentiator without affecting the integrity of the zero crossings of the frequency deviation function. At the output of the FM filter, the IF frequency is subtracted to provide a frequency deviation function precisely centered at zero as shown in Figure 4.

The zero-crossings are estimated from a linear interpolation of the sampled values of the frequency deviation function. These zero-crossings are utilized to provide the detected data that are needed for the computation of the ideal phase function.

As discussed in Section II, the zero-crossings are compensated so they will occur at integer multiples of T_b. Furthermore, as discussed previously, these zero crossings are used in a least-squares algorithm to provide an accurate estimate of the transmitter clock phase.

Figure 15



The procedure for detecting data and estimating the clock phase from zero-crossings of the frequency deviation function, as described above, will work well only if the signal-to-noise ratio (SNR) of the signal from the transmitter is high. It is not anticipated that the method for testing the phase and amplitude accuracy of a transmitter would be used for low SNR situations. In case it were, however, a more sophisticated clock phase and carrier frequency and phase estimator, and an optional demodulator employing the Viterbi algorithm could be incorporated in software.

The next step involves performing a discrete-time cross-correlation of the detected data sequence and the known midamble to establish the time reference necessary to synchronize the synthesized and measured phase trajectories. The measured amplitude profile is utilized to limit the range of bits over which the cross-correlation must be performed.

After the timing reference has been established and the data sequence detected, the ideal phase trajectory is computed as given by equation (14). The synthesized phase is subtracted from the measured phase that was stored in an array to provide the phase difference as given by equation (17). The phase difference is fit to a linear regression curve as given by equation (20) to provide estimates of carrier frequency error and instantaneous phase error.

The above described method has been utilized to test a prototype signal generator that produces an RF signal with GMSK.3 modulation. Information determined by the method is shown plotted in Figures 16 through 18. In Figure 16, the measured phase difference on a bit-by-bit basis is plotted versus time as curve 103. Curve 103 shows the difference in the phase between the ideal phase function and the transmitted phase function for each data bit in a signal burst. Curve 101 is the linear regression of the phase difference plotted versus the data bit number for a data burst. The slope of the linear regression curve 101 represents the frequency error of the transmitted signal. The curve in Figure 17 is a plot of the instantaneous phase error versus time (bit number) for the data bits in a signal burst and represents the instantaneous phase error of the transmitted signal when compared to the ideal signal. Figure 18a, 18b and 18c are plots of the measured signal amplitude versus bit number for a signal burst. Curve 121 is the amplitude of the signal burst. Curves 123 and 125 are the upper and lower bounds allowed for the amplitude. Figure 18b presents an expanded plot of the rise time of the transmitted signal amplitude and Figure 18c presents an expanded plot of the fall time of the transmitted amplitude.

Figure 16

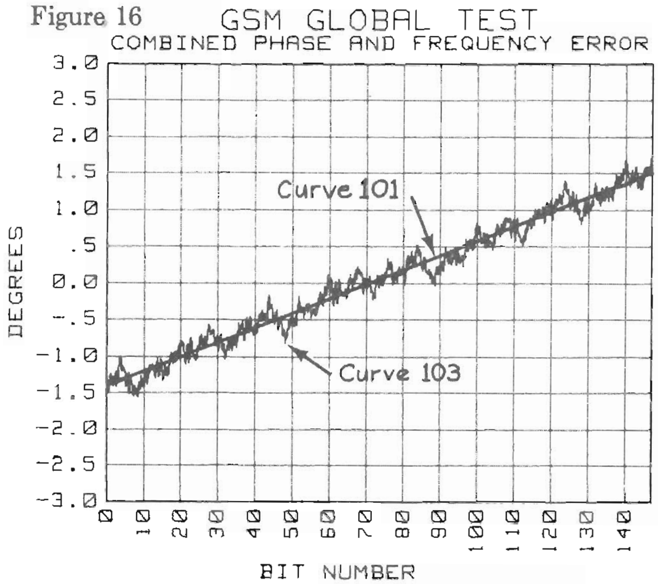


Figure 17

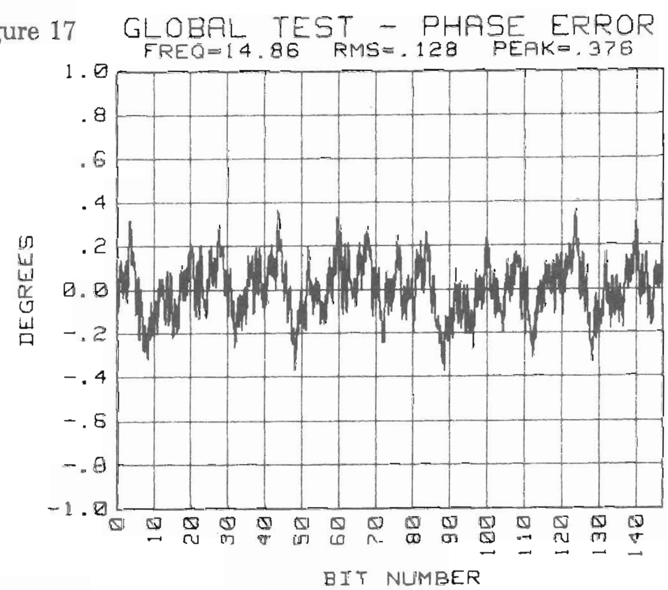


Figure 18a

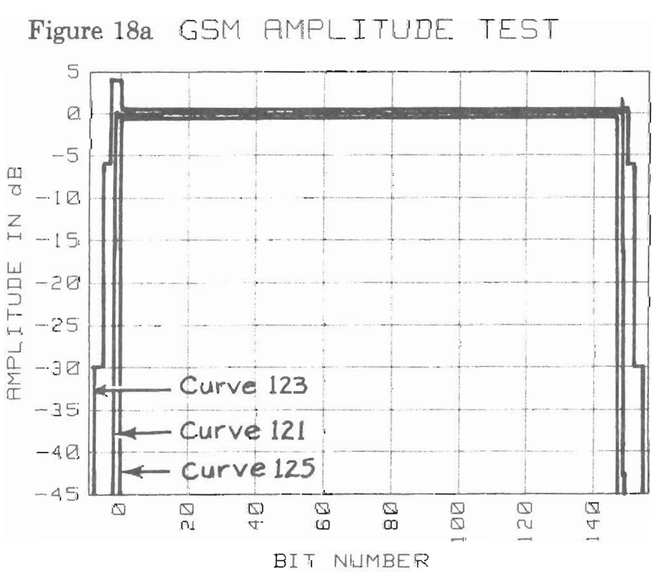


Figure 18b

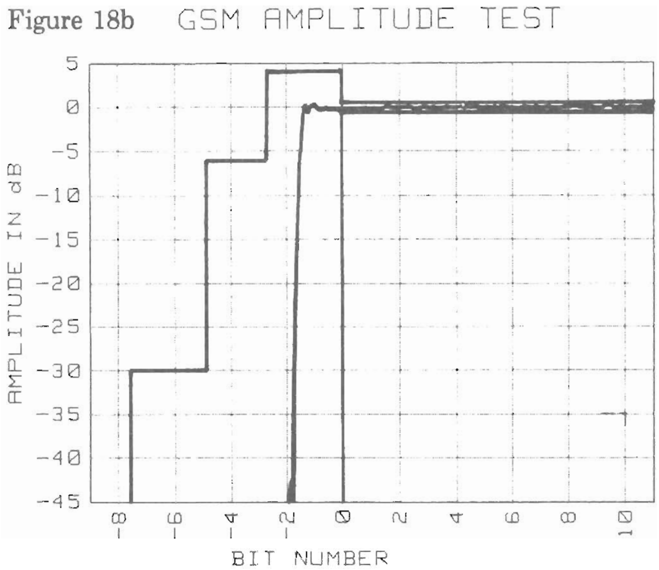
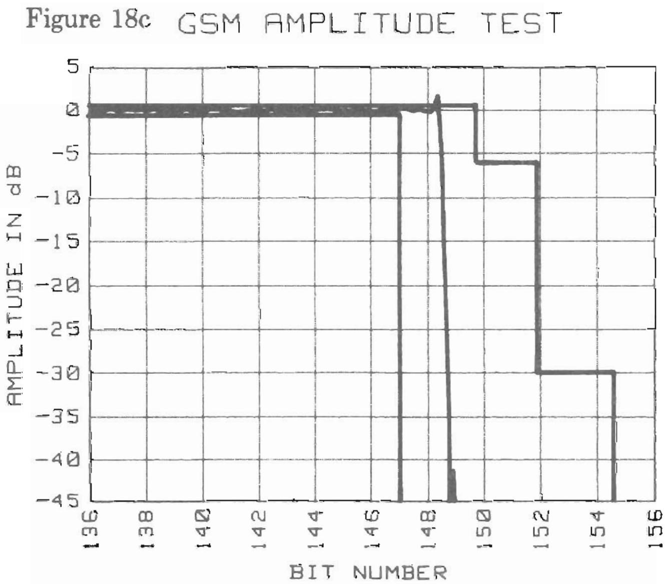


Figure 18c



IV. Summary and Conclusions

A method and apparatus for measuring the phase accuracy and amplitude profile of CPM signals was described. The method utilizes a down-converter to provide a relatively low-frequency IF signal which is filtered and sampled by an ADC. The digitized signal is fed to a digital signal processor to produce measured amplitude and phase functions of time of the transmitted signal. From the measured function and a known synchronizing signal, an ideal phase function is synthesized. The synthesized and measured functions are compared to provide estimates of frequency, phase and amplitude errors of the transmitter as a function of time.

A mathematical description of the method was given including considerations of quantization effects, estimates of transmitter clock phase and frequency, and detection of data.

An implementation of the method utilizing an HP 70700A digitizer and HP 9000-350 computer was described. Included is a description of the filters and signal processing operations used in the implementation. The results of measurements of frequency, phase and amplitude errors of a signal generated by a prototype transmitter determined by the method were included.

The algorithms in this paper are under protection of a patent pending in the U.S.A. and elsewhere.

Acknowledgement:

The author wishes to acknowledge the contributions made by Richard Ryan and Ken Thompson. Rick designed the downconverter and developed the signal processing software utilized to implement and demonstrate the feasibility of the technique presented in this paper. Ken added many user-friendly features to the software from which the results presented in this paper were provided.

TECHNIQUE FOR MEASURING PHASE ACCURACY AND AMPLITUDE PROFILE OF CONTINUOUS-PHASE-MODULATED SIGNALS

Application to GMSK.3

Dr. Raymond A. Birgenheier

A METHOD FOR MEASURING THE ACCURACY OF CONTINUOUS - PHASE MODULATED SIGNALS WILL BE DESCRIBED IN WHICH

- Transmitter Signal is Down Converted and Digitized
- Instantaneous Amplitude and Phase Functions of Transmitted Signals are Computed
- Data and Clock Phase are estimated from Zero-crossings of Frequency Deviation Function
- Ideal Phase Function is Synthesized
- Synthesized and Measured Functions are Compared to Provide Estimates of Transmitter Frequency Error, Phase Error and Amplitude Profile.

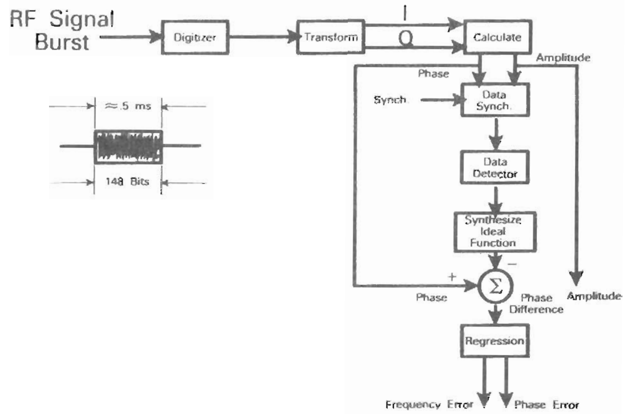
A MATHEMATICAL DESCRIPTION WILL BE GIVEN INCLUDING CONSIDERATIONS OF

- Quantization Effects
- Estimates of Transmitter Clock Period and Phase
- Detection of Data

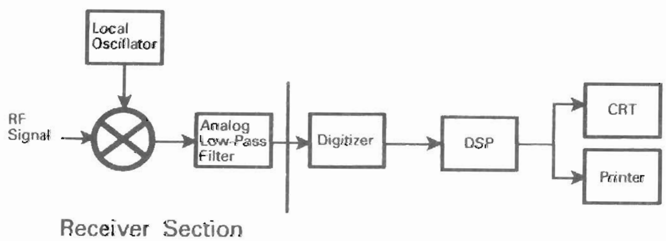
AN IMPLEMENTATION OF THE METHOD EMPLOYING AN HP70700A DIGITIZER AND HP9000-350 COMPUTER WILL BE DESCRIBED.

- Results of Measurements of frequency, Phase and Amplitude Errors of a CPM Signal will be included

Measuring Phase, Frequency, and Amplitude Profile of a CPM Signal



Apparatus for Measuring Phase, Frequency, and Amplitude Profile of CPM Signals



Definition of Terms

Input Signal

$$y(t) = \tilde{A}(t)\cos\left[(\omega_o + \Delta\omega)t + \tilde{\phi}(t;\underline{a}) + \phi_o\right]$$

where

$\tilde{A}(t)$ is the received signal amplitude.

ω_o is the nominal IF signal frequency.

$\Delta\omega$ is the frequency uncertainty

$\tilde{\phi}(t;\underline{a})$ is the received signal modulation function

ϕ_o is unknown carrier phase.

Digitized Signal

$$y[k] = \tilde{A}[k]\cos\left[(\Omega_o + \Delta\Omega)k + \tilde{\phi}(k;\underline{a}) + \phi_o\right]$$

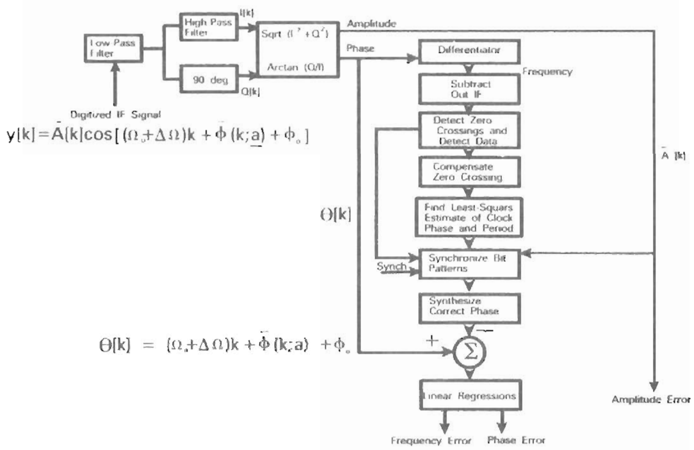
where

$t = k\,T_s$

$\Omega_o = \omega_o\,T_s$

$\Delta\Omega = \Delta\omega\,T_s$

Signal Flow Diagram



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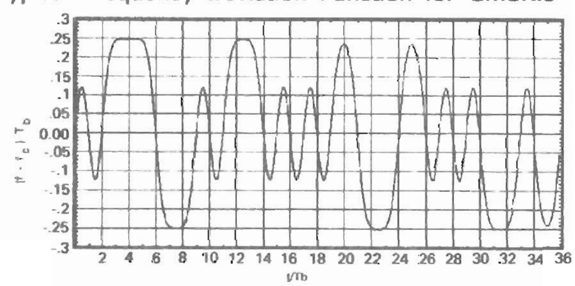
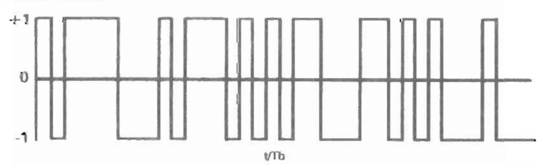
Typical Frequency Deviation Function for GMSK.3



Detected Data



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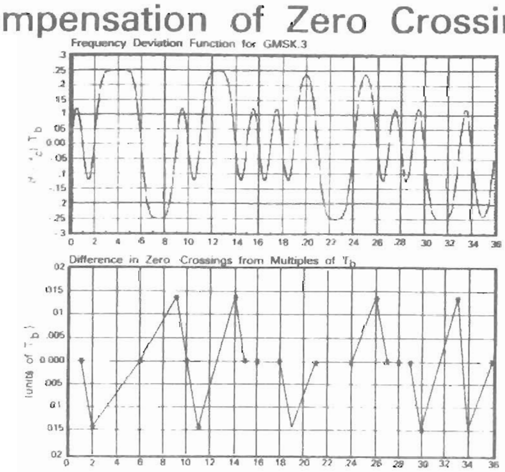
Compensation of Zero Crossings



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Estimation of Clock Period and Phase

True Transmitter Clock:

$$t_k = kT + b\ ,\ k = 0, 1, 2, \ldots$$

Measured and Compensated Zero-Crossing of Frequency Deviation Function:

$$s_i\ ,i = 1, 2, 3, \ldots, N$$

Estimate of Zero Crossings Spaced by Integer Multiples of $\hat{T}$:

$$\hat{s}_i = k_i \hat{T} + \hat{b}$$

where

$$k_i = \mathrm{INT}\left[(s_i - \epsilon_1)/\tilde{T} + .5\right]$$

$\tilde{T}$ is a priori clock Period.

$\hat{T}$ is an estimate of clock period.

$\hat{b}$ is an estimate of clock phase.

ϵ_1 is time reference zero-crossing near center of burst.

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Estimation of Clock Period and Phase (continued)

Minimize Mean-Square Error:

$$\overline{\epsilon^2} = \frac{1}{N}\sum_{i=1}^{N}(s_i - k_i\hat{T} - \hat{b})^2$$

Estimates:

$$\hat{T} = \frac{\sum_{i=1}^{N} s_i k_i - \frac{1}{N}\left(\sum_{i=1}^{N} s_i\right)\left(\sum_{i=1}^{N} k_i\right)}{\sum_{i=1}^{N} k_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} k_i\right)^2}$$

$$\hat{b} = \frac{1}{N}\left[\sum_{i=1}^{N} s_i - \hat{T}\sum_{i=1}^{N} k_i\right]$$

Estimated Clock:

$$t_k = k\hat{T} + \hat{b};\ k = 0, 1, 2, \ldots$$

If T is not estimated then $\hat{T} = \tilde{T}$

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Synthesis of Ideal Modulated Phase

$$\phi(k;\underline{a}) = 2\pi\sum_i h_i a_i q[(k-i)T_b]$$

where $\underline{a} = (\ldots, a_{-1}, a_0, a_1, a_2, \ldots)$

with $a_i = \pm 1, \pm 3, \ldots, \pm M-1$ is the data sequence

h_i is the modulation index

$q(t)$ is the phase pulse shape function

where

$$q(t) = 0,\ t < 0$$
$$= \frac{1}{2},\ t > T_b$$

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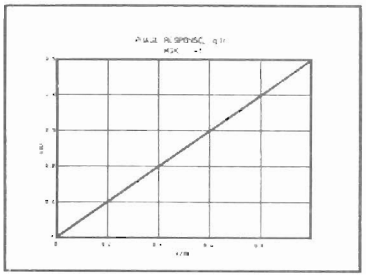
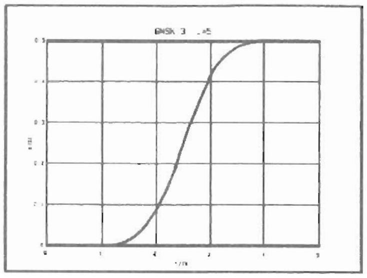
Phase Pulse-Shape Function for MSK and GMSK.3





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Comparison of Ideal and Measured Phase

Ideal Phase:

$$\phi[k;\underline{a}]$$

Measured Phase:

$$\Theta[k] = (\Omega_o + \Delta\Omega)\, k + \tilde{\phi}[k;\underline{a}] + \phi_1$$

Phase Difference:

$$\Delta\Theta[k] = \Theta[k] - \Omega_o k - \phi[k;\underline{a}]$$

$$= \Delta\Omega\, k + \tilde{\phi}[k;\underline{a}] - \phi[k;\underline{a}] + \phi_1$$

$$= \Delta\Omega\, k + \epsilon_\phi[k] + \phi_1$$

where Phase Error is

$$\epsilon_\phi[k] = \tilde{\phi}[k;\underline{a}] - \phi[k;\underline{a}]$$

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Estimated Phase Error

Phase Difference:

$$\Delta\Theta[k] = \Delta\Omega\, k + \epsilon_\phi[k] + \phi_1$$

where $k = 1, 2, 3, \ldots, K$

$\Delta\Omega$ is frequency error

ϕ_1 is unknown constant phase

Fit to Linear Regression Curve:

$$\widehat{\Delta\Theta}[k] = \widehat{\Delta\Omega}\, k + \hat{\phi}_1$$

Estimate of Phase Error:

$$\hat{\epsilon}_\phi[k] = \epsilon_\phi[k] + (\Delta\Omega - \widehat{\Delta\Omega})\, k + (\phi_1 - \hat{\phi}_1)$$

$k = 1, 2, 3, \ldots, K$

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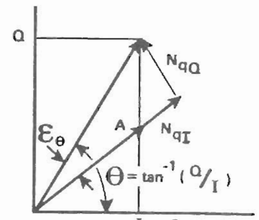
Quantization Effects to Phase Measurement Accuracy

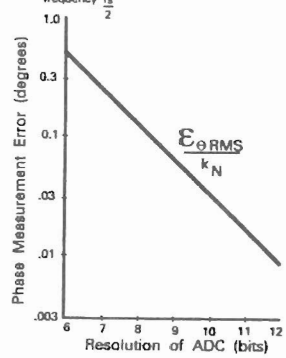


$$E\{N_{qQ}^2\} = \frac{A^2}{3 \cdot 2^{2n}} \cdot k_N^2$$

$$\mathcal{E}_\theta \simeq \tan^{-1}\left(\frac{N_{qQ}}{A}\right) \simeq \frac{N_{qQ}}{A}$$

$$\mathcal{E}_{\theta RMS} \simeq \frac{k_N}{\sqrt{3}\ 2^n}$$

k_N^2 is the noise bandwidth of the digital Low-Pass filter following the ADC relative to the folding frequency $\frac{fs}{2}$



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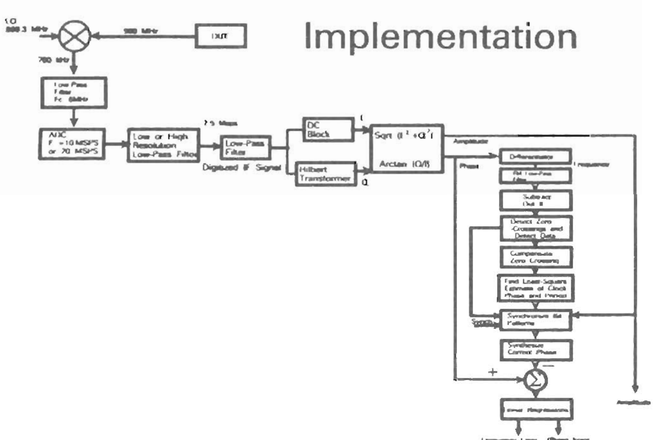
Implementation



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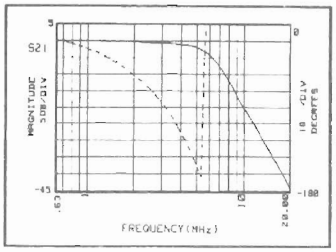
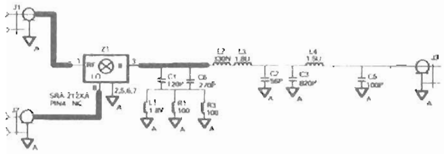
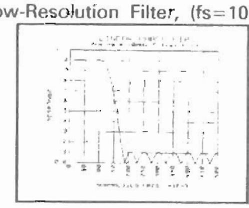
Down Converter



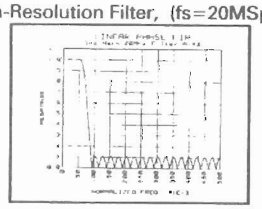


Low-Pass Digital Filters

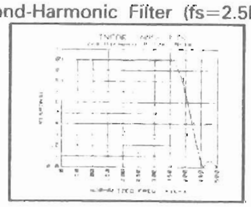
Low-Resolution Filter, (fs=10MSps)



High-Resolution Filter, (fs=20MSps)

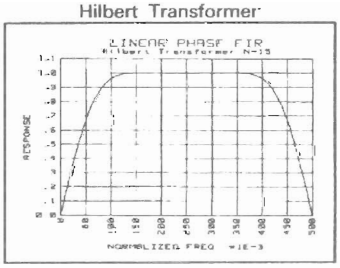


Second-Harmonic Filter (fs=2.5MSps)

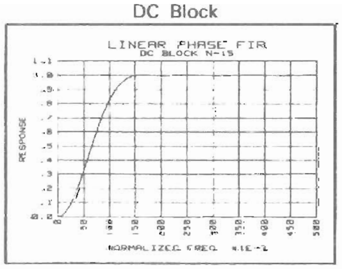


Quadrature Filters

Hilbert Transformer

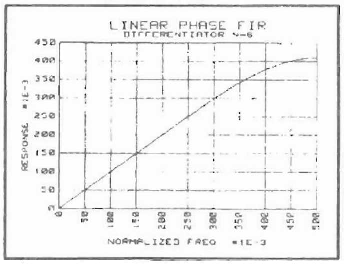


DC Block

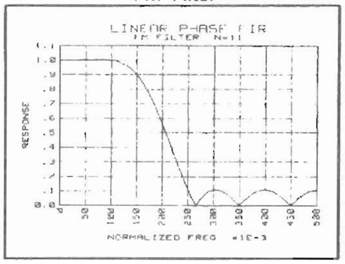


Differentiator and FM Filter

Differentiator

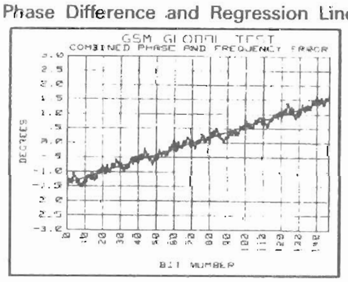


FM Filter

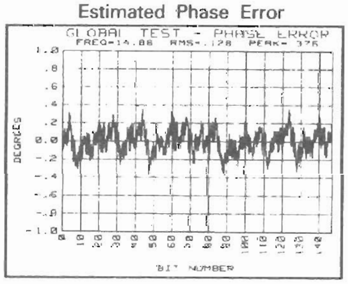


Phase Error

Phase Difference and Regression Line

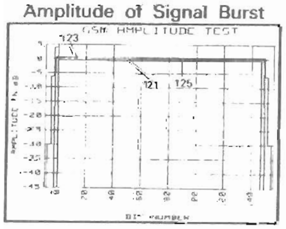


Estimated Phase Error

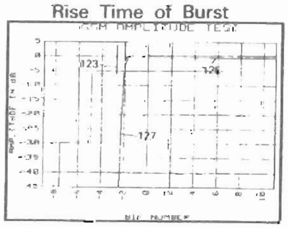


Amplitude Profile

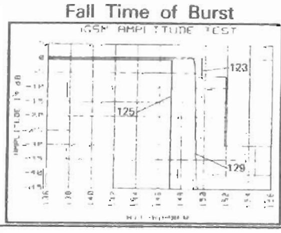
Amplitude of Signal Burst



Rise Time of Burst



Fall Time of Burst



SUMMARY AND CONCLUSIONS

A METHOD FOR MEASURING THE ACCURACY OF CPM SIGNALS WAS DESCRIBED IN WHICH:

- Transmitter Signal is Down Converted and Digitized.
- Instantaneous Amplitude and Phase Functions of Transmitted Signals are computed
- Data and Clock Phase are estimated from Zero crossings of Frequency Deviation Function.
- Ideal Phase Function is Synthesized
- Synthesized and Measured Functions are compared to provide estimates of Transmitter Frequency Error, Phase Error and Amplitude Profile.

AN IMPLEMENTATION EMPLOYING AN HP70700A DIGITIZER AND HP9000-350 WAS DESCRIBED.

For more information, call your local HP sales office listed in the telephone directory white pages.

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