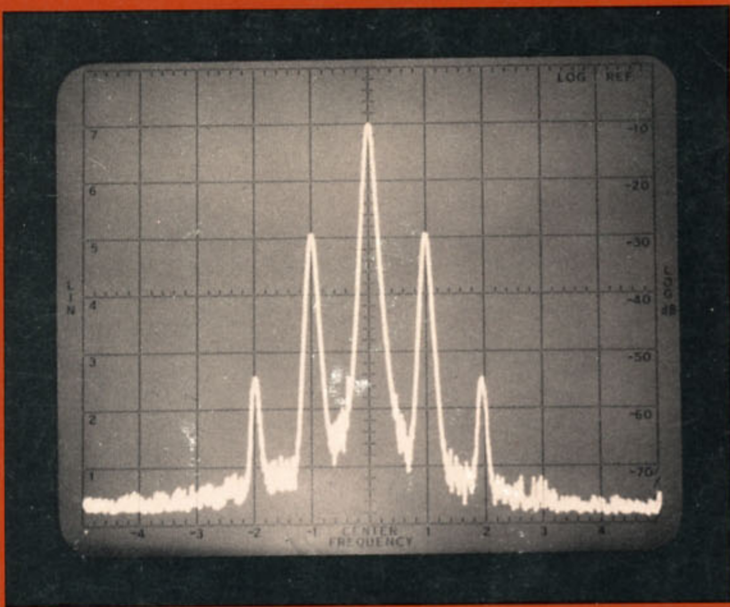
Spectrum Analyzer Series

APPLICATION NOTE 150–1



Spectrum Analysis
Amplitude and Frequency Modulation

HEWLETT hp PACKARD

November 1971

APPLICATION NOTE 150-1

Spectrum Analysis
Amplitude and Frequency Modulation

Printed NOVEMBER 1971

HEWLETT hp PACKARD

CONTENTS

CHAPTER 1
MODULATION METHODS

Since RF signals have been used for communication, the problems of modulating the carrier with the message to be transmitted are of basic interest. A variety of modulation methods has been developed to find the optimum performance for a specific requirement.

In each type of modulation, some property (amplitude, frequency, or phase) of the carrier wave is modulated (varied) in proportion to the instantaneous amplitude of the information-bearing waveform to be transmitted. The usual equation describing the unmodulated carrier wave is:

$$e = A \cos(\omega t + \phi)$$

where: e is the instantaneous amplitude of the carrier at time t;

A is the peak amplitude of the carrier.

ω is the angular velocity corresponding to the carrier frequency, f (e.g., $\omega = 2\pi f$).

ϕ is the initial phase displacement of the carrier with respect to some arbitrary reference.

Two basic properties available for modulation are the amplitude characteristics and angular characteristics of the carrier. Angular modulation is ordinarily subdivided into the more familiar categories of frequency modulation and phase modulation. In all these modulation systems the deviation of the modulated property of the carrier (with respect to the value of that property in the unmodulated carrier) is made proportional to the instantaneous amplitude of the modulating signal (e.g., the information to be transmitted).

CHAPTER 2
AMPLITUDE MODULATION

MODULATION DEGREE AND SIDEBAND AMPLITUDE

Amplitude modulation of a sine or cosine carrier results in a variation of the carrier amplitude in proportion to the amplitude of the modulating signal. In the time domain (i.e., amplitude versus time), amplitude modulation of a sinusoidal carrier by another sinusoid would appear as in Figure 1A. The mathematical expression for this complex wave shows that it is the sum of three sinusoids of different frequencies. One of these has the same frequency and amplitude as the unmodulated carrier. The second sinusoid is at a frequency equal to the sum of the carrier frequency and the modulation frequency; this component is called the upper sideband. The third sinusoid is at a frequency equal to the carrier frequency minus the modulation frequency; this component is called the lower sideband. These two sideband components have equal amplitudes; the amplitude is proportional to the amplitude of the modulating signal. Figure 1B shows the carrier and sideband components of the amplitude modulated wave of Figure 1A as they would appear in the frequency domain (i.e., amplitude versus frequency).

A measure of the amount of modulation is m, the **degree of modulation.** Usually this is expressed as a percentage, called the percent modulation. In the time domain, degree of modulation for sinusoidal modulation is calculated as in Figure 2A.

$$m = \frac{E_{max} - E_c}{E_c}$$

Since the modulation is symmetrical:

$$E_{\max} - E_c = E_c - E_{\min}$$

and

$$\frac{E_{\max} + E_{\min}}{2} = E_c$$

From this, it is easy to show that:

$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} \quad \text{for sinusoidal modulation}$$

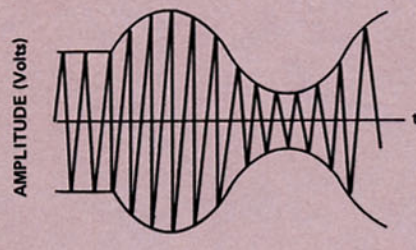


Figure 1A. Time domain (oscilloscope) display of an amplitude modulated carrier.

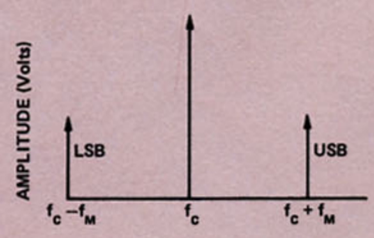


Figure 1B. Frequency domain (spectrum analyzer) display of an amplitude modulated carrier.

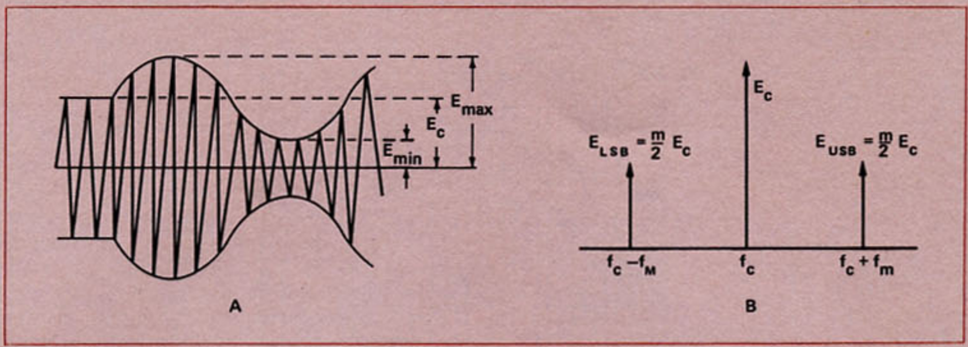


Figure 2. Calculation of degree of amplitude modulation from time domain and frequency domain displays.

When all three components of the modulated signal are in phase, they add together linearly, and form the maximum signal amplitude E_{max} (Figure 2):

$$E_{max} = E_c + E_{USB} + E_{LSB}$$

$$m = \frac{E_{max} - E_c}{E_c} = \frac{E_{USB} + E_{LSB}}{E_c}$$

and, since $E_{USB} = E_{LSB} = E_{SB}$

$$m = \frac{2\ E_{SB}}{E_c}$$

For 100% modulation ($m = 1.0$), the amplitude of each sideband will be one-half of the carrier amplitude (voltage). Thus, each sideband will be 6 *dB* less than the carrier, or one-fourth of the power of the carrier. Since the carrier component is not changed with amplitude modulation, the total power in the 100% modulated wave is 50% higher than in the unmodulated carrier.

Although it is easy to calculate modulation percentage M ($= m \cdot 100\%$) from a linear presentation in frequency or time domain, the logarithmic display on the spectrum analyzer offers some advantages, especially at low modulation percentages. Due to the high dynamic range of up to 70 *dB* the spectrum analyzer allows accurate measurements of modulation percentage M as low as 0.06%. This can easily be seen in Figure 3, where $M = 2\%$; i.e., the sideband amplitudes are only 1% of the carrier amplitude. Figure 3A shows a time domain display of an amplitude modulated carrier with $M = 2\%$. It is difficult to measure M on this display. Figure 3B shows the signal in frequency domain with the logarithmic display. The sideband amplitudes can easily be measured in *dB* below the carrier and then converted into M. (Vertical 10 *dB*/Div.)

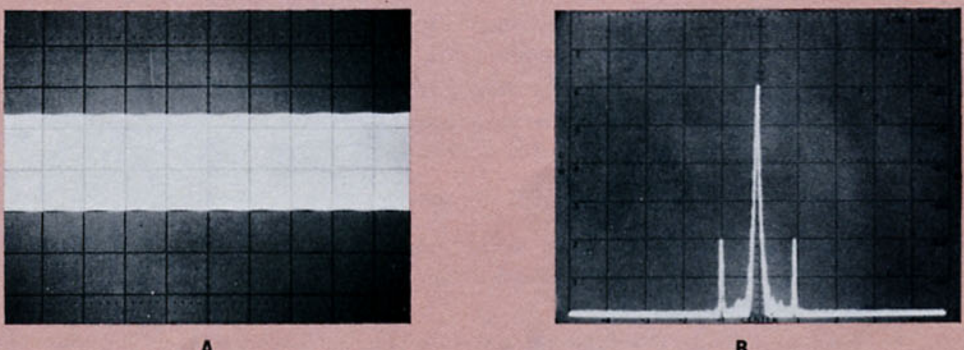


Figure 3. Time and frequency domain views of low level (2%) AM.

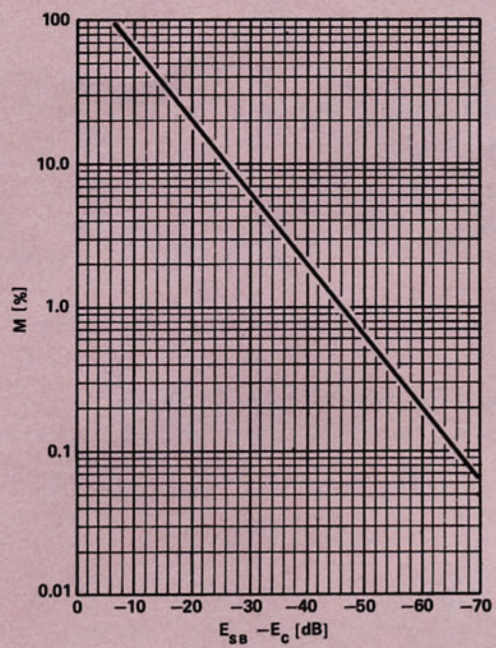


Figure 4. Modulation percentage *M* vs. Sideband level (Log Display).

The relationship between m and the logarithmic display can be expressed as:

$$E_{SB(dB)} - E_{c(dB)} = 20 \log \frac{m}{2}$$

or

$$E_{SB(dB)} - E_{c(dB)} + 6dB = 20 \log m$$

Figure 4 allows an easy conversion of a log display (lin dB scale) into modulation percentage M.

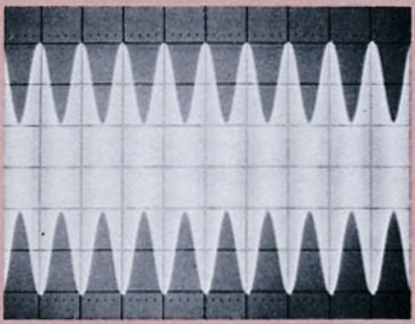
Figure 5 and 6 show typical displays of a carrier modulated by a sine wave at different modulation levels in time and frequency domain.

Using the analyzer as a manually tuned receiver ("zero scan"), we can recover the amplitude modulation of a transmitter by tuning the signal into the center of the IF bandpass filter and display the detector output in time domain on the analyzer's CRT. In this operation mode it is important to select an IF bandwidth which is at least twice the highest modulation frequency of interest. Also the analyzer must be operated in linear mode. Due to the slow scan speed used in this application, a variable persistence display (storage) is very helpful. Figure 7 shows an example.

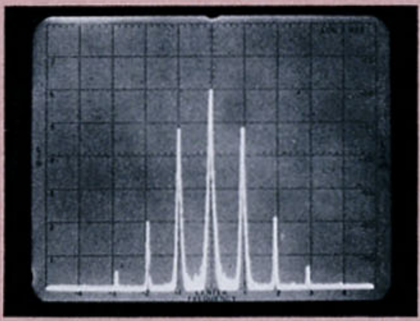
Using the formula $m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$, the maximum modulation degree during a ten-second time interval is calculated to $m = 0.83$ or $M = 83\%$.

Note: Figure 7 shows the variations of the modulating signal, not the modulation envelope of an RF carrier.

We also can listen to the demodulated signal by connecting a (high impedance) set of headphones to the "Vertical Output."

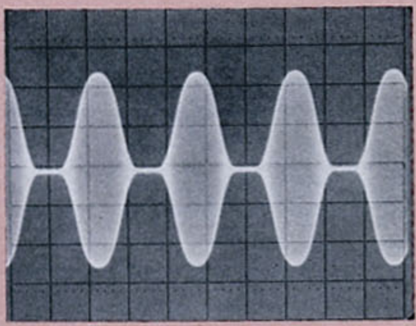


5A

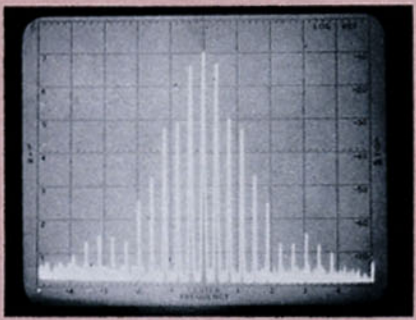


5B

Figure 5. **A** shows a time domain photograph of an amplitude modulated carrier. The percent modulation is: $M = (6-2)/(6+2) = 4/8 = 50\%$. (Scope calibration 0.1 msec/division, 50 *mV*/division.) The same waveform is measured in the frequency domain in *B*. Since the carrier and sidebands differ by 12 *dB*, $M = 50\%$. Frequency scan is 10 *kHz*/division centered at 60 *MHz*, and the log reference level is +10 *dBm*. You can also measure the 2nd and 3rd harmonic distortion on this waveform. 2nd harmonic sidebands at $f_c \pm 2 f_m$ are 28 *dB* down.



6A



6B

Figure 6. **A** shows an overmodulated ($M > 100\%$) 30 *MHz* signal in time domain, $f_m = 2$ *kHz*. (0.2 *ms*/Div, 50 *mV*/Div). The carrier is cut off at the modulation minima. **B** is the frequency domain display of the signal. Note that the first sideband pair is only 4 *dB* lower than the carrier. Also the occupied bandwidth is much greater because of severe distortion of the modulating signal. (5 *kHz*/Div, 10 *dB*/Div, *BW* 100 *Hz*.)

Figure 5.

Figure 6.

Figure 7. Picture of a voice modulated AM transmitter, Zero Scan, IF bandwidth 10 *kHz*, Display Sweep Time 1 sec/Div, Linear Display.

SPECIAL FORMS OF AMPLITUDE MODULATION

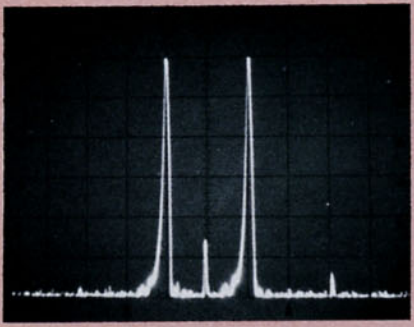
We know that a change in the degree of modulation of a particular carrier does not change the amplitude of the carrier component itself. It is the amplitude of the sidebands that is changed, thus changing the amplitude of composite wave. Since the amplitude of the carrier component does not change, all the transmitted information must be contained in the sidebands. Therefore, the rather considerable power transmitted in the carrier is essentially wasted. For improved power efficiency, the carrier component may be suppressed (usually by the use of a balanced modulator circuit), so that the transmitted wave consists only of the upper and lower sidebands. This type of modulation is called Double Sideband-Suppressed Carrier, or DSB-SC. The carrier must be reinserted at the receiver, however, to recover this type of modulation. In the time and frequency domains, DSB-SC modulation appears as in Figure 8.

Single Sideband

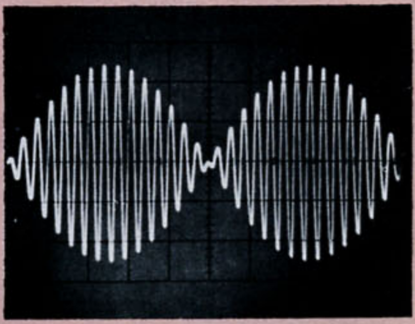
For today's communication, the most important type of amplitude modulation is single sideband with suppressed carrier (SSB). Either the upper or lower sideband can be transmitted, giving either SSB-USB or SSB-LSB. (The SSB prefix may also be omitted.) Since each sideband is displaced from the carrier by the same frequency, and the two sidebands have equal amplitudes, it follows that any information contained in one must also be in the other. Eliminating one of the sidebands cuts the power requirements by half and also halves the transmission bandwidth (frequency spectrum width) required to transmit the signal. This is essential for long range communication links in the crowded short-wave bands.

SSB is also used extensively throughout the telephone systems to combine many separate messages into a composite signal (baseband) by frequency multiplexing. By this method, up to several thousand 4 *kHz* wide channels containing voice, routing signals, and pilot carriers are combined. This composite signal can either be sent directly via coaxial lines or used to modulate microwave link transmitters.

The SSB signal is commonly generated at a fixed frequency by filtering or by phasing techniques. This necessitates mixing and amplification to get the desired transmitting frequency and output power. These stages following the SSB generator must be extremely linear to avoid any distortion of the signal resulting in unwanted in-band and out-of-band intermodulation products which can introduce severe interference in adjacent channels.



Vertical: 10 *dB*/Div.
Horizontal: 1 *MHz*/Div.



Vertical: Uncalibrated.
Horizontal: 100 *ns*/Div.

Figure 8. Frequency and time domain presentations of balanced modulator output. Note suppression of carrier (> 40 *dB*). (Inputs: $fc = 50$ *MHz* at 5 *mW*, $fm = 1$ *MHz* at 140 *mV*.)

Thus intermodulation measurements have become a vital requirement for designing, manufacturing, and maintaining multi-channel communication networks. The most commonly used type of measurement is the so-called "Two Tone Test". Two sine wave signals in the audio frequency range (300–3100 *Hz*) with low harmonic content and a few hundred hertz apart are used to modulate the SSB generator. The output of the system is then examined for intermodulation products with the aid of a selective receiver. The spectrum analyzer displays all intermodulation products simultaneously, thus decreasing measurement and alignment time substantially.

Figure 9 shows an intermodulation test of an SSB transmitter.

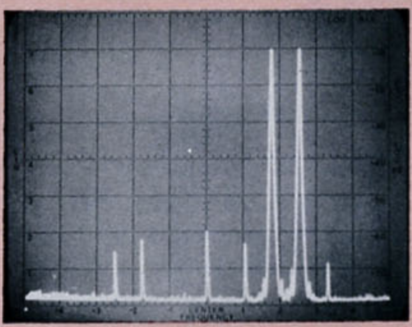


Figure 9A. SSB generator modulated with 2 sine wave signals of 1800 and 2600 *Hz*. The 20 *MHz* carrier (Display Center) is suppressed 50 *dB*, lower sideband signals and intermodulation products are more than 50 *dB* down. Log ref −30 *dBm*. Scan 1 *kHz*/Div, 10 *dB*/Div, Bandwidth 30 *Hz*.

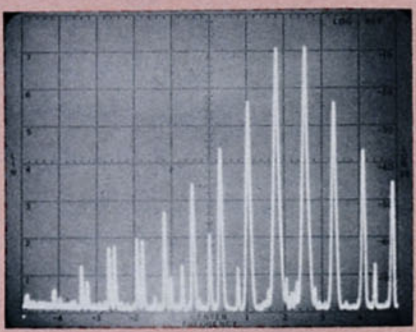


Figure 9B. Same signal after passing through an amplifier with 1 *dB* compression. Intermodulation products are penetrating into the suppressed sideband. Log ref +10 *dBm*. Scan 1 *kHz*/Div, 10 *dB*/Div, Bandwidth 30 *Hz*.

CHAPTER 3
ANGULAR MODULATION

DEFINITIONS

The two basic kinds of angular modulation are frequency modulation and phase modulation. A basic equation applicable to both kinds of modulation is:

$$\Delta\phi_{\text{peak}} = m = \frac{\Delta f_{\text{peak}}}{f_m}$$

where:

$\Delta\phi_{\text{peak}}$ = peak phase deviation of carrier (radians)

m = modulation index

Δf_{peak} = peak frequency deviation of carrier (Hz)

f_m = modulation frequency (Hz)

Note that none of these terms, as defined, contain any reference to the amplitude of the modulating signal. Definitions of the two kinds of angular modulation relate the proportionality of the amplitude of the modulating signal to either the phase or frequency characteristics of the carrier, as follows:

Frequency Modulation: The instantaneous frequency deviation of the modulated carrier with respect to the frequency of the unmodulated carrier is proportional to the amplitude of the modulating signal. (The ratio of the peak frequency deviation of the carrier to the frequency of the modulating signal is m, the modulation index. The modulation index is equal to the peak phase deviation of the carrier in radians.)

Phase Modulation: The instantaneous phase deviation of the modulated carrier with respect to the phase of the unmodulated carrier is proportional to the amplitude of the modulating signal. (The peak phase deviation of the carrier, in radians, is the modulation index. As in frequency modulation, the modulation index is equal to the ratio of the peak frequency deviation of the carrier to the frequency of the modulating signal.)

It should be noted that the definitions of these forms of modulation are independent of the modulating frequency. In each case, the modulated property of the carrier is deviated in proportion to the instantaneous amplitude of the modulating signal, regardless of the rate at which that amplitude may be changing. In angular modulation, however, the frequency of the modulating signal is important and appears in the expression for modulation index.

Comparing the basic equation with the definitions of both modulation forms, we can find two essential facts:

1. A carrier sine wave modulated with a single sine wave of constant frequency and amplitude will give the same resultant signal properties (i.e., the same spectrum display) for frequency and phase modulation. A distinction in this case can be made only by a direct comparison of the signal with the modulating wave as shown in Figure 10.
2. Phase modulation can generally be converted into frequency modulation by choosing the frequency response of the modulator so that its output voltage is proportional to $1/f_m$ (integration of the modulating signal).

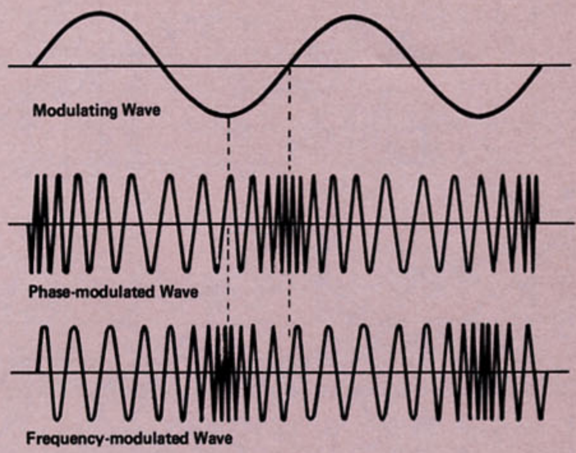


Figure 10. Phase and frequency modulation of a sine wave carrier by a sine wave signal.

Because phase modulation can be applied in an amplifier stage of a transmitter, a very stable crystal-controlled oscillator can be used. Thus, "indirect FM" is commonly used in VHF and UHF communication stations where high stability of the carrier frequencies are required.

It can also be seen that the amplitude of the modulated signal always stays constant, regardless of modulation frequency and amplitude. The modulating signal adds no power to the carrier as it does with AM.

A mathematical treatment shows that, in contrast to AM, an angular modulation of a sine wave carrier with a single sine wave yields an infinite number of sidebands spaced by the modulation frequency, f_m. For a distortion-free detection of the modulating signal, all sidebands must be transmitted. The spectral components (including the carrier component) change their amplitudes when m is varied. The sum of the squares of these components always yields a composite signal with an average power which is constant and equal to the average power of the unmodulated carrier wave.

The curves of Figure 11 show the relation (Bessel function) between the carrier and sideband amplitudes of the modulated wave as a function of the modulation index m. Note that the carrier component J_0 and the various sidebands J_n go to zero amplitude at specific values of m.

From these curves we can get the amplitudes of the carrier and the sideband components in relation to the unmodulated carrier. As an example, we find for a modulation index of $m = 3$ the following amplitudes:

Carrier	$J_0 = 0.27$
First order sideband	$J_1 = 0.33$
Second order sideband	$J_2 = 0.48$
Third order sideband	$J_3 = 0.33$ etc.

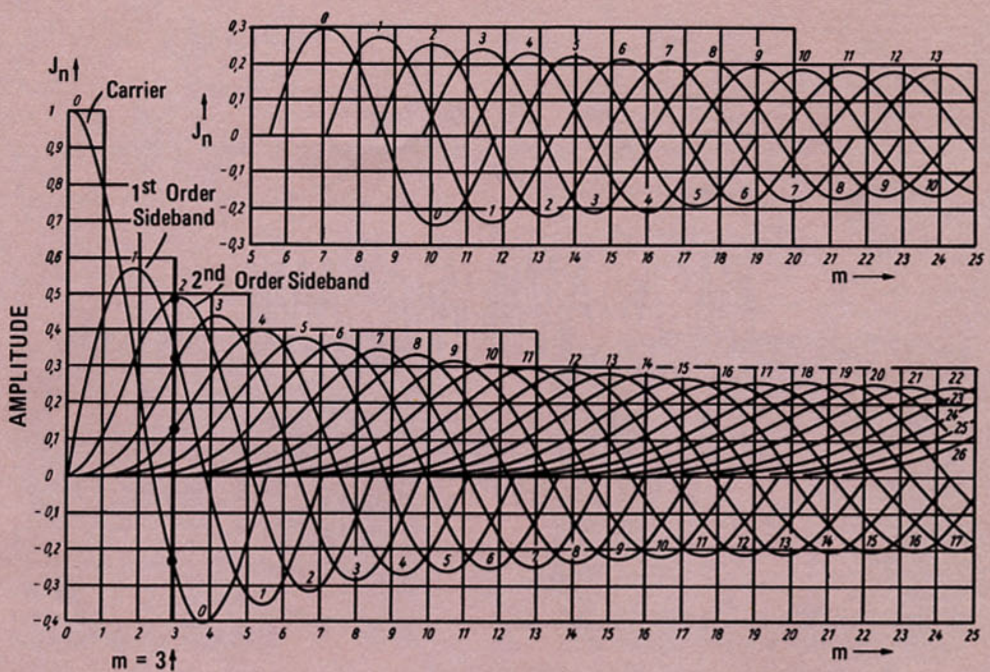


Figure 11. Carrier and sideband amplitudes for angle-modulated signals.

The sign of the values we get from the curves is of no significance since the spectrum analyzer displays only the absolute amplitudes.

The exact values for the modulation index corresponding to each of the carrier zeros are listed in Table I.

Table I. Values of Modulation Index for Which Carrier Amplitude Is Zero

Order of Carrier Zero	Modulation Index
1	2.40
2	5.52
3	8.65
4	11.79
5	14.93
6	18.07
n $(n > 6)$	$18.07 + \pi(n-6)$

BANDWIDTH OF FM SIGNALS

In practice the spectrum of an FM signal is not infinite. The sideband amplitudes become negligibly small beyond a certain frequency range from the carrier, depending on the magnitude of m. We can determine the bandwidth required for a low distortion transmission by counting the number of significant sidebands. With the word "significant" we usually mean all those sidebands which have a voltage at least 1 percent (-40 dB) of the voltage of the unmodulated carrier.

We will now investigate the spectral behavior of an FM signal for different values of m. In Figure 12 we see the spectra of a signal for $m = 0.2, 1, 5, 10$. The sinusoidal modulating signal has the constant frequency f_m, so that m is proportional to its amplitude. In Figure 13 the amplitude of the modulating signal is held constant and, therefore, m is varied by changing the modulating frequency.

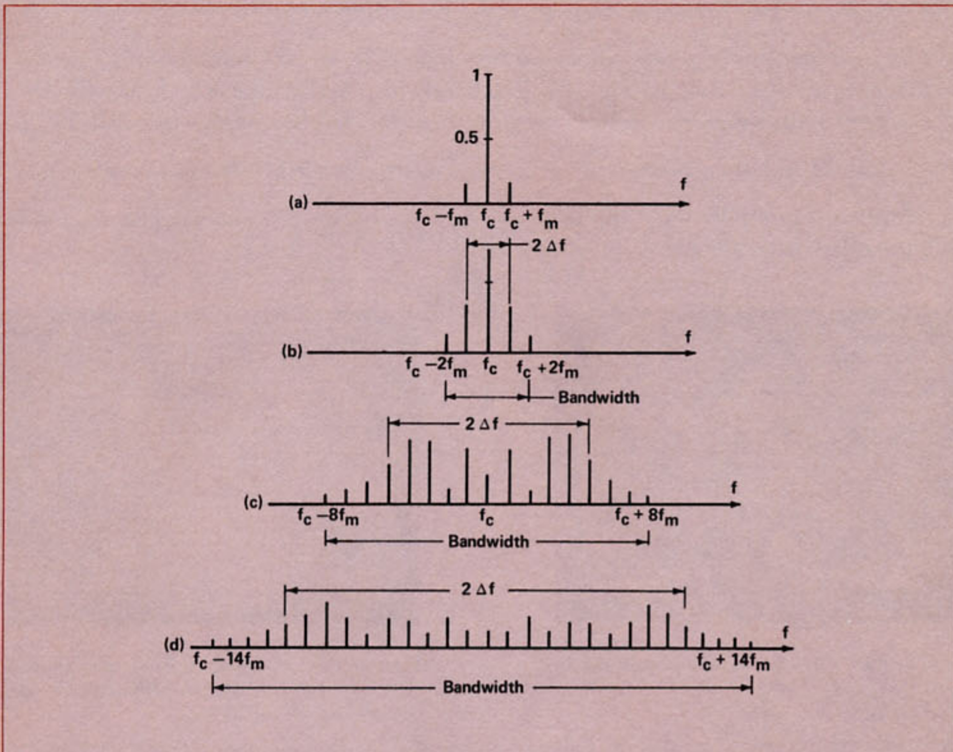


Figure 12. Amplitude-frequency spectrum, FM signal (sinusoidal modulating signal, f_m fixed, amplitude varying). (*a*) $m = 0.2$; (*b*) $m = 1$; (*c*) $m = 5$; (*d*) $m = 10$.

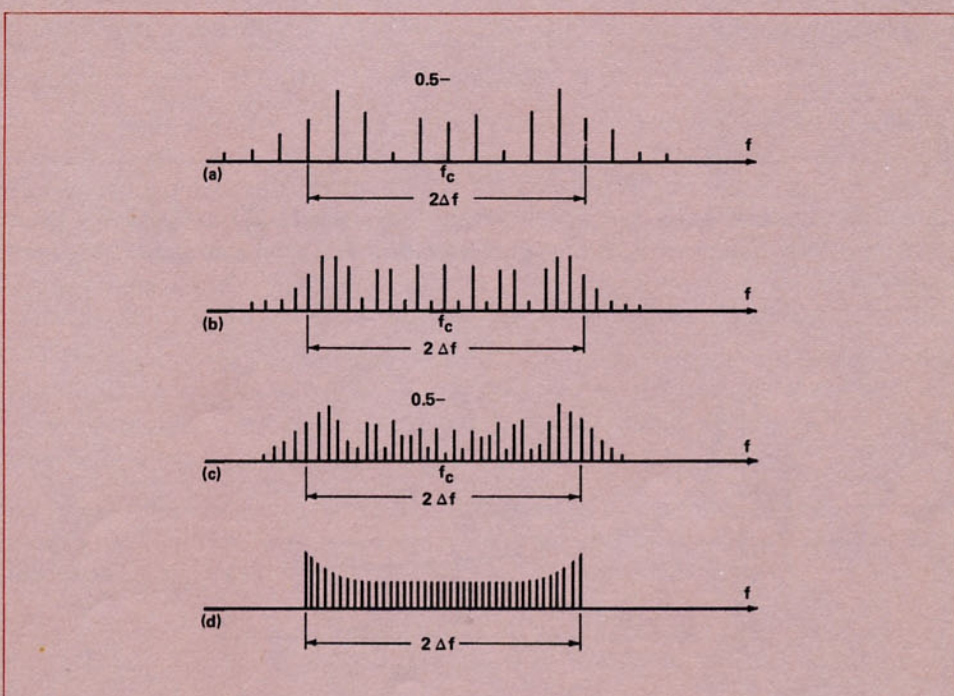


Figure 13. Amplitude-frequency spectrum, FM signal (amplitude of Δf fixed, f_m decreasing. (*a*) $m = 5$; (*b*) $m = 10$; (*c*) $m = 15$; (*d*) $m \rightarrow \infty$.

We can see two important facts from the preceding pictures:

1. For very low modulation indices (m less than 0.2), we get actually only one significant pair of sidebands. The required bandwidth in this case is $2 \cdot f_m$ as with AM.
2. For very high modulation indices (m more than 100), the bandwidth equals $2 \cdot \Delta f_{peak}$.

For values of m between these margins we have to count the significant sidebands.

Figures 14 and 15 show the analyzer displays of two FM signals, one with $m = 0.2$, the other with $m = 95$.

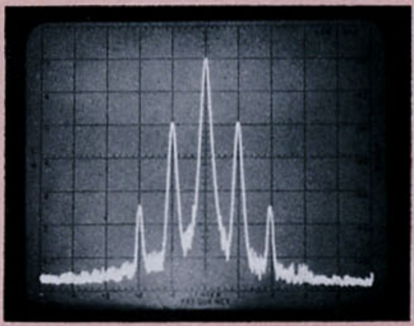


Figure 14. 50 *MHz* carrier modulated with $f_m = 10$ *kHz* and $m = 0.2$. 10 *kHz*/Div, 10 *dB*/Div, $B = 1$ *kHz*.

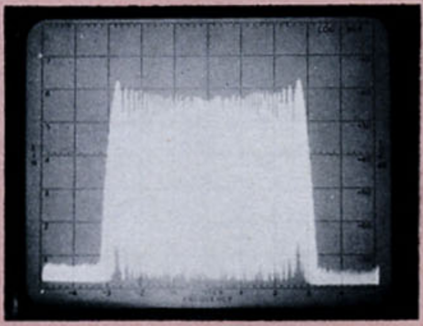


Figure 15. 50 *MHz* carrier modulated with $f_m = 1.5$ *kHz* and $m = 95$. 50 *kHz*/Div, 10 *dB*/Div, $B = 100$ *Hz*.

Figure 16 shows the bandwidth requirements for a low distortion transmission in relation to m:

For voice communication a higher degree of distortion can be tolerated; i.e., all sidebands with less than 10% of the carrier voltage (−20 *dB*) can be neglected in this case. We can calculate the necessary bandwidth B with the approximation:

$$B = 2\Delta f_{peak} + 2 f_m$$

or

$$B = 2 f_m (1 + m)$$

All the above discussion on FM sideband frequencies and bandwidth has been based on a single sine wave as the modulating signal. The extension to more complex

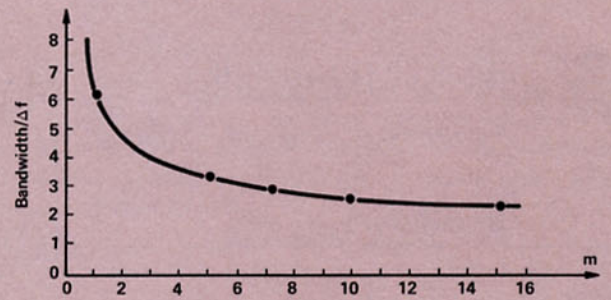


Figure 16. Bandwidth requirements vs. modulation index, m.

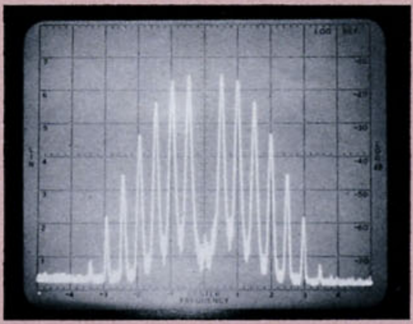


Figure 17. This is the spectrum for an FM signal at 50 *MHz*. The deviation has been adjusted for the first carrier null. f_m is 10 *kHz*, therefore, $\Delta f_{peak} = 2.4 \times 10\ kHz = 24\ kHz$. 20 *kHz*/Div, 10 *dB*/Div, $B = 1\ kHz$.

and more realistic modulating signals is very difficult. However, the single tone method can provide useful information as the following example shows:

An FM broadcast station has a maximum frequency deviation (which is determined by the maximum amplitude of the modulating signal) of $\Delta f_{peak} = 75\ kHz$. The highest modulation frequency $f_m = 15\ kHz$. This yields a modulation index $m = 5$, and the resulting signal will have eight significant sideband pairs. Thus the required bandwidth can be calculated to: $2 \times 8 \times 15\ kHz = 240\ kHz$. For modulation frequencies below 15 *kHz* (same amplitude assumed), the modulation index increases above 5 and the bandwidth eventually approaches $2\ \Delta f_{peak} = 150\ kHz$ for very low modulation frequencies.

Thus we can calculate the required transmission bandwidth using the highest modulation frequency and the maximum frequency deviation Δf_{peak}.

FM MEASUREMENTS WITH THE SPECTRUM ANALYZER

The spectrum analyzer is a very useful tool for measuring Δf_{peak}, m, and for fast and accurate adjustments of FM transmitters. It is also frequently used for calibration of frequency deviation meters.

A signal generator or transmitter is adjusted to a precise frequency deviation with the aid of the spectrum analyzer using one of the carrier zeros and selecting the appropriate modulating frequency. In Figure 17, a modulation frequency of 10 *kHz* and a modulation index of 2.4 (first carrier null) necessitate a carrier peak frequency deviation of exactly 24 *kHz*. Since the modulation frequency can be easily set accurately with the aid of a frequency counter, and the modulation index is also known accurately, the frequency deviation thus generated is equally accurate.

Table II is a useful chart that provides the modulation frequency to be set on the counter for commonly used values of deviation for the various orders of carrier zeros.

Table II. List of Modulation Frequencies to Be Used to Set Up Certain Convenient FM Deviations.

Order of Carrier Zero	Modulation Index	Commonly Used Values of FM Peak Deviation										
		7.5 *kHz*	10 *kHz*	15 *kHz*	25 *kHz*	30 *kHz*	50 *kHz*	75 *kHz*	100 *kHz*	150 *kHz*	250 *kHz*	300 *kHz*
1	2.40	3.12	4.16	6.25	10.42	12.50	20.83	31.25	41.67	62.50	104.17	125.00
2	5.52	1.36	1.81	2.72	4.53	5.43	9.06	13.59	18.12	27.17	45.29	54.35
3	8.65	.87	1.16	1.73	2.89	3.47	5.78	8.67	11.56	17.34	28.90	34.68
4	11.79	.66	.85	1.27	2.12	2.54	4.24	6.36	8.48	12.72	21.20	25.45
5	14.93	.50	.67	1.00	1.67	2.01	3.35	5.02	6.70	10.05	16.74	20.09
6	18.07	.42	.55	.83	1.88	1.66	2.77	4.15	5.53	8.30	13.84	16.60

The procedure for setting up a known deviation is as follows:

1. Select the column with the appropriate deviation required, such as, for example, 250 *kHz*.
2. Select an order of carrier zero number which gives a frequency in the table that is commensurate with the normal modulation bandwidth of the generator to be tested. For example, if an audio modulation circuit is provided in the 250 *kHz* example above, it will be necessary to go to the 5th carrier zero to get a modulating frequency within the audio passband of the generator (16.74 *kHz*).
3. Set the modulating frequency to 16.74 *kHz*. Monitor the generator output spectrum on the analyzer and adjust the amplitude of the audio modulating signal until the carrier amplitude has gone through four zeros and stop when the carrier is at its fifth minimum. With the modulating frequency of 16.74 *kHz* and the spectrum at its fifth zero, then a unique 250 *kHz* deviation is being provided by the setup. The modulation meter may then be calibrated. You can make a quick check by moving to the adjacent carrier zero and resetting the modulating frequency and amplitude (i.e., 13.84 *kHz* at the sixth carrier zero in the above example).

Other intermediate deviations and modulation indexes are settable using various orders of sideband zeros but these are influenced by incidental amplitude modulation. Since it is known that amplitude modulation does not cause the carrier to change but instead puts all the modulation power into the sidebands, incidental AM will not affect the carrier zero method above.

If it is not possible or desirable to alter the modulation frequency to get a carrier or sideband null, there are other ways to obtain usable information about frequency deviation and modulation index. One method is to calculate m by using the amplitude information of five adjacent frequency components in the FM signal. These five measurements are used in a recursion formula for Bessel functions to form three calculated values of a modulation index. Averaging yields m with practical measurement errors taken into consideration. Because of the number of calculations necessary, this method is only applicable by using a computer. A somewhat easier method consists of two measurements.

First, the sideband spacing of the modulated carrier is measured by using a sufficiently small IF bandwidth (BW), thus getting the modulation frequency f_m. Second, the peak frequency deviation Δf_{peak} is measured by selection of a convenient scan width and an IF bandwidth wide enough to cover all significant sidebands. Modulation index m can then be calculated easily.

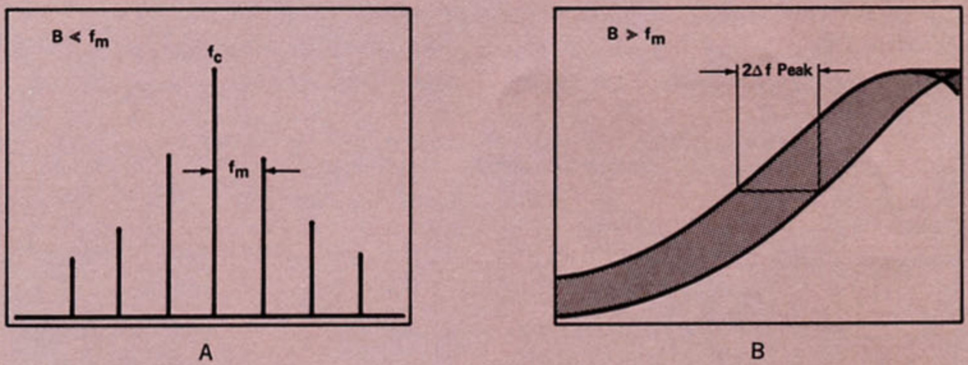
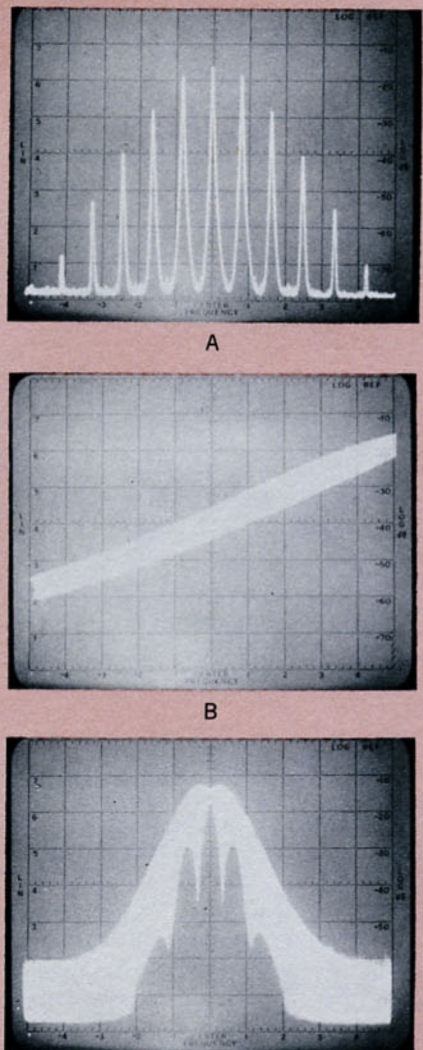


Figure 18. Measurement of f_m and Δf_{peak}.



A

Picture shows a frequency modulated carrier. Sideband spacing is measured to 8 *kHz*. 10 *kHz*/Div, 10 *dB*/Div, $B = 300$ *Hz*.

B

The peak-to-peak frequency deviation of the same signal is measured to 20 *kHz*. 10 *kHz*/Div, 10 *dB*/Div, $B = 30$ *kHz*.

C

Insufficient Bandwidth: $B = 10$ *kHz*.

Figure 19. m can be calculated to $m = \frac{20\ kHz}{2 \times 8\ kHz} = 1.25$.

Note that Figure 18B shows the peak-to-peak deviation. An example of this type of measurement is shown in the photographs, Figure 19, A and B.

Another possibility to measure Δf is to use a modified IF section of the analyzer system which has a 3 *MHz* IF output. The output signal can thus be fed into an external frequency discriminator. For this application, the spectrum analyzer must be used as a manually tuned receiver (zero scan) with sufficient bandwidth. A typical setup is shown in Figure 20.

The spectrum analyzer can also be used to monitor FM transmitters; e.g., broadcast or communication stations. Here the statistical nature of the modulation must be considered. Thus the signal must be observed for a sufficiently long time to increase the probability of catching the peak frequency deviations. This necessitates a storage display or at least a camera.

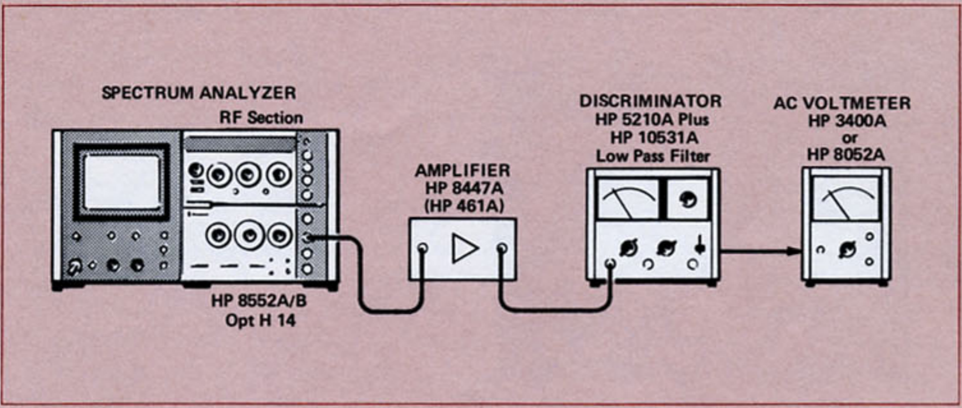


Figure 20. A typical setup.

Figures 21 and 22 show FM broadcast stations, one modulated with a mono signal, and the other with stereo multiplex. Note that in the latter case the spectrum envelope resembles an FM signal with low modulation index. This is due to the fact that the stereo modulation signal contains additional information in the frequency range of 23 to 53 *kHz*, far beyond the audio frequency limit of 15 *kHz*. Since the occupied bandwidth must not exceed the bandwidth of a transmitter modulated with a mono signal, the maximum frequency deviation of the carrier must be kept substantially lower.

Although the HP Spectrum Analyzers have no built-in FM discriminators, it is possible to recover the modulating signal with sufficient linearity by slope detection. Here the analyzer is used as a manually tuned receiver (zero scan) with sufficient IF bandwidth. But in contrast to AM the signal is not tuned into the passband center but to one slope of the filter curve as illustrated in Figure 23.

At the slope, the frequency variations of the FM signal are converted into amplitude variations (FM to AM conversion). The resultant AM signal is then detected with the envelope detector. The detector output can then be displayed in time domain and is also available at the vertical output.

A disadvantage of this method is that the detector also responds to amplitude variations of the signal.

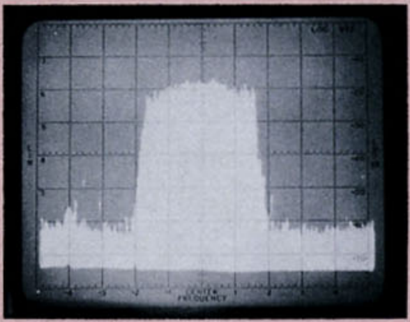


Figure 21. FM broadcast transmitter modulated with a mono signal. 50 *kHz*/Div, 10 *dB*/Div, $B = 3$ *kHz*, scan 50 *ms*/Div, approx. 50 scans.

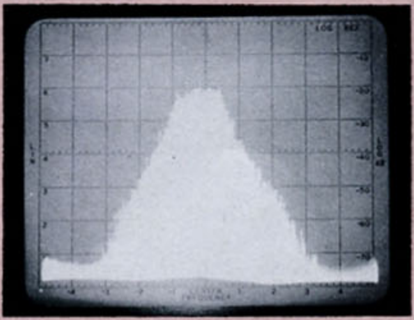


Figure 22. FM broadcast transmitter modulated with a stereo signal. 50 *kHz*/Div, 10 *dB*/Div, $B = 3$ *kHz*, scan 50 *ms*/Div, approx. 50 scans.

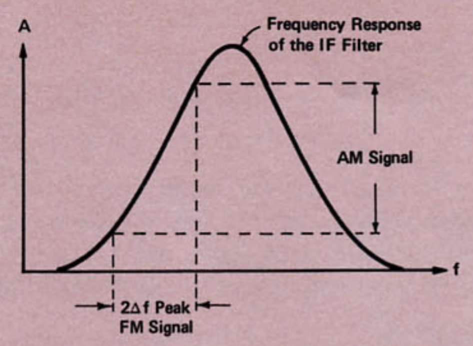


Figure 23. Slope detection of an FM signal.

AM PLUS FM (Incidental FM)

Although AM and angular modulation are different modulation methods, they have one property in common: they always produce a symmetrical sideband spectrum.

In Figure 24 we see a modulated carrier with unsymmetrical sidebands. The only way that we can have one sideband larger than the corresponding other is for both AM and FM or phase modulation to exist simultaneously and at the same modulating frequency. The reason is that the phase relations between carrier and sidebands are different for AM and angular modulation (see Appendix). Since the sideband components of both modulation types add together geometrically, the resultant amplitude of one sideband is reduced. The amplitude of the other is increased respectively. The spectrum analyzer does not retain any phase information and thus displays the absolute magnitude of the result.

For relatively low amounts of FM the modulation degree of the AM component can be calculated with acceptable accuracy by taking the average amplitude of the first sideband pair. The amount of incidental FM can only be calculated if the phase relation between the AM and FM sideband vector is known. It is not possible to measure Δf_{peak} of the incidental FM using the slope detection method, because of the simultaneously existent AM.

The best way is to use the 3 *MHz* IF output of a modified IF section (analyzer in "zero scan" mode), and to feed the output signal into a limiting amplifier to remove the AM component and measure the frequency deviation with a frequency discriminator. For higher sensitivity the frequency of the output signal can be multiplied, thus increasing the frequency deviation by the multiplication factor. However, for low deviations the residual FM of the analyzer itself has to be taken into consideration.

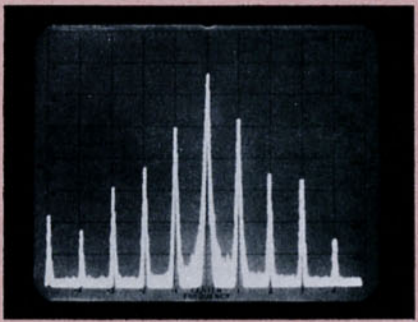


Figure 24. Pure AM or FM signals always have equal sidebands, but when the two are present together, the modulation vectors usually add in one sideband and subtract in the other. Thus unequal sidebands are an indication of simultaneous AM and FM. This CW signal is amplitude modulated 80% at a 10 *kHz* rate. The harmonic distortion and incidental FM are clearly visible.

APPENDIX I

AMPLITUDE MODULATION

A sine wave carrier can be expressed by the general equation:

$$e_{(t)} = A \cdot \cos(\omega_c t + \phi_0) \qquad (1\text{-}1)$$

In AM systems only A is varied. A basic assumption is that the modulating signal varies slowly compared with the carrier. This means that we can talk of an envelope variation or variation of the locus of the carrier peaks. The carrier, amplitude modulated with the function $f_{(t)}$, has the form (carrier angle ϕ_0 arbitrarily set to zero):

$$e_{(t)} = A\,[1 + m \cdot f_{(t)}] \cdot \cos \omega_c t \qquad (m = \text{modulation degree}) \qquad (1\text{-}2)$$

for $f_{(t)} = \cos \omega_m t$ (single sine wave) we get:

$$e_{(t)} = A\,(1 + m \cdot \cos \omega_m t) \cdot \cos \omega_c t \qquad (1\text{-}3)$$

or

$$e_{(t)} = A \cos \omega_c t + \frac{m \cdot A}{2} \cos(\omega_c + \omega_m)t + \frac{m \cdot A}{2} \cos(\omega_c - \omega_m)t \qquad (1\text{-}4)$$

We get three steady state components:

a) $A \cdot \cos \omega_c t$ Carrier

b) $\frac{m \cdot A}{2} \cos(\omega_c + \omega_m)t$ Upper Sideband

c) $\frac{m \cdot A}{2} \cos(\omega_c - \omega_m)t$ Lower Sideband

We can represent these components by three phasors rotating at different angular velocities (Figure I-1A). We assume the carrier phasor to be stationary to obtain the angular velocities of the sideband phasors in relation to the carrier phasor A (Figure I-1B).

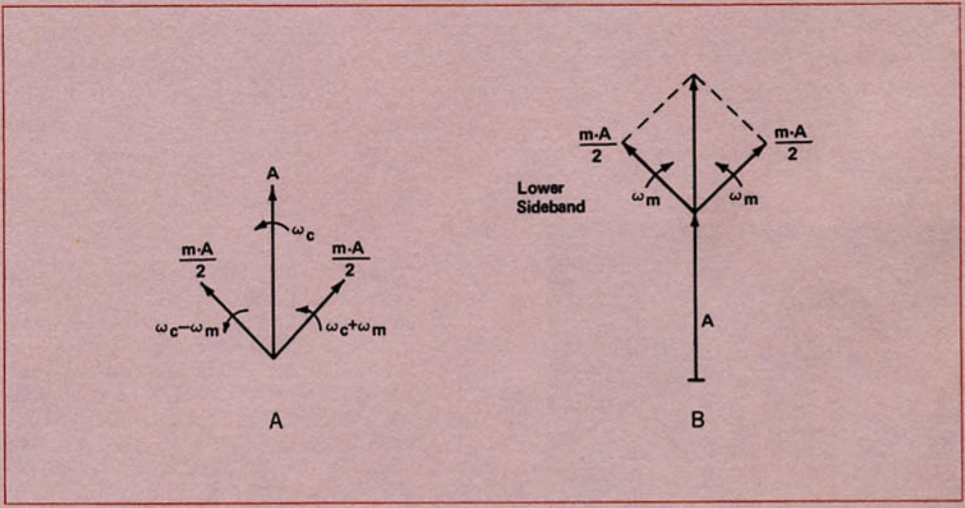


Figure I-1.

Figure I-2 shows the composition of the envelope of an AM signal by phasors:

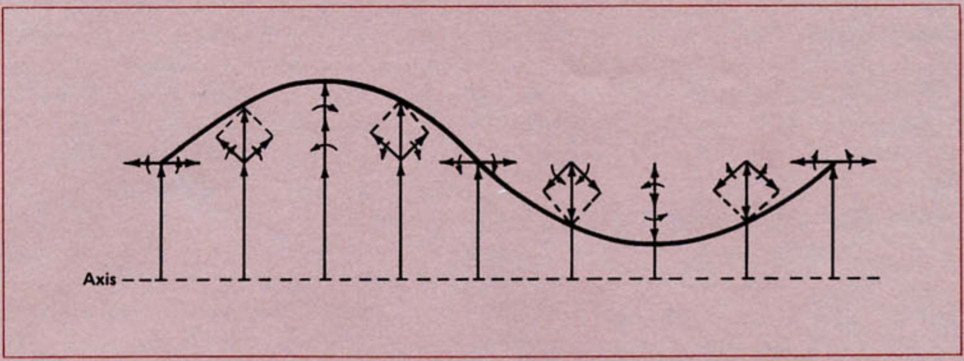


Figure I-2.

It can be observed that the phase of the vector sum of the sideband phasors is always collinear with the carrier component. It can also be seen from Eq. (1-3) and Figure I-1 that the modulation degree m cannot exceed the value of unity for linear modulation.

ANGULAR MODULATION

Definition of instantaneous frequency:
The usual expression for a sine wave of the frequency f_c is:

$$f_{c(t)} = \cos \phi_{(t)} = \cos (\omega_c t + \phi_o) \tag{2-1}$$

We define the instantaneous radian frequency ω_i to be the derivative of the angle as a function of time:

$$f_{c(t)} = \cos \phi_{(t)}$$

$$\omega_i = \frac{d\phi}{dt} \tag{2-2}$$

This agrees with the usual use of the word frequency if $\phi_{(t)} = \omega_c t + \phi_o$.

If $\phi_{(t)}$ in Eq. (2-1) is now made to vary in some manner with a modulating signal $f_{(t)}$, we call the resulting form of modulation angular modulation.

Phase and Frequency modulation are both special cases of angular modulation.

PHASE MODULATION

In particular, where

$$\phi_{(t)} = \omega_c t + \phi_o + K_i \cdot f_{(t)} \tag{2-3}$$

we vary the phase of the carrier linearly with the modulating signal. K_i is a constant of the system.

FREQUENCY MODULATION

Now we let the instantaneous frequency, as defined in Eq. (2-2), vary linearly with the modulating signal,

$$\omega_i = \omega_c + K_2 \cdot f_{(t)}$$

Then

$$\phi_{(t)} = \int \omega_i dt$$

$$= \omega_c t + \phi_o + K_2 \cdot \int f_{(t)} dt \tag{2-4}$$

In the phase modulation case the phase of the carrier varies with the modulating signal and in the FM case the phase of the carrier varies with the integral of the modulating signal. There is thus no essential difference between phase and frequency modulation. We shall use the term FM generally to include both modulation types.

For further analysis we assume a sinusoidal modulation signal at the frequency f_m:

$$f_{(t)} = a \cdot \cos \omega_m t$$

The instantaneous radian frequency ω_i is

$$\omega_i = \omega_c + \Delta\omega \text{ peak} \cdot \cos \omega_m t \qquad \Delta\omega \text{ peak} << \omega_c \tag{2-5}$$

$\Delta\omega$ peak is a constant depending on the amplitude a of the modulating signal and on the properties of the modulating system.

The phase $\phi_{(t)}$ is then given

$$\phi_{(t)} = \int \omega_i dt = \omega_c t + \frac{\Delta\omega_{\text{peak}}}{\omega_m} \sin \omega_m t + \phi_o$$

We can take ϕ_o as zero by referring to an appropriate phase reference. The frequency modulated carrier is then expressed by

$$e_{(t)} = \text{A} \cdot \cos (\omega_c t + m \cdot \sin \omega_m t) \tag{2-6}$$

$$m = \frac{\Delta\omega_{\text{peak}}}{\omega_m} = \frac{\Delta f_{\text{peak}}}{f_m} \tag{2-7}$$

m is the modulation index and represents the maximum phase shift of the carrier, Δf_{peak} is the maximum frequency deviation of the carrier.

NARROWBAND FM

To simplify the analysis of FM, we first assume that $m << \frac{\pi}{2}$ (usually $m < 0.2$):

We have

$$\begin{aligned} e_{(t)} &= \text{A} \cdot \cos (\omega_c t + m \cdot \sin \omega_m t) \\ &= \text{A} [\cos \omega_c t \cdot \cos (m \cdot \sin \omega_m t) - \sin \omega_c t \cdot \sin (m \cdot \sin \omega_m t)] \end{aligned}$$

for $m << \frac{\pi}{2}$

$$\cos (m \cdot \sin \omega_m t) \doteq 1 \qquad \text{and} \qquad \sin (m \cdot \sin \omega_m t) \doteq m \cdot \sin \omega_m t$$

thus

$$e_{(t)} = \text{A} (\cos \omega_c t - m \cdot \sin \omega_m t \cdot \sin \omega_c t) \tag{2-8}$$

Written in the sideband form:

$$e_{(t)} = \text{A} \cos \omega_c t - \frac{m \cdot \text{A}}{2} \cos (\omega_c - \omega_m)t + \frac{m \cdot \text{A}}{2} \cos (\omega_c + \omega_m)t \tag{2-9}$$

This resembles the AM case in Eq. (1-4), but with the difference that in narrowband FM the phase of the lower sideband is reversed. Thus, the resultant sideband vector sum is always in phase quadrature with the carrier.

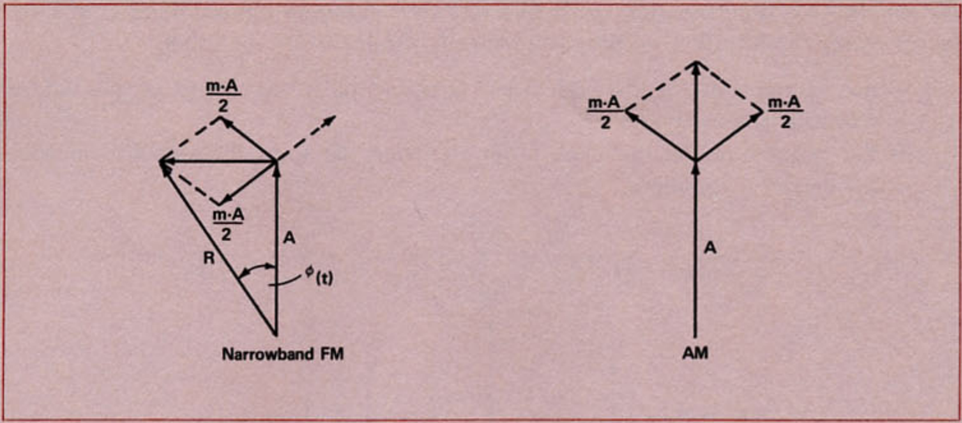


Figure I-3.

The FM case thus gives rise to phase variations with very small amplitude change ($m << \frac{\pi}{2}$), while the AM case gives amplitude variations with no phase deviation.

Figure I-4 shows the spectra of an AM and a narrowband FM signal. However, on a spectrum analyzer the FM sidebands appear as in AM because the analyzer does not retain phase information.

WIDEBAND FM

$$e_{(t)} = A \cos(\omega_c t + m \sin \omega_m t) \qquad m \text{ not small}$$

$$= A\,[\cos \omega_c t \cdot \cos(m \cdot \sin \omega_m t) - \sin \omega_c t \cdot \sin(m \cdot \sin \omega_m t)]$$

Using the Fourier series expansions:

$$\cos(m \cdot \sin \omega_m t) = J_0(m) + 2J_2(m) \cdot \cos 2\omega_m t + 2J_4(m) \cos 4\omega_m t + \ldots \qquad (2\text{-}10)$$

$$\sin(m \cdot \sin \omega_m t) = 2J_1(m) \sin \omega_m t + 2J_3(m) \sin 3\omega_m t + \ldots \qquad (2\text{-}11)$$

where $J_n(m)$ is the n^{th}-order Bessel function of the first kind, we get

$$\begin{aligned} e_{(t)} = J_0(m) \cos \omega_c t - J_1(m)\,&[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] \\ &+ J_2(m)\,[\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t] \\ &- J_3(m)\,[\cos(\omega_c - 3\omega_m)t - \cos(\omega_c + 3\omega_m)t] \\ &+ \ldots\ldots\ldots \qquad (2\text{-}12) \end{aligned}$$

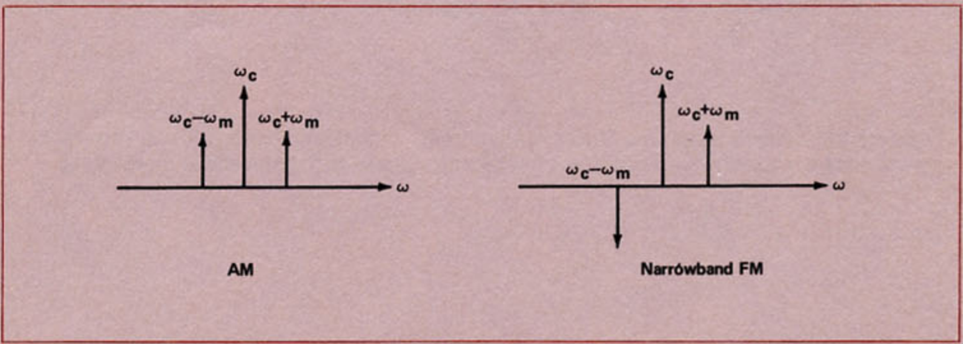


Figure I-4.

We thus have a time function consisting of a carrier and an infinite number of sidebands, whose amplitudes are proportional to $J_n(m)$. It can be seen that:

a) the vector sums of the odd order sideband pairs are always in quadrature with the carrier component.

b) the vector sums of the even order sideband pairs are always collinear with the carrier component.

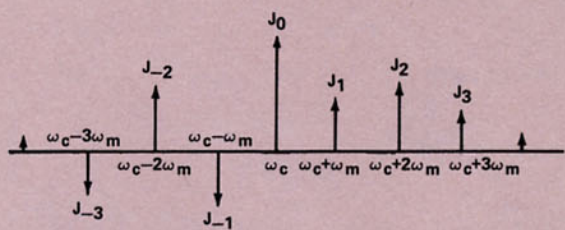


Figure I-5. Composition of an FM wave into sidebands.

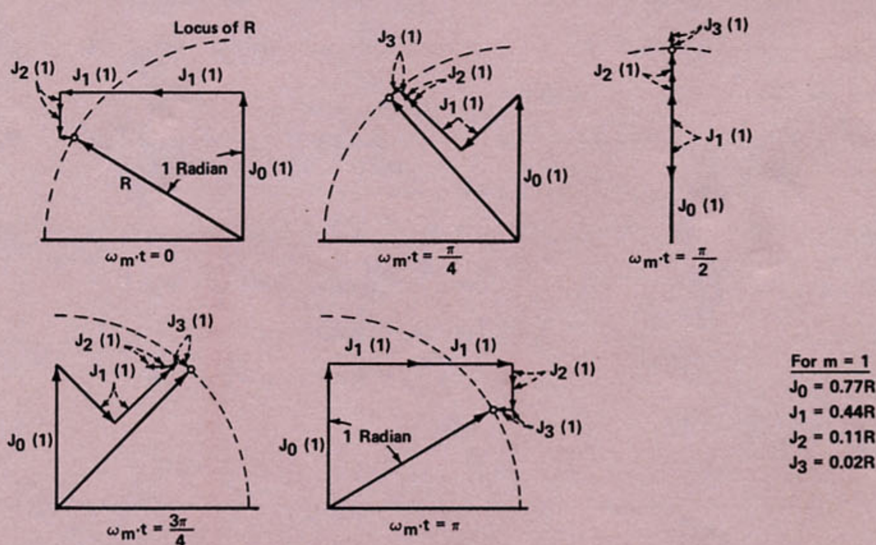


Figure I-6. Phasor diagrams of an FM signal with a modulation index $m = 1$. Different diagrams corresponding to different points in the cycle of the sinusoidal modulating wave.

HEWLETT hp PACKARD

5952-1051

PRINTED IN U.S.A.