WAYNE KERR

THE TRANSFORMER RATIO-ARM BRIDGE

By Raymond Calvert

WAYNE KERR MONOGRAPH No. 1

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By R. Calvert

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Introduction

The value of an impedance Z is given by the ratio E/I where E is the voltage across the impedance, and I the current flowing through it. The most direct way of measuring impedance is therefore to measure E and I and compute the ratio. Unfortunately, it is extremely difficult to make absolute measurements of voltage or current with any degree of accuracy. In practice it is far easier to make a comparison measurement, the unknown being compared with a standard impedance by means of a bridge circuit. Thus only ratios are involved, which can be established more easily and accurately than absolute quantities.

Let the subscripts u and s refer to the unknown and standard respectively. Then:

$$Z_u = E_u/I_u$$

and

$$Z_s = E_s/I_s$$

By division we have:

$$Z_u = \frac{E_u}{E_s} \cdot \frac{I_s}{I_u} \cdot Z_s \quad . \quad . \quad . \quad (1)$$

This is the fundamental equation for the measurement of impedance, whatever method is employed, and it is seen that there are two ratios E_u/E_s and I_s/I_u which govern the range of measurement with a standard Z_s.

The Conventional Bridge

The arrangement of the conventional method of measurement, based on the Wheatstone Bridge principle, is shown in simplified form in Fig. 1.

Z_s is a variable standard impedance, and Z_1 and Z_2 are known fixed standards.

At balance, the unknown impedance is given by:

$$Z_u = \frac{Z_1}{Z_2} \cdot Z_s \quad . \quad . \quad . \quad (2)$$

The usual procedure is to balance the bridge by the adjustment of Z_s and the "ratio-arms" Z_1 and Z_2 are so arranged that the ratio Z_1/Z_2 is adjustable in decade steps to provide a multiplying factor.

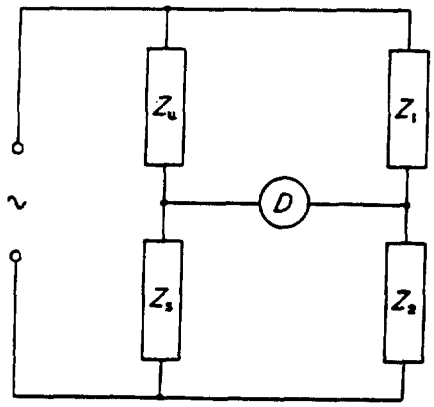


FIG. 1

Since the current in the standard and unknown is the same, only one factor Z_1/Z_2 is adjustable to give the range of measurement within the limits of the variable standard Z_s.

This conventional method of measurement has a number of disadvantages:

1. Since the bridge relies entirely upon the ratio of impedance standards, the range is severely restricted because very low and very high standard impedances are extremely difficult to manufacture with precision.
2. The theoretical conventional circuit cannot be achieved in practice, except over a very limited range of measurement. To ensure any degree of accuracy, a Wagner Earth arrangement is essential, requiring a double balance. This method is very inconvenient in practice and only two-terminal measurements are possible over any range. As discussed in further detail later, this is a severe restriction.
3. For accurate measurements decade standards covering several decades of resistance

and reactance must be used. At least 4 standards per decade are necessary, and the associated switches and wiring introduce spurious impedances that are difficult to handle. The standards are bulky and costly and can only be made effectively pure over a small range.

4. With so many standards involved in the decade boxes and ratio-arms, calibration is difficult and often uncertain.

The Transformer Ratio-Arm Bridge

The basic circuit of the transformer bridge is shown in Fig. 2. The subscripts *u* and *s* refer to the unknown and standards side of the bridge respectively.

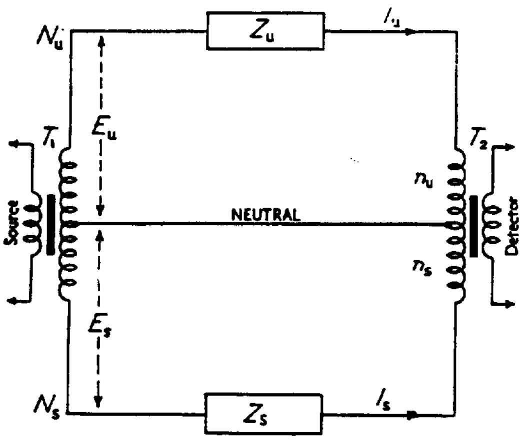


Fig. 2

T_1 is a voltage transformer to the primary of which the source is connected. The secondary winding is tapped to give sections having N_u and N_s turns.

T_2 is a current transformer, the primary of which is tapped to give n_u and n_s turns, and the secondary winding is connected to the detector.

To simplify the illustrations in this monograph, the relative sense of transformer windings is not shown. In all instances the windings are so arranged that, at balance, there is zero core flux in the current transformer.

Assume at this stage that the transformers are ideal and that Z_s is adjusted to give null indication in the detector. Under these conditions zero flux is produced in the current transformer, and there is therefore no voltage drop across its windings. The detector sides of both the unknown and standard impedances are therefore at neutral potential. The voltages across the unknown and standard are then E_u and E_s respectively.

Therefore $\quad I_u = E_u / Z_u$

and $\quad I_s = E_s / Z_s$

For conditions of zero core flux in the current transformer, the algebraic sum of the ampere-turns must be zero.

Therefore $I_u n_u = I_s n_s$

Substituting for I_u and I_s we have:

$$\frac{E_u}{Z_u} \cdot n_u = \frac{E_s}{Z_s} \cdot n_s$$

or

$$Z_u = \frac{E_u}{E_s} \cdot \frac{n_u}{n_s} \cdot Z_s \quad . \quad . \quad (3)$$

For an ideal transformer the voltage ratio is equal to the turns ratio, therefore

$$Z_u = \left(\frac{N_u}{N_s} \cdot \frac{n_u}{n_s} \right) Z_s \quad . \quad . \quad . \quad (4)$$

It will be seen from equation 4 that unlike the conventional bridge, two ratios $\frac{N_u}{N_s}$ and $\frac{n_u}{n_s}$ are available. Thus it is possible, by means of a suitable combination of tappings on the two transformers, to produce a very high ratio product permitting a very wide range of measurement.

The Practical Transformer

Before proceeding further, it is necessary to justify the statement that the actual transformers used in the bridge may be considered ideal.

Firstly, the transmission loss between the primary and secondary of the voltage transformers is of no consequence. The only effect of this loss is to reduce the sensitivity of the bridge, and this can be compensated by increasing the gain of the detector. The important factor is the actual voltage ratio between the unknown and standard, both of which are tapped across the secondary windings of the voltage transformer. This voltage ratio across the bridge windings is dependent upon three factors:

(a) The turns ratio
(b) The flux linkage
(c) The effective series impedance of the windings compared with that of the load.

The voltage induced in a coil is proportional to the number of turns multiplied by the rate of change of flux. Therefore, provided all the turns of the bridge windings embrace the same flux, the ratio of induced voltages is equal to the turns ratio. The windings are, in fact, wound with precision on a common core of high permeability material. The ratio of core flux to air flux is of the order of 1000 : 1, and the geometrical arrangement of the windings is such that the air flux is largely common to the two windings. Even if the windings were so badly arranged that none of the air flux was common, the error between the induced voltage ratio and the turns ratio would be only 0·01%. If necessary this error can be reduced to a few parts in a million.

The only error which need be considered is that caused by the voltage drop in the windings. Ignoring spurious shunt impedances for the moment, the load current in the unknown windings is the current in the impedance being measured. In the measurement of impedances of 10 to 100 megohms, the total series impedance of both the voltage and current transformer windings is approximately 100 ohms. Even if all this impedance were concentrated in one arm of the bridge the worst error would only be ·001 %. In practice this error is negligible.

It will be shown later that the effects of shunt loading can be effectively compensated, so that it is reasonable to presume that for all practical purposes the transformers are ideal.

PRACTICAL ARRANGEMENT OF THE BRIDGE

Independent Tapping of Resistance and Reactance Standards

In Fig. 3, the unknown and standard impedances have been divided into their resistive and reactive components.

Since at balance the in-phase and quadrature ampere-turns must separately sum to zero, the conductance standard G_s and the reactance standard X_s may be connected to different tappings on the voltage transformer to balance out the currents caused by the unknown impedance.

This independence of the components of current may be put to advantage in many ways. The effect of impurities in the standards (for example, dielectric loss in capacitors) can be balanced out by the insertion of compensating components across the ' unknown ' side of the bridge. The leakage and capacitance across test jigs etc. can also be removed in this manner.

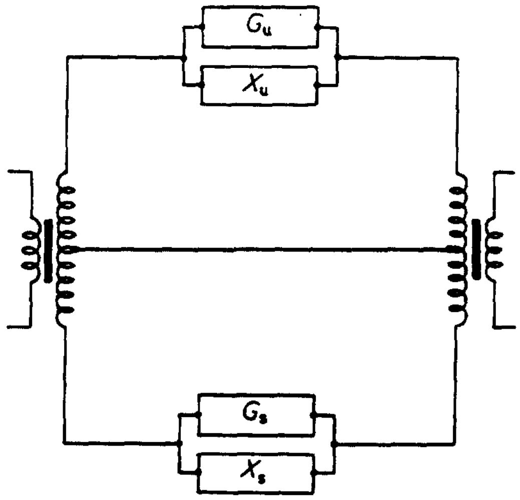


Fig. 3

The Decade Switches

Fig. 4 shows a bridge arrangement in which the voltage transformer winding feeding the standards has one hundred turns and is tapped at every ten turns, the tappings being brought out to a switch and connected to the standard. For convenience only the conductance standard is shown.

The equation for balance conditions may be written:

$$G_u = (G_s N_s) \frac{n_s}{N_u n_u} \quad . \quad . \quad . \quad (5)$$

If the product term inside the bracket is taken to represent the standard, it is of no importance whether the number of turns is fixed and the conductance given ten different values (the normal procedure with a decade box), or the conductance fixed and the number of turns given ten different values. By employing the latter method only one standard per decade is necessary.

In Fig. 5 the tappings on the standard side are taken to four switches wired in parallel.

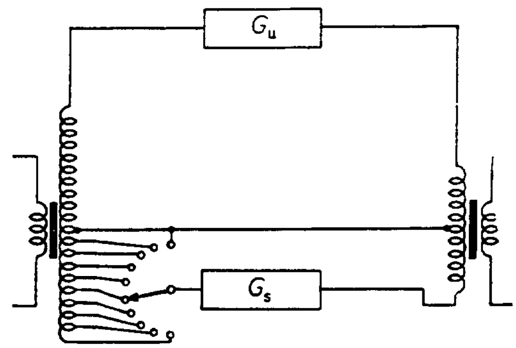


FIG. 4

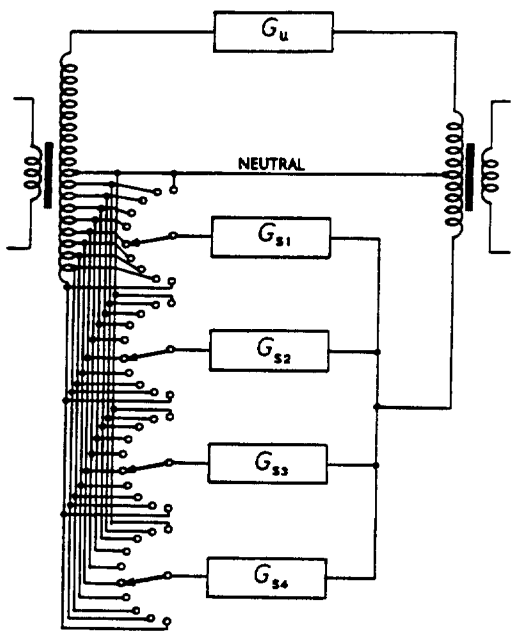


FIG. 5

Each switch connects through a fixed standard to a common tap on the current transformer. The conductances are given the relative value 1, 10, 100 and 1,000. The effect is exactly the same as that obtained by having a transformer of fixed voltage and a four-dial decade box.

Continuing with tappings to four more switches, each of which is connected through a fixed standard capacitor to the current transformer, the effect of adding a four-dial decade capacitor is produced.

Continuously Variable Controls

So far only the step-by-step adjustment of the standards has been considered. It has been shown how the decade boxes for resistance and capacitance standards normally associated with accurate bridge measurements can be replaced by a few fixed standards and banks of switches fed from taps on the voltage transformer.

Decade switching is essential when accuracies of better than 1% are required. However the provision of such switches alone makes the balancing of the bridge a slow and often laborious process, especially if the unknown is a complex impedance. The provision of a pair of continuously-variable direct-reading controls would make the bridge quick and easy to handle, but calibration difficulties would limit the accuracy of such an arrangement to 1–2%. Fortunately, it is possible with the transformer ratio bridge for continuously variable controls to be added without detracting from the accuracies of the decade standards.

A continuously variable reactance is conveniently provided by an air-dielectric capacitor whose residual capacitance C_o is balanced by a pre-set trimmer connected to a transformer winding of opposite sense. This arrangement is shown in Fig. 6.

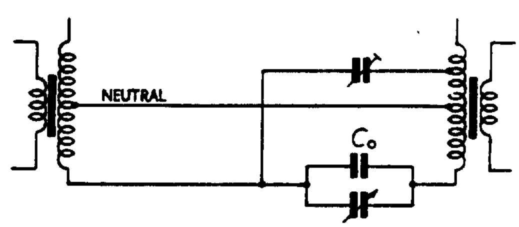


FIG. 6

A continuously variable conductance control is provided by combining a potentiometer with a fixed resistor as shown in Fig. 7.

The best practical compromise for accuracy and ease of operation is to use a single decade and a continuously variable control for accuracies of the order of 1%. A second decade need only be introduced when it is necessary to increase the accuracy to 0·1%, or when it is required to obtain very high discrimination at balance to measure small changes in impedance.

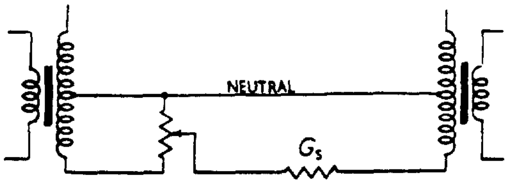


FIG. 7

Correction of Standards

Since only one fixed standard per decade is required, it is a simple matter to make each one effectively pure. The technique adopted is illustrated in Fig. 8, applied to a capacitance standard.

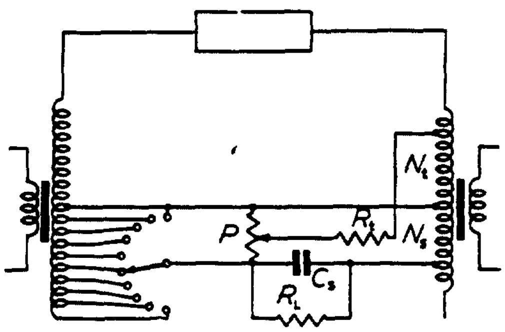


FIG. 8

At a given frequency a capacitor can be represented by a pure capacitance C_s shunted by a resistance R_L. The problem is to remove the effect of R_L leaving only the pure quadrature component C_s. It is therefore necessary to cancel the ampere-turns produced by R_L in the current transformer. This is done by feeding a current through a fixed trimming resistor R_t into the opposite side of the transformer. The current is adjusted to give exact cancellation by adjusting the voltage at the wiper of the pre-set potentiometer P. If the voltage applied to the capacitor is E and a fraction of this, kE, is applied to the trimming resistor, the effect of R_L is completely removed when

$$k = N_s/N_t \,.\, R_t/R_L \quad . \quad . \quad . \quad (6)$$

The adjustment is independent of the voltage, and therefore holds for all positions of the tapping switch.

The effect of a reactance term associated with the conductance standard can be removed in the same way. At a given frequency the standard can be regarded as a pure conductance shunted by a reactance. The reactance causes an unwanted quadrature current to flow in the current transformer. This can be cancelled by a quadrature current of opposite sign, the ampere-turns being made equal. It is generally convenient to use a capacitor for the trimming control and to give it the correct sense by connecting it either to the unknown or standard side of the transformer, according to the sign of the spurious reactance.

Standards Multipliers

To keep the number of standards to a minimum the in-phase and quadrature standards are chosen to cover approximately the same range of impedance at the operating frequency of the bridge. Occasionally, it is required to measure a complex impedance the in-phase and quadrature components of which are of quite different orders. It is therefore convenient to be able to shift the effective impedance of one standard with respect to the other. Since the voltage transformer is tapped to provide the decade adjustment of each standard the current transformer must be tapped to shift the relative ranges of the standards. This is shown diagrammatically in Fig. 9. A pair of taps at, say, 10 turns and 100 turns will give a 10 : 1 shift.

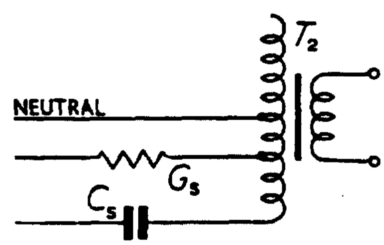


FIG. 9

This facility must however be used with caution. If a measurement were made where the quadrature currents are in the ratio of say 1,000 : 1, perfect standards would be necessary and there would have to be no losses or stray capacitances associated with the wiring.

Any small out-of-phase current introduced by the major component standard could give rise to a considerable error in the measurement of the minor component of the unknown impedance. This will be dealt with in more detail under the subject of 3-terminal measurements.

Range Multipliers

It has been shown how the decade steps are obtained by suitably tapping the standards winding of the voltage transformer. The range of the Bridge can now be extended above and below that of the standards by means of taps on both the voltage and current transformers; as shown in Fig. 10.

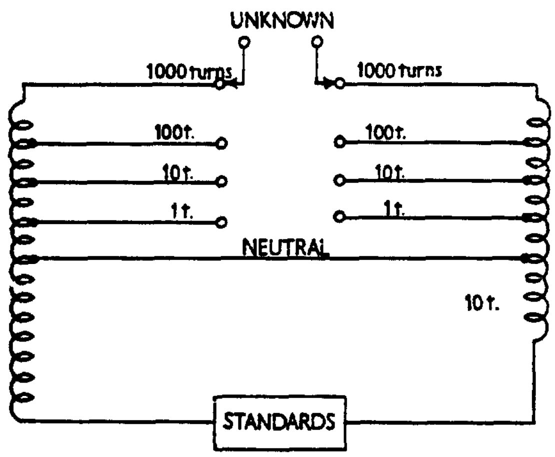


Fig. 10

In the case of capacitance measurements a convenient standard is 0–0·01 microfarads connected between the 100 turn and 10 turn windings respectively. Tapping the unknown windings at 1,000, 100, 10 and 1 turns gives the ranges shown in the table.

Voltage Transformer Tap	*Current Transformer Tap*	*Range*
1000	1000	0–10 pF*
100	1000	0–100 pF
100	100	0–1000 pF
10	100	0–0·01 µF
10	10	0–0·1 µF
1	10	0–1·0 µF
1	1	0–10 µF

* 1 picofarad (pF) = 1 micro microfarad (µµF)

The standard multiplier will extend the low capacitance range to 0–1 pF. If the standard comprises two decade switches and a continuously variable control, the variable will cover the range 0–0·01 pF and a 1% discrimination on this is 0·0001 pF. If proper attention is paid to the screening of the leads and connections, and if the standards are carefully trimmed, the Bridge will measure from 0·0001 pF to 10 microfarads with an accuracy of ±0·1% ±0·0001 pF.

Set Zero Controls

The flexibility of the Bridge is improved by adding a pair of set-zero controls, taking advantage of the fact that at balance there is no interaction between the current paths leading into the current transformer. For instance it may be necessary to measure the grid-anode capacitance of a number of pentode valves. A valve holder is mounted in a suitable jig for connection to the Bridge. The spurious capacitance and leakage paths are removed by connection to neutral, but there remains capacitance and leakage between the grid and anode pins themselves. The set-zero controls permit these to be balanced out so that a direct measurement of valve capacitance can be made.

A small variable capacitor provides a suitable set-zero for reactance and a fixed resistor, combined with a potentiometer, provides the control for conductance. They are adjusted to bring the Bridge to balance with all the standards set to zero, before the unknown is applied.

3-Terminal Measurements

It is often convenient to be able to measure an impedance *in situ*, without disconnecting any other components which may be associated with it. Moreover, with certain test jigs it is often impossible to 'disconnect' the effective shunt and stray capacitances to the base of the jig.

Generally speaking, any circuit can be resolved into a three-terminal network, the arrangement being as shown in Fig. 11.

The impedance to be measured is Z_u, but the effect of impedances Z_{AC}, Z_{BC} must be removed before a measurement of Z_u can be made. Fig. 12 shows the 3-terminal network applied to the Bridge.

The arrangement can be considered as a π – network, where Z_{EN} shunts the voltage transformer and Z_{IN} shunts the current transformer. It is assumed for the moment that the transformers are ideal.

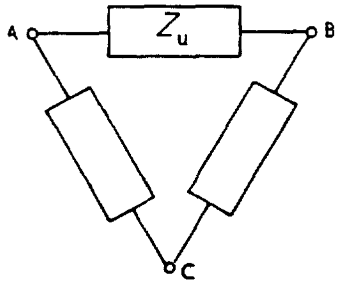


FIG. 11

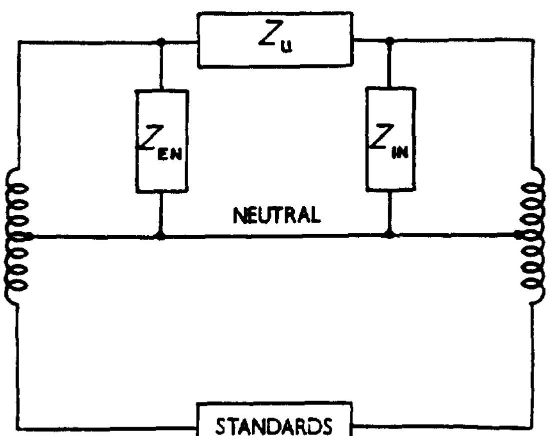


FIG. 12

At balance there is no voltage across the current transformer, and therefore the only effect of Z_{IN} is to reduce the input impedance to the detector, and consequently the off balance sensitivity. If necessary this can be compensated by increasing the detector gain.

Z_{EN} shunts the unknown winding of the voltage transformer and has the full voltage across it at balance. It causes a voltage drop determined by the ratio of its transformed impedance at the transformer primary to the source impedance. However, a voltage drop also occurs in the standards side of the voltage transformer in proportion to the turns ratio. The Bridge is therefore unaffected and no balance error is introduced. The only effect is again to reduce the sensitivity, which can be restored either by increasing the source voltage or the detector gain.

In practice the transformers have a finite short-circuit impedance and shunt loading can cause errors. Difficulty with shunt loading is most likely to arise on the high impedance ranges. The error caused by the loading of the unknown itself is greatest at the low impedance end of each range, amounting to approximately 1 part in 10^5 (a typical figure is 100 ohms total effective series impedance for both transformers in the measurement range 10 to 100 megohms). Shunts having a hundredth of the impedance of the unknown would therefore cause errors of 0·1%, a thousandth of the impedance of the unknown 1%, and so on. In the case quoted one would expect shunt impedances of 10,000 ohms to cause errors of about 1% when using the Bridge on the 10 to 100 megohm range.

It is only when the shunt impedances are very low compared with the impedance under measurement that errors arising from them are likely to be important, and in these circumstances they can be corrected by a simple calculation. Since the unknown has an insignificant loading effect, it can be neglected and the arrangement of Fig. 13 used for computation.

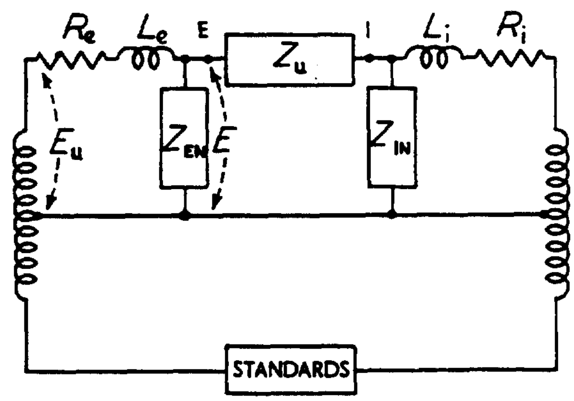


FIG. 13

R_e represents the effective series resistance of the voltage transformer winding and L_e the leakage inductance. Let $Z_e = R_e + j\omega L_e$. Z_i, R_i and L_i are the equivalent components of the current transformer winding.

Since the impedance to be measured is very high compared with the shunt impedances, it can be considered as an open-circuit and the shunting effect on each side of the unknown may be considered separately.

Without any loading, the normal balance equation for the Bridge applies. We have therefore:

$$Z_u = \frac{E_u}{E_s} \cdot Z_s \quad . \quad . \quad . \quad . \quad (7)$$

Due to the effect of the shunt Z_{EN}, the voltage applied to the unknown is reduced to E. To restore balance Z_s must be changed to, say, Z_m.

Therefore $$Z_u = \frac{E}{E_s} \cdot Z_m \quad . \quad . \quad . \quad . \quad (8)$$

But $$E = E_u \cdot \frac{Z_{EN}}{(Z_{EN} + Z_e)} \quad . \quad . \quad (9)$$

Therefore $$Z_u = \frac{Z_{EN}}{(Z_{EN} + Z_e)} \cdot \left[\frac{E_u}{E_s} \cdot Z_m\right] \quad . \quad (10)$$

where $\left[\frac{E_u}{E_s} \cdot Z_m\right]$ is the value read on the Bridge.

For simplicity, consider unity ratio of the voltage transformer,

Then: $$Z_u = \frac{1}{\left(1 + \frac{Z_e}{Z_{EN}}\right)} \cdot Z_m \quad . \quad . \quad . \quad (11)$$

This may be expressed as a series as follows:

$$Z_u = Z_m\left(1 - \frac{Z_e}{Z_{EN}} + \left(\frac{Z_e}{Z_{EN}}\right)^2 - \text{etc.}\right) \quad . \quad (12)$$

For almost all practical cases the second and higher order terms are so small that they may be neglected. An adequate approximation is then given by:

$$Z_u = \left(1 - \frac{Z_e}{Z_{EN}}\right) Z_m \quad . \quad . \quad . \quad (13)$$

A similar argument applies to the error arising from loading the current side of the Bridge. Under these conditions it can be shown that:

$$Z_u = \left(1 - \frac{Z_i}{Z_{IN}}\right) Z_m \quad . \quad . \quad . \quad (14)$$

Therefore the true value of the unknown impedance with both voltage and current transformer shunting is given by:

$$Z_u = \left(1 - \frac{Z_e}{Z_{EN}}\right)\left(1 - \frac{Z_i}{Z_{IN}}\right) Z_m \quad . \quad . \quad . \quad (15)$$

or $$Z_u = \left(1 - \left[\frac{Z_e}{Z_{EN}} + \frac{Z_i}{Z_{IN}}\right] + \frac{Z_e\, Z_i}{Z_{EN}\, Z_{IN}}\right) Z_m \quad (16)$$

Since the short circuit impedance of the transformers is small compared with the shun impedances, the term $\frac{Z_e\, Z_i}{Z_{EN}\, Z_{IN}}$ may be neglected, giving a final approximation:

$$Z_u = Z_m\left[1 - \left(\frac{Z_e}{Z_{EN}} + \frac{Z_i}{Z_{IN}}\right)\right] \quad . \quad . \quad . \quad (17)$$

Where Z_u is the true value of the unknown impedance and Z_m the value read on the bridge.

Z_e and Z_i are constants for the bridge and can be measured quite simply on each range by measuring the apparent change in value of a 2-terminal impedance when known shunts are connected across the transformer windings.

Accurate Measurement of Minor Components of Complex Impedances

There is one type of measurement which must be handled with great care, and precautions taken to keep the loading to an absolute minimum: this is when an accurate measurement is required on a minor component. Suppose for example it is required to measure power factors of the order of 0·0001. The minor component is only 1 part in 10^4 of the major component, and very small phase shifts indeed will cause errors of this magnitude. The modulus of the voltage and current can be regarded as unchanged, because a quadrature component of 1 part in 10^4 produces a change in modulus of only $\frac{1}{2}$ part in 10^8. Phase is the important factor.

Consider the voltage side of the bridge. The new voltage caused by the loading is given by the factor $1 - \frac{Z_e}{Z_{EN}}$ to the nearest approximation.

Let $\frac{Z_e}{Z_{EN}} = |K| \underline{/x}$. Then

$$1 - \frac{Z_e}{Z_{EN}} = (1 - |K| \cos x) - j|K| \sin x$$

If the unknown is a pure capacitance of value C, the current through it will be $\omega C|K| \sin x + j\omega C (1 - |K| \cos x)$. But this is equivalent to removing the loading and shunting the capacitor with a resistance $R = \frac{1}{\omega C|K| \sin x}$ which would give it a power factor of $|K| \sin x$, approximately. $|K| \sin x$ therefore represents the error term caused by loading the voltage transformer when measuring power-factor. A similar error is caused by loading the current transformer and the total error is the sum of the two. The worst condition occurs when the loading impedance is in quadrature with the series impedance of the winding. Then $x = \frac{\pi}{2}$ and the error is $\frac{Z_e}{Z_{EN}}$ on the voltage side and $\left(\frac{Z_i}{Z_{IN}}\right)$ on the current side.

Measurement of Network Characteristics

The facility of 3-terminal measurements and the readiness with which either or both the conductance and reactance standards can be made effectively negative by merely switching to a winding of reverse sense, make the transformer bridge a most efficient instrument for measuring the characteristics of networks. Transfer admittance, for example, is simply measured by the arrangement shown in Fig. 14.

The input of the network is connected between the voltage terminal and neutral. The output is connected in series with its terminating resistance R_T between the current terminal and neutral. Let the transfer admittance, defined as the current flowing in the terminating resistance for unit input voltage, be $A\underline{/x}$. Then, writing turns for volts, the ampere-turns on the unknown side of the current transformer are $N_u n_u A\underline{/x}$. (Note: at balance the network is properly terminated because the low potential end of R_T is at neutral potential). The ampere-turns on the standards side are $N_s n_s (\pm G \pm j\omega C)$ where G is the conductance and C the capacitance. Equating the two we have:

$$A\underline{/x} = \frac{N_s n_s}{N_u n_u} (\pm G \pm j\omega C)$$

Therefore $$A = \frac{N_s n_s}{N_u n_u} \sqrt{G^2 + \omega^2 C^2}$$

and $$x = \tan^{-1} \pm \omega C/G$$

In practice $N_s n_s/N_u n_u$ is the bridge ratio, the reading of which is given directly on the range switch. The calculations are therefore quickly made.

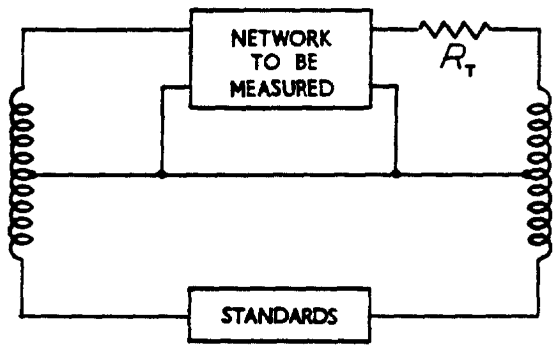


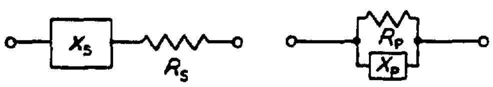
Fig. 14

PARALLEL TO SERIES CONVERSION

Because the bridge sums currents, it measures impedance as a parallel combination of the in-phase and quadrature components. For some purposes this is inconvenient and it is necessary to know the equivalent series components. The relevant equations are given in the table overleaf.

To avoid the awkward factor 2π appearing in all the calculations, the source frequency on certain bridges is fixed at 1592 c/s, when $\omega = 10{,}000$. This not only simplifies the transformation of parallel to series circuits, but makes it possible to read reactance in ohms directly from the capacitance scale. By reversing the sense of the capacitance standard connections to

SERIES CIRCUIT — PARALLEL CIRCUIT



$$Q = \frac{X_s}{R_s} = \frac{R_p}{X_p}$$

$$R_s = R_p \frac{1}{(1+Q^2)} \qquad R_p = R_s(1+Q^2)$$

$$X_s = X_p \frac{Q^2}{(1+Q^2)} \qquad X_p = X_s \frac{(1+Q^2)}{Q^2}$$

WHEN $Q \gg 1$:

$$R_s = \frac{R_p}{Q_p^2} \qquad R_p = Q^2 R_s$$

$$X_s = X_p \qquad X_p = X_s$$

WHEN $Q \ll 1$:

$$R_s = R_p \qquad R_p = R_s$$

$$X_s = Q^2 X_p \qquad X_p = \frac{X_s}{Q^2}$$

the current transformer, it is possible to measure inductance, the reading of which is simplified if $\omega = 10{,}000$.

EXTENDING THE RANGE OF MEASUREMENT

Although the bridge in its existing form covers an extremely wide range the accuracy will deteriorate rapidly when measuring impedances lower than about 10 ohms, when the series resistance of the leads or switches become significant. For the measurement of very low impedances a different technique must be adopted.

So far, a voltage has been applied across the unknown and the resulting current compared with that in a standard. With impedances of the order of 10 ohms and below, it becomes increasingly difficult to apply a precise voltage, owing to the uncertainty of the voltage drop in the connections. It is now preferable to pass a current through the unknown and to measure the voltage drop across it. This cannot be done directly with the existing bridge arrangement, but it can be done by making the unknown the shunt element of a T-network (the series arms being fixed standard resistors) and measuring the transfer admittance.

The method is a general one and can be applied to any complex impedance in the range zero to, say, 10 ohms. For simplicity it is shown in Fig. 15 applied first to a pure resistance and then to a pure reactance, the unknown being r.

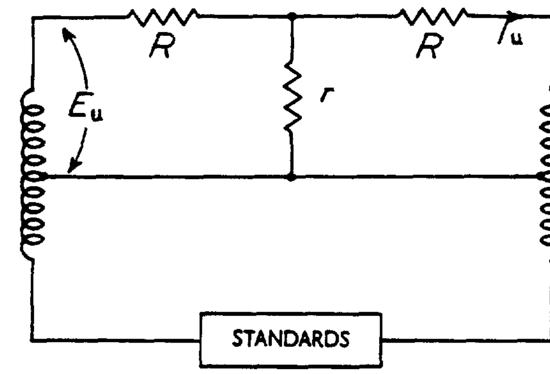


Fig. 15

The two resistors R are standards of known value. Let the current leaving the network be I_u, remembering that at balance there is zero voltage across the current transformer. The voltage across r is I_uR. The current through r is I_uR/r. The current through the first resistor of the network is $I_uR/r + I_u$. The input voltage E_u is therefore:

$$E_u = I_uR(1 + R/r) + I_uR$$

and $$E_u/I_u = R^2/r + 2R$$

If R is very large compared with r (say 1000 times), the second term can be ignored and we have a close approximation:

$$E_u/I_u = R^2/r$$

But E_u/I_u is the ratio that would be obtained by connecting a resistor R_u straight across the bridge instead of the T-network. That is to say the bridge will measure the T-network as though it were a resistor of value R_u ohms. So we can put

$$r = R^2/R_u$$

where R_u is the resistance read on the bridge. Alternatively, if the bridge reads conductance directly, we have

$$r = R^2G$$

Several ranges can be provided by switching R in pairs, or by range changing on the transformer taps, or both. In this way it is possible to make measurements down to a few microhms without undue difficulty. If r is replaced by an inductance L and the above argument repeated, we have:

$$E_u/I_u = R^2/i\omega L$$

In this case the bridge measures the T-network as though it were a capacitance of value L/R^2, so we have

$$L = R^2C$$

where C is the capacitance measured on the bridge. Again, it is a simple matter to arrange a number of switched ranges, and inductance down to the order of one millimicrohenry can be measured.

CONCLUSION

It has been shown that the attributes of the transformer ratio-arm bridge are such that complicated measurements can be quite simply made which would be very difficult, if not impossible, with a conventional bridge arrangement.

The advantages of the transformer bridge may be summarized as follows:

1. The measurement is dependent upon the product of the current and voltage ratios for each fixed standard used. Both of these ratios are easily obtained with great accuracy, increasing the possible range of measurement to a figure well beyond the scope of impedance arms. Voltage and current ratios each of 1,000 : 1 are possible, giving an overall ratio of one million to one.

2. Since the ratios depend only upon the number of turns on the transformer windings, they are permanent and calibration is unnecessary.

3. Only one standard is required for each decade. Furthermore, the transformer ratios may be used to set the standard in one decade against that in another, so that only two fixed standards of known accuracy are required, one resistive and one reactive.

4. The standards need not be pure. The effects of a resistive term associated with a reactance standard and of a reactive term associated with a resistive standard can be removed entirely at a given frequency.

5. The measurement of very small capacitances by conventional means is extremely difficult, due to the effect of stray capacitance and the difficulty in obtaining a standard of sufficiently low value. By means of the transformer bridge, it is possible to measure capacitors of the order of ·0002 pF at the ends of leads having some thousand times this capacitance.

6. As the sign of a voltage or current can be reversed simply by reversing the connections to the relevant transformer winding, measurements can be made in all four quadrants of the complex plane. The bridge can therefore be used to measure transfer impedance as well as direct impedance.

7. The bridge will measure the impedance between any pair of terminals of a 3-terminal network. *In situ* measurements are possible on impedances remote from the bridge and on components wired into a circuit. The capacitance of long connecting leads and spurious impedances connected to the unknown terminals are completely counteracted by the appropriate use of the third terminal of the bridge.