Plane-Shear Measurement with Strain Gages

Introduction

Loading a specimen as shown in Figure 1a produces shear stresses in the material. An initially square element of the material, having vertical sides parallel to the direction of loading, is distorted by these stresses into a diamond shape as illustrated in Figure 1b, where the distortion is greatly exaggerated for pictorial clarity. Shear strain is defined as the magnitude of the change in the initial right angle of the element at the X-Y origin. That is,

$$\gamma = \frac{\pi}{2} - \varphi \qquad (1)$$

Since shear strain is a change in angle, its natural units are radians, although it can also be expressed in terms of in/in [*m*/*m*] and percent. From Equation (1), the sign of the shear strain is positive when the initial right angle of the element is reduced ($\varphi<\pi/2$). Reversing the directions (sign) of the shear stresses in Figure 1 causes the initial right angle to increase and results in a negative shear strain.

Normal strains cause dimensional changes in the grid of a strain gage, changing its electrical resistance. Pure shear strains merely rotate the grid, and do not cause the elongation or contraction necessary to vary the resistance. Fortunately, shear and normal strains are related through mechanics principles, allowing strain gages to provide a direct indication of shear strain. Properly orienting gages on a strained surface and properly connecting them in a Wheatstone bridge circuit yields an instrument indication that is directly proportional to the surface shear strain.

This Tech Note first develops an expression for determining the surface shear strain in any given direction from two normal-strain measurements. Next is a discussion of strain gage and Wheatstone bridge arrangements for direct indication of shear strains. The shear-strain magnitude varies sinusoidally around a point in a biaxial strain field. Since the maximum strain values are usually of primary interest in stress analysis, Mohr's strain circle is used to obtain an expression for the maximum shear strain at a point in a biaxial strain field. Practical examples of shear measurement with strain gages are given for both isotropic materials (e.g., metals) and orthotropic materials such as wood and fiber-reinforced composites.

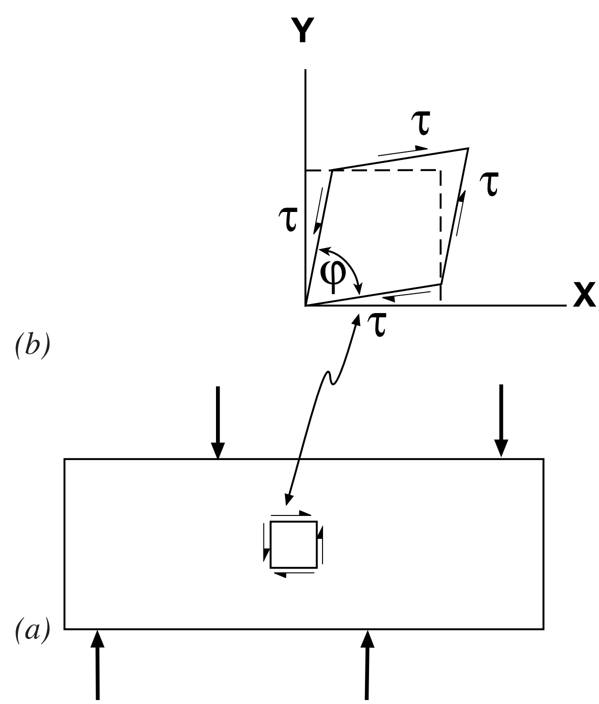

Figure 1 – (a) Shear loads applied to specimen; (b) enlarged shear deformation of an initially square element of the material.

Shear Strain from Normal Strains

Consider an array of two strain gages oriented at arbitrarily different angles with respect to an X-Y coordinate system which, in turn, is arbitrarily oriented with respect to the principal axes, as in Figure 2 (following page). From elementary mechanics of materials, the strains along the gage axes can be written as:

Plane-Shear Measurement with Strain Gages

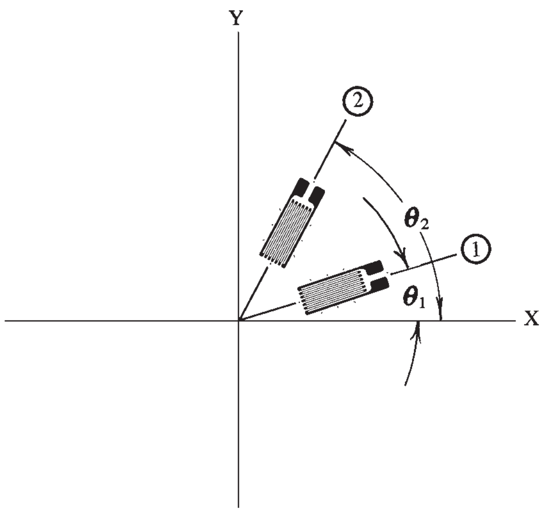

Figure 2 – Arbitrarily oriented strain gages in a biaxial strain field.

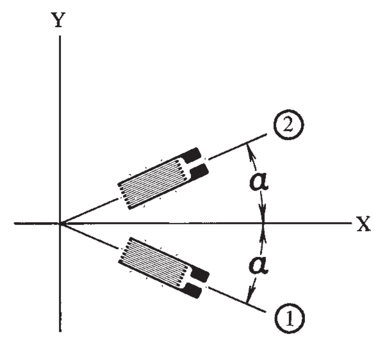

Figure 3 – Defining X axis as bisector of angle between gage axes, $\gamma_{xy} = (\varepsilon_1 - \varepsilon_2)/\sin 2\alpha$.

$$\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_1 + \frac{\gamma_{xy}}{2}\sin 2\theta_1 \qquad (2)$$

$$\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_2 + \frac{\gamma_{xy}}{2}\sin 2\theta_2 \qquad (3)$$

Subtracting (3) from (2) and solving for γ_{xy}

$$\gamma_{xy} = \frac{2(\varepsilon_1 - \varepsilon_2) - (\varepsilon_x - \varepsilon_y)(\cos 2\theta_1 - \cos 2\theta_2)}{\sin 2\theta_1 - \sin 2\theta_2} \qquad (4)$$

It is now noticeable that if $\cos 2\theta_1 \equiv \cos 2\theta_2$, the term in ε_x and ε_y vanishes, and

$$\gamma_{xy} = \frac{2(\varepsilon_1 - \varepsilon_2)}{\sin 2\theta_1 - \sin 2\theta_2} \qquad (5)$$

Since the cosine function is symmetrical about the zero argument, and about all integral multiples of π, $\cos 2\theta_1 \equiv \cos 2\theta_2$ when, for an arbitrary angle α,

$$\theta_1 + \alpha = -\pi/2, 0, \pi/2, \pi \ldots \frac{n\pi}{2} = \theta_2 - \alpha \qquad (6)$$

It is thus evident that, if the gage axes are oriented symmetrically with respect to, say, the X axis (Figure 3),

$$\theta_1 = -\theta_2 = \alpha$$

and,

$$\gamma_{xy} = -\frac{\varepsilon_1 - \varepsilon_2}{\sin 2\theta_2} = \frac{\varepsilon_1 - \varepsilon_2}{\sin 2\theta_1} = \frac{\varepsilon_1 - \varepsilon_2}{\sin 2\alpha} \qquad (7)$$

The preceding results can be generalized as follows: The difference in normal strain sensed by any two arbitrarily oriented strain gages in a uniform strain field is proportional to the shear strain along an axis bisecting the strain gage axes, irrespective of the included angle between the gages.

When the two gages are 90 degrees apart, the denominator of Equation (7) becomes unity and the shear strain along the bisector is numerically equal to the difference in normal strains. Thus, a conventional 90-deg two-gage rosette constitutes an ideal shear half bridge because the required subtraction, $\varepsilon_1 - \varepsilon_2$, is performed automatically for two gages in adjacent arms of the bridge circuit (Figure 4a). When the gage axes of a two-gage 90-deg rosette are aligned with the principal axes, the output of the half bridge is numerically equal to the maximum shear strain. A full shear-bridge (with twice the output signal) is then composed of four gages as shown in Figure 4b. The gages may have any of several configurations, including the cruciform arrangement and the compact geometry illustrated in the figure.

Principal Strains

It should be kept in mind that with the shear-bridges described above, the indicated shear strain exists along the bisector of any adjacent pair of gage axes, and it is

Plane-Shear Measurement with Strain Gages

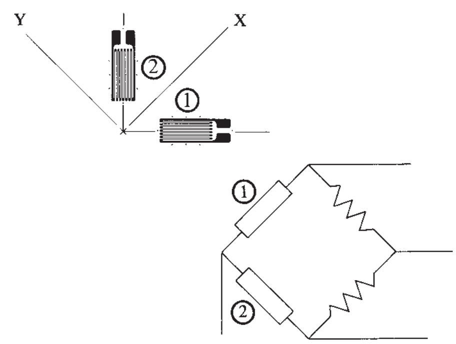

Figure 4a – 90-deg rosette for direct indication of shear strain γ_{xy}.

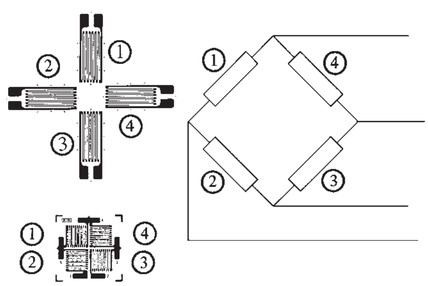

Figure 4b – Full shear-bridge.

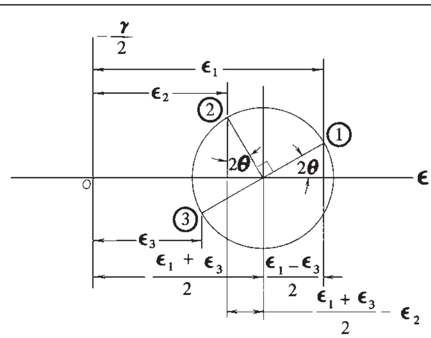

Figure 5 – Mohr's circle for strain used to determine maximum shear strain.

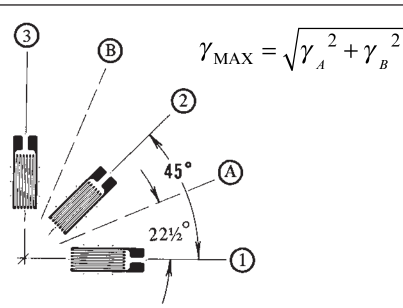

Figure 6 – 45-deg rosette used to determine γ_A and γ_B, the shear strains in the Ⓐ and Ⓑ directions, respectively.

not possible to determine the maximum shear strain or the complete state of strain from any combination of gage outputs unless the orientation of the gage axes with respect to the principal axes is known. In general, when the directions of the principal axes are unknown, a three-gage 45-deg rectangular rosette can be used.

Referring to Mohr's circle for strain (Figure 5) it is apparent that the two shaded triangles are always identical for a 45-deg rosette, and therefore the maximum shear strain is equal to the "vector sum" of the shear strains along any two axes which are 45 degrees apart on the strained surface. Looking at the 45-deg rosette as shown in Figure 6, it can be seen that the shear strains along the bisectors of the gage pairs ①–② and ②–③ are in fact 45 degrees apart and, thus, the maximum shear strain is,

$$\gamma_{MAX} = \sqrt{\gamma_A^{\ 2} + \gamma_B^{\ 2}}$$

and, considering Equation (7),

$$\gamma_{MAX} = \sqrt{\left(\frac{\varepsilon_1 - \varepsilon_2}{\sin 45^o}\right)^2 + \left(\frac{\varepsilon_2 - \varepsilon_3}{\sin 45^o}\right)^2}$$

or,

$$\gamma_{MAX} = \sqrt{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2} \qquad (8)$$

and, from Mohr's circle again, the principal normal strains are obviously:

$$\varepsilon_p, \varepsilon_q = \frac{\varepsilon_1 + \varepsilon_3}{2} \pm \frac{1}{\sqrt{2}}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2} \qquad (9)$$

Correction for Transverse Sensitivity

Up to this point, the effect of transverse sensitivity on shear-strain indication has been ignored. However, correction for this effect is particularly simple when all strain gage grids, in either a two-element tee rosette or a three-element 45-deg rectangular rosette, have the same transverse sensitivity. For such cases, correction consists of merely multiplying the indicated shear strain by the factor $(1–\nu_0 K_t)/(1–K_t)$, where K_t represents the common transverse sensitivity of the rosette grids and ν_0 is the Poisson's ratio of the beam material on which the manufacturer measured the gage factor of the gages. When the rosette grids do not have the same transverse sensitivity, the error is a function of the strain state, and the indicated strain from each grid must be corrected separately. Relationships for this purpose are provided in our Tech Note TN-509, "Errors Due to Transverse Sensitivity in Strain Gages."

Plane-Shear Measurement with Strain Gages

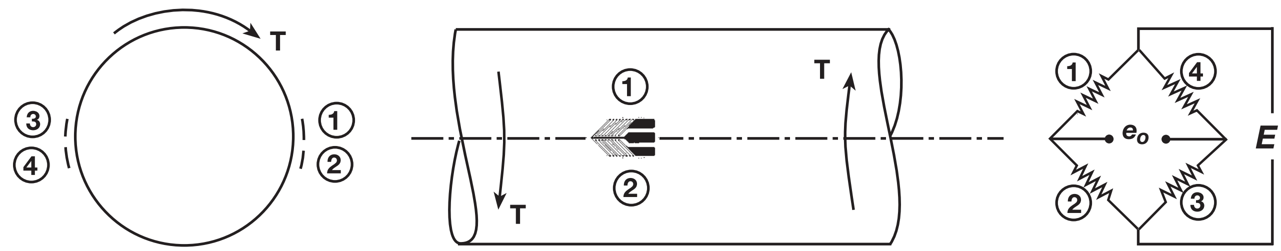

Figure 7 – Torque transducer based on shear measurement in shaft.

Applications

The area of application for shear strain measurement can be divided into two categories by the type of material (isotropic or orthotropic) on which the measurement is made. As a rule the same categorization also divides the applications according to the purpose of the measurement. In the case of materials which can be treated as isotropic (e.g., the structural metals), the usual reason for measuring shear strain is to determine the magnitude of an applied shear stress or load. In contrast, shear strain measurements on materials such as wood and unidirectionally reinforced plastics (orthotropic materials) are most commonly made for the purpose of determining a mechanical property of the material — namely, its shear modulus or modulus of rigidity.

An example of shear strain measurement on metals occurs in shear-buckling studies. When thin panels of steel or aluminum alloy are loaded in shear, there is ordinarily a critical load at which the material buckles, forming one or more waves, generally parallel to the maximum principal stress direction. In studies evaluating the relative merits of different structural configurations, a common practice is to install strain gage rosettes near the center of the panel to determine the maximum sustainable shear stress or applied load prior to buckling.

A more common application for shear measurement on metals is the torque transducer. A cylindrical shaft in torsion is a case of essentially pure shear, and the applied torque can be readily determined from two tee rosettes positioned diametrically on the shaft surface, and oriented so that their gridlines are at 45 degrees to the shaft axis. Specially configured tee rosettes (Figure 7) are ordinarily used for this purpose. When the rosettes are connected to form a full Wheatstone bridge as indicated in the figure, the bridge output is doubly sensitive to shaft torque, but insensitive to bending and axial loads.

The most frequent application of shear strain measurement in transducers is the shear-beam load cell, indicated schematically in Figure 8. The load cell consists of a short, stiff cantilever beam with the material recessed in one area to form a thin “shear web”. Tee rosettes are installed on both sides of this web to produce an output proportional to the vertical shear force on the beam. Since the vertical shear force is necessarily constant throughout the length of the beam, the transducer output tends to be independent of the position (along the beam axis) of the applied load. The tee rosettes are connected in a full-bridge circuit as indicated to render the output insensitive to side loads and off-axis load components on the beam.

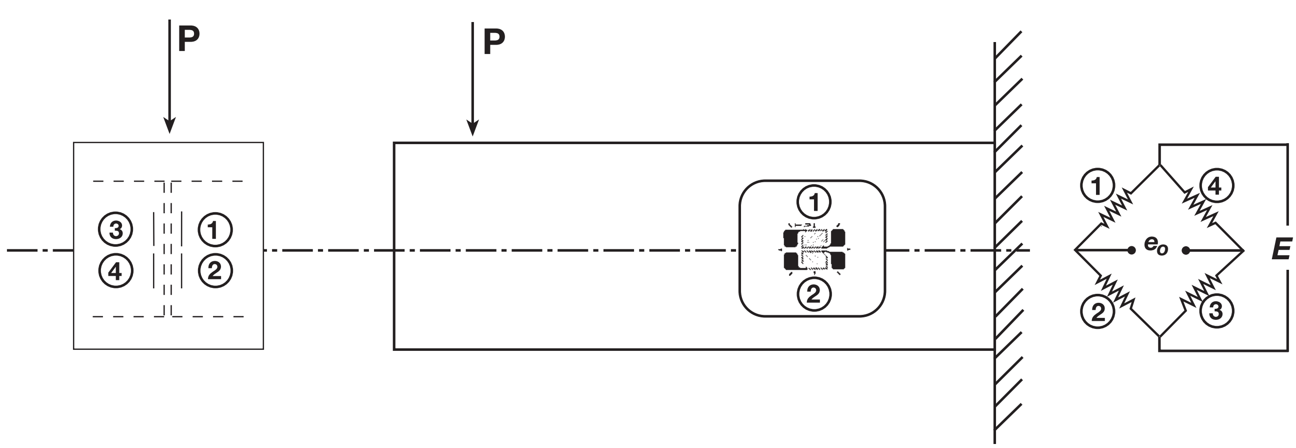

Figure 8 – Schematic representation of shear-beam load cell.

Plane-Shear Measurement with Strain Gages

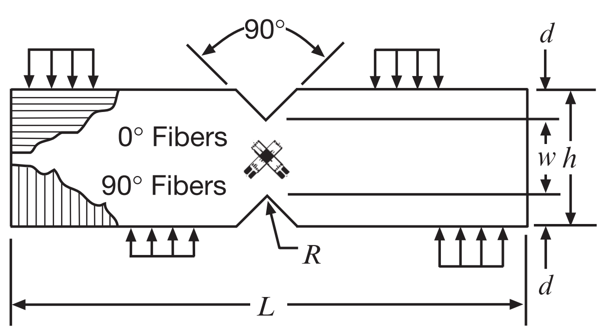

Figure 9 – Iosipescu V-notch specimen for measurement of shear modulus.

Experimental determination of the shear modulus of a metal is uncommon, since this property can usually be calculated with sufficient accuracy from the relationship: $G = E/[2(1+\nu)]$. In orthotropic materials, however, the shear modulus is an independent mechanical property which must be measured for each different material. The usual procedure for doing so is to establish a specimen geometry and loading system which produces a state of pure shear with respect to the principal material directions. Tee rosettes are installed on both sides of the specimen for determining the shear strain (γ) under load. The corresponding shear stress (τ) is obtained from the measured load divided by the cross-sectional area in shear.

The shear modulus is then calculated from: $G_{12} = \tau_{12}/\gamma_{12}$, where the 1-2 subscripts refer to the principal material directions.

Although many different shear test methods have been used on composite materials, the Iosipescu method (Figure 9) is widely favored and is the subject of an ASTM standard (D 5379). The standard specifies measurement of the shear strain by tee rosettes installed on the horizontal centerline of the specimen, while the shear stress is calculated from the load divided by the cross-sectional area between the notches. It has been demonstrated, however, that the shear stress distribution between the notches is far from uniform. Moreover, the nonuniformity in shear stress distribution varies markedly according to whether the length of the specimen is parallel or perpendicular to the major principal material direction (i.e., a 0-deg or 90-deg specimen). As a result, it is necessary in each case to adjust the calculated shear modulus with a different empirical correction factor.

Actually, the true average shear stress on the cross section between the notch roots is, by definition, equal to the load divided by the cross-sectional area. If the average shear strain over the same area were measured, the true shear modulus of the material could be obtained directly, without the need for correction factors. But every strain gage gives an output corresponding to the average strain under its grid length. Thus, the appropriate tee rosette for the Iosipescu specimen is one which spans the complete depth, from notch root to notch root as shown in Figure 10. When these special shear gages (Micro-Measurements C032 and C085 patterns) are employed with the Iosipescu specimen, the observed shear modulus is the same for the 0-deg and 90-deg specimens (without correction factors), as it should be for the validity of the mechanics of orthotropic materials.

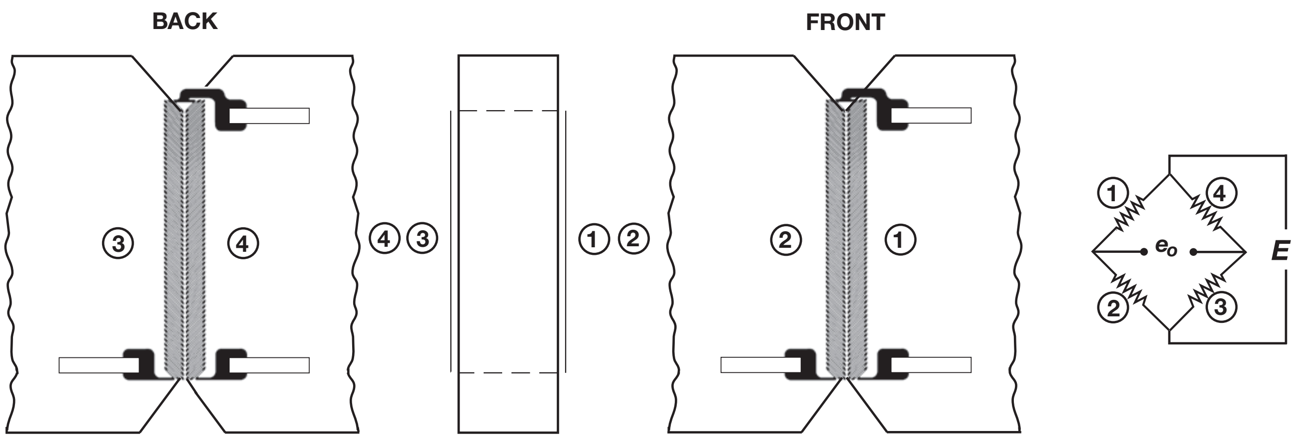

Figure 10 – Iosipescu specimen with tee rosette shear gages installed on both faces.

Plane-Shear Measurement with Strain Gages

Some Cautions and Limitations

Since shear strains are inferred from normal strain measurements, accurate results require the same care and attention to detail as for any other strain measurement task. There are, however, several additional considerations when determining shear strains or analyzing data from strain gage rosettes. The data-reduction relationships used with rosettes are derived from the strain-transformation equations [Equations (2), (3)], which are, in turn, based on "small-strain" theory. The latter equations are precise only for infinitesimal strains, but they are adequately accurate for typical strain measurements on metal test objects where the strains are usually less than 1 percent. With plastics and composite materials, however, larger strains may be encountered, and the rosette data-reduction relationships can become increasingly inaccurate at strain levels greater than 1 percent.

Another consideration is the uniformity of the strain field under the area covered by the rosette, which is necessarily greater than for a single grid of the same gage length. For typical stress analysis purposes (i.e., to obtain the stress at a point in a homogeneous material) the state of strain under the rosette should be nearly uniform. When a significantly nonuniform strain field is expected, the rosette should be small enough relative to the strain gradient to satisfactorily approximate measurement of the strain at a point. This restriction does not apply when there is a valid reason to integrate or average the strain over a specific area, as in the previously cited case of the Iosipescu shear-test specimen. Similarly, for any strain measurements on composite materials, the rosette dimensions should usually be large relative to the distance between inhomogeneities (reinforcing fiber spacing) to provide an accurate indication of the macroscopic strain state.

Since the determination of shear strain is accomplished by measuring two or three normal strains, it is always necessary to make certain that the indicated shear strain is unaffected by nonshearing load components such as bending, twisting, or axial loads. This requirement can usually be satisfied quite easily in the case of geometrically symmetric test objects and specimens. With shear-test specimens, for example, it is imperative that strain gage rosettes be installed on both sides of the specimen at corresponding points. When the rosettes are connected in the Wheatstone bridge circuit as indicated in Figure 10, strains due to out-of-plane bending or twisting are canceled within the bridge circuit. Cancellation of undesired strain components within the Wheatstone bridge circuit is a widely applicable technique, and standard practice in the technology of strain gage based transducers (see Figures 7 and 8). However, the intrinsic capacity for removing undesirable strain components through bridge circuits should not be used in place of good load alignment. Carried to the extreme, shear strains could theoretically be extracted from even predominantly off-axis loading. To achieve the best accuracy, however, it is always preferable to load the test specimen through its centroidal axis.