A 1Oppm ACCURATE DIGITAL AC MEASUREMENT ALGORITHM

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ABSTRACT

Digital sampling has been used for a number of years to make specialized and general purpose RMS AC voltage measurements. An algorithm is described that can be implemented using commercially available equipment to achieve a 1 year absolute accuracy approaching 10ppm. The implementation is usable for up to 1% distorted sinewaves with frequencies below 0.01 Hz and up to 1kHz with voltages from 10mV to 700V. Also, the error analysis serves as a tutorial on the limitations of integrating voltmeters used in a digitizing application.

INTRODUCTION

A digital sampling algorithm was developed and optimized for precision low frequency AC RMS measurement using the Hewlett Packard model HP3458A voltmeter. It's accuracy at high speed, precision time base, level triggering, and frequency measurement function made it ideal for this application. While optimized for this specific voltmeter, the principles discussed herein should be transportable to other digitizing equipment.

A program written in HP BASIC implementing this algorithm is listed in the paper's appendix. Operating instructions and an address for obtaining more information and program copies are also included.

Voltmeters have been used as samplers to digitally measure low frequency AC for many years (albeit in a somewhat handicapped fashion). By definition, an RMS (Root MEAN Squared) measurement involves averaging, which for low frequencies requires large amounts of time. For periodic waveforms, this can be substantially reduced if the sampling interval is exactly one or more periods. But due to the voltmeter's timebase quantization, this can normally be done only for specific frequencies. For example, if the voltmeter can space samples at multiples of 0.1s, it's pretty easy to see that 100 samples can exactly sample 10 periods of a 1 Hz waveform. If the waveform had a frequency of 1.3Hz, however, things wouldn't be quite so simple. In general, sampling over integral number of periods can only be achieved to a precision of 1/2 the voltmeter's time quantization. This imprecision can be a source of significant measurement error.

Another source of measurement error is aliasing. A pure sinewave can be perfectly reproduced (and measured) if the sample rate is greater than twice it's frequency (Nyquist theorem). For RMS measurement, a looser restriction that allows undersampling applies. An undersampled sinewave still appears as a sampled sinewave - but at a lower frequency. The RMS operation involves squaring this aliased sinewave which generates higher frequencies that may themselves alias down to some lower frequency. Errors will manifest if the lowest aliased frequency component is inside the passband of the RMS averaging filter. With care and a pure sinewave, it's usually possible to avoid this. But what about a distorted sinewave? Distortion appears in the frequency domain as higher frequency harmonics that can each contribute alias errors when undersampled. To complicate matters further, the desire to sample over integral number of periods tends to force a sample rate that is an exact multiple of the distorted sinewave's frequency and practically guaranteeing aliasing problems with it's harmonics.

This sampling algorithm uses the DCV (DC voltage) function of the HP3458A to digitize a low frequency waveform (from less than 0.1Hz up to 1kHz) and compute it's ACV and ACDCV RMS voltage. Errors due to sampling over inexact integral multiples of the waveform's period are eliminated with an algorithm involving the HP3458A's level trigger. Sample timing is selected to reduce aliasing of higher frequency harmonics. In addition, a number of errors introduced by the voltmeter (most notably that due to time integration) are backed out.

When the program in the appendix is run, an estimate of the total measurement uncertainty is displayed. Also displayed are measurement bandwidth, the sampling parameters, and the fundamental frequency of the input signal. Next, several intermediate results are displayed and then the final answer is computed. An example is shown in figure 1.

```
SIGNAL FREQUENCY(Hz)= 99.9991047572
Number of samples in each of 6 bursts= 1070
Sample Spacing(sec)= .0008411
A/D Aperture(sec)= .0008111
Measurement bandwidth(Hz)= 616.4
SINEWAVE MEASUREMENT UNCERTAINTY(ppm)= 13
ADDITIONAL ERROR FOR 1% DISTORTION(ppm)= 8

The 6 intermediate results:
        1
        . 999975
        .999979
        .999996
        .999976
        .999978

AC RMS VOLTAGE= .999984

ACDC RMS VOLTAGE= .999984
```

Figure 1 - Typical program output (see appendix)

THEORY of OPERATION

The RMS equation is:

IN GENERAL: $T \rightarrow \infty$

$$\sqrt{\int_{t-T}^{t} \frac{V^2(t)}{T}\, dt} \qquad (1)$$

This is usually done in less than the infinite time shown above. The shortened equation is shown below where the averaging interval is T. If s(t) is periodic of period T then the result is exactly the same as T=infinity above.

IF PERIODIC:

$$\sqrt{\int_{t-NT}^{t} \frac{V^2(t)}{NT}\, dt} \qquad (2)$$

T= PERIOD, N INTEGER

If s(t) is a sinewave (sin(wt)), the above equation reduces to:

$$\sqrt{\int_{t-T}^{t} \frac{\sin^2(2\pi Ft)}{T}\, dt} \qquad (3)$$

Expressed as an error from the ideal result of 1/SQR(2), the above is approximately:

!--scale factor--! !--time varying ripple--!
Err= sin(wT)/(2wT) * sin(2wt) **(4)**

This equation is bounded by the scaling factor 1/(4*PI*n) where n is the number of periods that are averaged over.

If T was exactly an integral number of periods of the input signal the above Err would be exactly zero. In practice, this can only be done with some uncertainty. Call this uncertainty dt. Then the scaling factor in the above equation reduces to:

Err= sin(w*Tperfect+wdt)/(2wT)= approx.= dt/(2wT)= dt/(2T) **(5)**

The algorithm uses the HP3458A to attempt to take a burst of Num samples spaced Tsamp apart where Num*Tsamp is an integral multiple of periods. The sample spacing of the HP3458A is quantized at 100ns. Therefore the sample spacing Tsamp can deviate from the ideal value needed by as much as 50ns. dt above then accumulates as the number of samples increases.. T also increases as Num increases so the error term becomes:

Err= dt/(2T)= 50ns*Num/(2*Tsamp*Num)= 50ns/(2*Tsamp) **(6)**

This error is reduced still further if Num is large. The timing error (50ns*Num) can not increase forever. Num is selected so that whatever the timing error of Tsamp, the worst case deviation from the ideal of Tsamp*Num-integral periods is Tsamp/2. Therefore the error is bounded by:

Err= dt/(2T)= (Tsamp/2)/(2*Tsamp*Num)= 1/(4*Num) **(7)**

The actual error term is therefore bounded by the smaller of:

Err= smaller of [50ns/(2*Tsamp) or 1/(4*Num)] **(8)**

The algorithm first reduces ripple by reducing the scale factor of equation 4. This is achieved by selecting Num*Tsamp as close to ideal as possible so that the resultant error is that expressed by equation 8. The next level of reduction is done by synchronizing the burst of Num samples off of the zero crossing of the input waveform. The internal level trigger in the HP3458A is used to start the burst an amount of time equal to Delay from the zero crossing of the input signal. Multiple measurements will be identical without a time varying "ripple". The ripple component is "frozen" in time at a value of:

!---Frozen in time--!
Ripple= sin(wT)/(2wT) * sin(2*w*Delay) Note: w = 2*PI*Freq **(9)**

Now, these level triggered bursts are repeatably stable, but they still have an error equal to the "frozen" ripple. If multiple bursts of Num samples are taken with different values of Delay, it can be seen in equation 9 that differing values of ripple will be created. It can be conceptualized that the ripple term is being sampled in equivalent time at the difference of the different Delay values. If one ripple period is sampled at two or more equally spaced points, it is possible to average out the frozen ripple. For example, if 4 bursts are taken with each burst delayed relative to the previous by 1/(4*Freq), the resulting errors will cancel when the measurements are averaged. Figure 2 illustrates this technique.

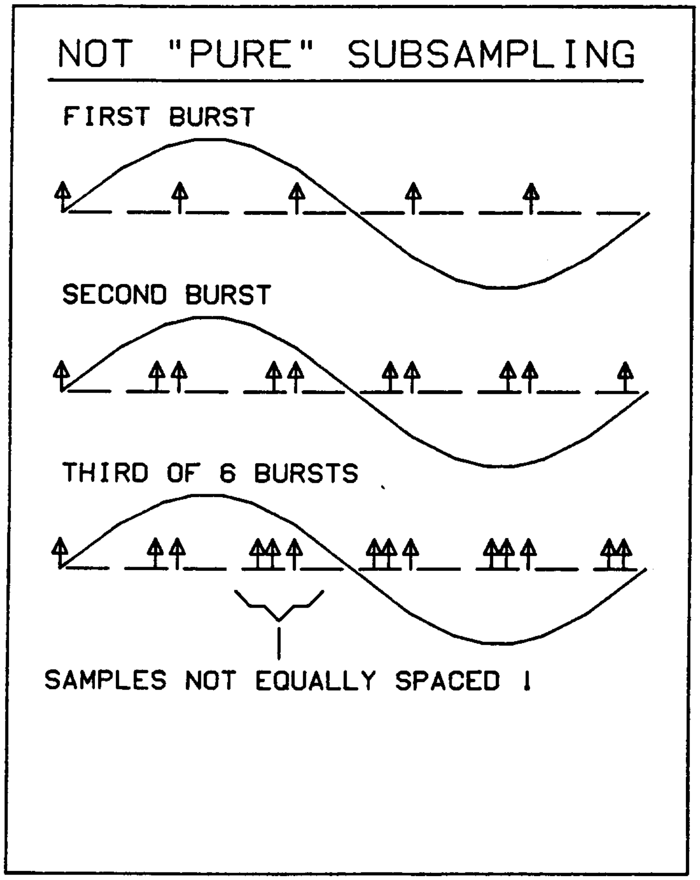

Figure 2 - Multiple bursts to average "frozen" ripple

The main ripple frequency in equation 9 is 2*Freq. Nyquist theory requires that at least 2 samples per period are needed for full characterization. Therefore the maximum value of Delay is 1/(4*Freq) and the minimum value for the number of bursts is 4 if the ripple waveform is to be sampled over one period (1/Freq). If the input signal isn't a pure sinewave, then the ripple will have higher frequency components which require smaller delays and larger number of bursts if aliasing is to be avoided. It can be shown that 6 bursts will average out 2nd harmonic distortion and that 8 bursts will average up to the 3rd harmonic. Since the algorithm is restricted to sinewaves of less than 1% distortion, the ripple due to harmonics is small relative to the main ripple defined in equation 8. And because for many waveforms energy decreases at higher frequencies, it may be appropriate to ignore these errors. Under this situation, 6 or 8 bursts are adequate. The following derivation illustrates this point. If the input signal is a sinewave with 1% distortion at the 3rd harmonic (D=.01), the RMS equation (3) becomes:

$$\sqrt{\int_{t-T}^{t}\frac{(\sin(2\pi Ft)+D\sin(6\pi Ft))^2}{T}\,dt} \qquad (10)$$

It reduces (sin(w1t)sin(w2)= cos((w1-w2)t)/2 - cos((w1+w2)t)/2) to :

$$\sqrt{\frac{1}{T}\int_{t-T}^{t}\left[\frac{1-\cos(4\pi Ft)}{2}+D\cos(4\pi Ft)-D\cos(8\pi Ft)\right]dt} \qquad (11)$$

Reducing to a form similar to equation 4, equation 11 becomes:

Err= Dsin(wT)/wT * sin(2wt) - Dsln(2wT)/wT * sin(4wt) **(12)**

A similar form of equation 8 can be derived for the 4*Freq ripple in equation 12. The ripple error is the smaller of:

Err= smaller of [2*D*50ns/Tsamp or D/Num **]** **(13)**

The purpose of equation 12 is to show that third harmonic distortion generates ripple components of 2*Freq and 4*Freq and that this ripple is much smaller than the main ripple. Equation 13 shows that with 1% distortion (D=.01), the magnitude of this ripple with a sample time of 1ms is only 1ppm. This is the maximum error that would exist if this distortion term was undersampled and aliased. To summarize then, under most circumstances, there isn't much need for the number of bursts to be any higher than 4 or 6.

INTEGRATING VOLTMETER LIMITATIONS and CORRECTIONS

The actual program includes various enhancements to the theory described above. Most are means to compensate for various deficiencies of the HP3458A.

Reference is made to the program listing contained in the appendix.

A/D aperture correction:

The DCV function of the HP3458A uses an integrating analog to digital converter (A/D) that integrates the input signal over a very specific time aperture. This aperture can be selected between 5OOns and Is with 1OOns quantization and it's time accuracy is basically the accuracy of the crystal clock used to control the A/D (0.01%). In the time domain, a waveform is integrated over the aperture as defined by the following equation:

INTEGRATING A/D IN TIME DOMAIN:

$$\int_{t-Aper}^{t} V(t)/Aper \, dt \qquad \textbf{(14)}$$

The Fourier transform of equation 14 is sometimes a more convenient means for analyzing the A/D's behavior. This behavior is expressed in the frequency domain by a sinc function. The error relative to perfect sampling is shown below:

X= PI*Aper*Freq Err= sin(X)/X -1 **(15)**

Notice that at DC (X=0), the error is zero and at X=n*PI the error is exactly -100%. This behavior is very desirable for a DC voltmeter since common interference due to power line and power line harmonic pickup is rejected if the aperture is selected to be an integer multiple of the power line period. Of greater concern to this article is the fact that this error is very repeatable and even large errors can be corrected with high precision. For example, an integrating AN with a 1ms aperture sampling a 10OHz sinewave will introduce an error of -16368ppm in an RMS measurement. Since the A/D's aperture and the frequency of the measured waveform are known, equation 15 can be used as a correction factor to the computed RMS value of the sampled waveform. For the above example, the -16368ppm error can be corrected with an uncertainty of less than 3ppm. Referring to the appendix, line 830 to 1140 of the program show how this correction occurs.

For distorted sinewaves this correction is not perfect, however, since the distortion harmonics are attenuated more by the A/D's aperture than the fundamental frequency. The correction calculated for the fundamental frequency will then be insufficient for the harmonics. By limiting the distortion to some value (under 1% for example), the correction uncertainty. becomes quantifiable.

Frequency Measurement:

The input signal's frequency is measured by the HP3458A and used as an input to an algorithm that computes sample time (Tsamp) and the number of samples in a burst (Num). Ideally, the samples are picked such that Num*Tsamp is an integral multiple of (1/Freq) with an uncertainty implied by equation 8. However, the voltmeter's time base uncertainty could lead to a larger error. The HP3458A's data sheet shows it's time base error as 0.01% which is a reflection of the accuracy of an internal crystal clock. For example, 10000 samples programed to be spaced 1ms apart may actually take 10.001s to complete instead of the ideal 10s. If the desire was to exactly sample 10 periods of a 1Hz sinewave, an unanticipated error of 1ms may occur.

Because the same clock used for sample timing is also used to set the gate time in the internal frequency measurement, in principle this 0.01% time base error is invisible. For the above example where the timebase is off by 0.01%, the 1Hz sinewave will be measured as 1.OOO1Hz. Then only 9999 samples will be picked for Num and the total sample time for the Num samples will be the ideal 10s instead of 10.001s. Exactly 10 periods of the 1Hz sinewave will be sampled. This won't happen automatically, however. When the HP3458A is calibrated, the FREQUENCY function of the HP3458A is compared to an external frequency standard. A correction constant is stored in permanent memory and used to scale raw frequency measurements (but not the sample timings). Therefore a frequency measurement won't show the same errors as the time base unless there is some way to "uncalibrate" the voltmeter. This is done by querying the FREQUENCY calibration constant and backing it out of the frequency measurement. The function FNFreq (line 1470) illustrates how this is done.

Bandwidth correction:

The input signal is effected by the HP3458A's bandwidth. To a large extent, this error is repeatable from one HP3458A to another and can therefore be backed out.

On the .1, 1, and 10V ranges, the input signal is effected by a 1 pole low pass filter with a nominal bandwidth of 120kHz. This error term is the dominant error for the 1V and 10V ranges:

Err= SQR(1/(I+(Freq/l2OkHz)^2)) - 1 **(16)**

On the 100V and 1kV ranges, the input signal is effected by the bandwidth of the 10M ohm high voltage input attenuator which is about 36kHz. This error term is the dominant error term for these ranges:

Err= SQR(1/(l+(Freq/36kHz)^2)) - 1 **(17)**

The 1OOmV range has the same error component as the 1V and 1OV ranges plus an additional error term due to the input amplifier's bandwidth being substantially lower. At low frequencies the input amplifier is actually peaking with a 1 pole approximation frequency of 8200OHz. The 1OOmV range exhibits an error of:

Err= SQR((1+(Freq/82kHz)^2)/(1+(Freq/l20kHz)^2)) - 1 **(18)**

The function FNVmeter_bw (line 1720) performs the bandwidth correction calculations in the program. The error estimation routines assume that the bandwidths of the HP3458A are only known to +-30% and calculates uncertainties accordingly.

It should be noted that for frequencies below 20OHz, the bandwidth uncertainties are insignificant. They become significant to the 40ppm level at 1kHz on the 10V and lower ranges and significant to the 150ppm level at 1kHz on the higher voltage ranges.

ERROR ESTIMATION

Referring to the appendix, the subroutine Err_est (line 1900) calculates an estimate for total measurement uncertainty. The various components of this calculation are discussed below.

Basic I year accuracy:

The DCV function's ppm of reading specification for 1 year after an ACAL DCV operation is close to 10ppm for all ranges. The ppm of range specifications can be ignored since this specification is intended to cover offset variations which add in an RSS fashion for ACV RMS calculations. If the ppm of range errors were large enough they would have to be considered, but they are of the order of 1uV which adds less than .005ppm error to a 10MV measurement!

It should be noted that if less than a 1 year calibration cycle is used for the HP3458A, this error can be substantially lower. Line 1970 is where this error is located. This error is of a random nature and is appropriately handled in a statistical fashion for error analysis purposes.

Voltmeter bandwidth:

Equation 16, 17, and 18 illustrate how corrections for errors induced by the limited bandwidth of the HP3458A are made. For error analysis purposes it is assumed that the various bandwidths are only known to +-30%. Line 2060 shows how this calculation is made.

This error is random and is handled statistically for error analysis purposes.

A/D gain uncertainty for short apertures:

The basic 1yr accuracy discussed above is based on an A/D aperture of 100 power line cycles or greater. For shorter apertures, the accuracy of the HP3458A is reduced. This effect is shown in the data sheet on page 11 in the form of a graph. Line 2120 of the program reduces this graph to equation form (it is valid only for short apertures).

This error is random and handled statistically for error analysis purposes.

A/D aperture uncertainties in the frequency domain:

Equation 15 and the related discussion concerns the nature of backing out gain errors that are a function of A/D aperture and input signal frequency. If the A/D is programed for a particular aperture, the actual aperture is only known to the tolerance of the crystal clock oscillator used to control the A/D (0.01%). Also, various A/D switching effects add another 50ns of uncertainty. Line 2290 shows how this uncertainty is quantified.

This error is random and handled statistically for error analysis purposes.

Errors due to 1% distortion of the input signal:

As mentioned in the discussion pertaining to equation 15, backing out sinc(X) errors due to A/D aperture can only perfectly correct the fundamental frequency of a distorted sinewave. Other frequencies due to 1% signal distortion are not corrected properly. For error analysis purposes it is assumed that a distortion component equal to 1% of the fundamental frequency amplitude is present at the third harmonic. The error due to incorrect A/D aperture correction is quantified on line 2420.

This error is minimized if the A/D aperture is as small as possible. This error is returned separately so that if the user of the program does not have a distorted signal the error can be ignored.

Individual sample noise:

The HP3458A exhibits reading to reading variation that is a function of A/D aperture and voltage range. This 1 standard deviation measurement noise is specified in a graph on page 11 of it's data sheet. Line 2740 translates this graph into an equation that is valid only for short apertures. The data sheet's noise multiplier of 20 for the 1OOmV range is overly pessimistic for short apertures and the more realistic value of 7 is used on line 2790.

This noise is multiplied by 10 to reflect the worst case of operation at 1/10 full scale (line 2760) and then further scaled by 2 to reflect variations at the 2 standard deviation level.

Most of the measurement noise determined above is uncorrelated from sample to sample. Therefore, the total noise of a measurement composed of a multitude of samples is then reduced by the the square root of the number of samples taken (line 2750).

This error term is handled absolutely to reflect the fact that it will eventually be seen if measurements are repeated enough.

Dissipation factor error:

The input signal of the HP3458A is routed to a 1OMohm input attenuator on the 100V and 1kV ranges. The output resistance of this attenuator is 100kohms and is routed to the input of the main DC voltage amplifier. On the way to the amplifier this signal sees about 30pF of good low dissipation factor (D.F.) capacitance (FET inputs, ceramic capacitances, etc.) and about 15pF of poor D.F. printed circuit (pc) board capacitance. The pc board capacitance has a dissipation factor value of about 0.6%. The effective D.F. of the combined capacitance of 45pF is about 0.2% (.6*15/45).

For the low voltage ranges, the input signal is routed to a 10kohm resistor whose output drives about 120pF of good D.F. capacitance and 15pF of bad D.F. pc board capacitance. The effective D.F. of the combination is about 0.07% (.6*15/135).

A capacitor that has a dissipation factor of Df acts like it has a parallel resistance across it equal to 1/Df times its capacitive reactance. Thus it's parallel resistance is 1/(Df*2*PI*C*Freq). This resistance creates a resistive divider with the input resistance described above.

Line 3050 quantifies this error. This error is only significant on the 100V and 1000V ranges and is always negative. It is treated in an absolute sense for error analysis purposes.

Total error calculation:

Random errors are handled in a statistical fashion. Other errors are added in an absolute fashion. Line 3100 shows how the program adds the various errors.

PERFORMANCE VERIFICATION

One aspect of verification is repeatability. That is, how stable are multiple measurements of the same source over some time interval? At 7V and 40Hz, informal comparisons with a Datron model 4200 AC calibrator over a week in an environment stable to +-2 deg.C varied less than 10ppm. The same measurement over 10 minutes exhibited a standard deviation of 1.5ppm. 10

minute standard deviations when measuring a 7V, 40Hz sinewave sourced from a thoroughly warmed up Hewlett Packard HP3245 universal source are 0.4ppm.

Another aspect of verification is accuracy. For audio band frequencies, thermal AC/DC converters can be traced to national standards with better than 10ppm uncertainty. This traditional procedure, while marginal for verifying a 10ppm measurement, can be used at these frequencies to verify the algorithm's performance. At lower frequencies where available thermal converters become less accurate, a different approach is required.

The United States' National Institute of Standards and Technology (NIST) has developed a digital synthesized calculable AC standard suitable for this verification. It -uses an 8 bit digital to analog converter (DAC) in conjunction with sinewave look-up tables to generate stepped sinewave approximations. Varying the DAC's clock frequency allows the source's output frequency to vary from below 0.1Hz to above 20kHz (see Reference 1).

The accuracy of the AC standard is derived from DC measurements of the voltage steps composing the sinewave approximation. These steps are measured during a calibration procedure where the DAC's clock is paced off the "Measurement Complete" line of a precision DC voltmeter. Assuming that the voltages do not change at other clock frequencies or at a later time, the RMS voltage of the standard can be computed. The accuracy of this computation is relatively independent of the DAC's linearity.

Figure 3 is an illustration of the output of the AC standard (with the step size exaggerated for clarity).

64 STEP SINEWAVE (8 BIT)

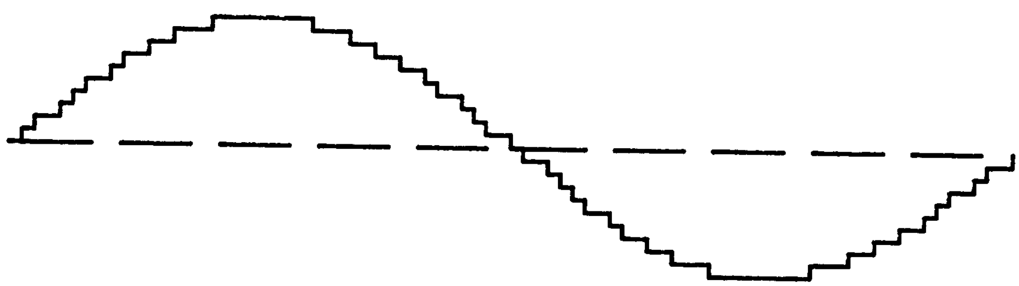

Figure 3 - NIST Calculable AC Standard Output (see reference 1)

The standard can be programmed to output sinewaves approximated with 64, 128, 256, or 512 steps per period. All of these approximations are distortions of a pure sinewave and therefore contain energy at higher frequencies. Most of this energy is located near the DAC's clock frequency, but there also is a dispersed energy due to it's 8 bit quantization noise.

Digressing somewhat, it should be noted that when measuring non-sinewaves, the bandwidth of the measuring device will effect the measurement. Two "perfect" AC voltmeters with different bandwidths may measure the same signal differently if it contains energy outside the passband of one of the meters. Since the bandwidth of this algorithm is very low, the verification procedure using the NIST AC standard must consider this effect.

The calculated RMS output of the AC standard is the value that would be measured by a very high bandwidth AC voltmeter. Lower bandwidth meters will measure a somewhat lower value. If this bandwidth is known, the expected deviation is calculable given the number of steps per period being output by the standard. The RMS voltage in the fundamental period of the standard is described by the equation:

RMS_fundamental= RMS_highfreq* SIN(PI/Steps)/(PI/Steps) **(19)**

For verification purposes, the algorithm was modified so that it's measurement bandwidth would exclude the standard's clock frequencies. Referring to the appendix, on lines 360 and 370 the variables were changed to Aper _ targ = 1 and Nharm = 10. These changes cause the algorithm to try and use the largest possible A/D aperture (I second) conditional with being able to measure the 10th harmonic of the signal (by sampling at least 20 times the signal's frequency). For most frequencies this will force the A/D aperture to be about 1/20 the signal's period which leads to a measurement 3dB bandwidth of 10 harmonics. With the crudest sinewave approximation of 64 steps per period, the sampling harmonics are near the 64th harmonic which is well outside the algorithm's bandwidth. The algorithm's expected measurement is then described by Equation 19 which leads to a table of expected deviations (figure 4).

STEPS per PERIOD	EXPECTED DEVIATION
64	-401.5 ppm
128	-100.4 ppm
256	-25.1 ppm
512	-6.3 ppm

Figure 4 - Expected algorithm deviation when measuring NIST AC standard

In addition to figure 4, there is an additional expected -4ppm deviation due to the dispersed 8 bit quantization noise of the NIST standard. This noise relative to a sinewave is 0.32% and contributes 5.1ppm to a wide band RMS AC measurement. About 4ppm of this is outside the bandwidth of the algorithm.

Backing out the above expected deviations, 5 comparisons with the NIST AC standard were made over a period of 2 days. The results are reported in figure 5.

(Format is mean + 3 standard deviations in ppm)

Voltage	Steps	0.1Hz	1.2Hz	76Hz
7 V	512	6.0+2.0 .	18+3.4	2.3+1.1
7 V	256	-2.7+1.4	-3.2+5.4	2.1+.8
7 V	128	-1.5+1.2	-1.2+1.5	2.1+2.6
7 V	64		-0.2+1.6	1.4+1.7
1 V	512			6.9+5.6
1 V	256			6.4+3.6
1 V	128			5.6+2.5
.1 V	512			7.2+0.1
.01 V	512			12 +6

Figure 5 - Agreement with NIST AC standard (ppm)(mean + 3 sigma)

CONCLUSION

The described algorithm, when used with commercially available equipment, advances the state of the art in low frequency AC measurement.

Feedback is desired from users of the algorithm. Interested parties are requested to contact the author for more information and program copies:

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attn: Ronald Swerlein CU323
phone (970) 679-2029
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Loveland, CO 80539

REFERENCES:

1 Oldham, Nile; Hetrick, Paul; Zeng, Xiangren, "A Calculable Transportable Audio Frequency AC Reference Standard", IEEE Transactions on Instrumentation and Measurement . Volume 38, Number 2, April 1989, pp. 368-371.

APPENDIX

Included are operating instructions and a listing of a program written in Hewlett Packard BASIC that incorporates the algorithm described previously.

Using the program:

The commonly modified variables are on LINE 200-280 and are the voltmeter address, the voltmeter DCV range, and the target measurement time. The voltmeter range should be picked so that a peak value of the input waveform will not overload the meter. The minimum voltage on a given range is what is necessary to operate the internal level trigger which has about 10% of range hysteresis. For example, the 10V range should be able to measure AC voltages from 1V to 7V. Measurement time is pretty self explanatory, longer times give higher accuracy. But experiment with measurement time since sometimes very short measurements are highly accurate.

When the program is run, the user is asked to apply the input signal and allow it to settle and then to press "CONT". If the input frequency is less than O.5Hz, there will be a prompt for it's frequency. This value should be known to within .02% or so. Next the program prints out this frequency, a list of the sampling parameters, and an estimate of measurement uncertainty. Then the intermediate and final results are printed. Pressing "CONT" will generate another measurement., but the sampling parameters will still be the same. This is useful for observing measurement repeatability. If a signal with a different frequency is to be measured the program should be re-run.

The less commonly modified variables are on line 290 -390. If Forcefreq is the program prompts the user for the input signal frequency instead of automatically looking. This is convenient for frequencies below 0.5Hz where the HP3458A can't measure frequency. Otherwise 1.5 sec is wasted before the program realizes that it can't measure it and prompts the user. If Force=1, the sampling parameters on lines 330-350 are forced. In general, Tsampforce*Numforce are set to be an integral multiple of the period of the input signal. Keep in mind that it is possible to generate inaccurate measurements by forcing the wrong sampling parameters. This can also occur if a wrong or inaccurate input frequency is entered after the frequency prompt.

Nharm (line 370) is the minimum number of input signal harmonics that will be passed without aliasing before the program 'automatically speeds up its sampling. (At least 2*Nharm samples are forced to be present in each period of the input signal). If Nharm is too high, at higher input signal frequencies, the A/D aperture will be forced to such a low value that the basic gain accuracy of the program will be degraded (the HP3458A is less accurate with a short aperture than a long one). If Nharm is too low, small amounts of distortion may generate alias errors that can show up as measurement drift or error. A test for lack of alias error is to change Aper-target or Nharm or Nbursts slightly and verify that the measurement does not significantly change. In general, one shouldn't get too concerned about alias error with this program, it was designed to be highly resistant. The sample rate is picked so that 1/2/Tsamp is offset slightly from Nharm*Freq so as to resist aliasing up to 10*Nham*Freq. Also, at these higher frequencies, the aperture of the A/D becomes an effective anti-alias filter. Nharm=6 is a good value.

Nbursts (line 380) selects the number of intermediate results that are used in computing the final result. Each intermediate result is computed from a burst of Num samples. Each burst of Num samples is delayed in time K/Freq/Nbursts from the input signal's zero crossing where K varies from 0 to Nbursts-1. Any value of Nbursts>=6 is good. Under some conditions, smaller Nbursts can be used. The purpose of using multiple bursts is to remove errors due to Num*Tsamp not being an exact integral number of periods in length and to further reduce sensitivity to aliasing.
Program listing: