Aids in the Design of Intermediate-Frequency Systems*

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Summary—A set of design curves has been developed by means of which the over-all selective performance of an intermediate-frequency amplifier may be predicted with the expenditure of little time or effort. The design chart is based on identical circuits, but with the appropriate conversions, variations in circuit Q can be accommodated readily by determination of the equivalent identical circuits. The design method permits a prediction of the allowable spacing between equal desired and undesired signals in the frequency spectrum to maintain the desired signal within 6 decibels of resonant output and the undesired signal at more than 60 decibels below resonant output, when variations in frequency due to such causes as modulation, drift, and setting are taken into account. Methods of arriving at a rapid approximation to the over-all curve, in those cases in which circuits of different coupling are cascaded, are discussed.

ONE OF the important devices with which the radio engineer works is the resonant circuit. Fortunately, in most of its applications, this is a linear device and its performance may be accurately determined by calculation. Furthermore, when resonant circuits are used with class A amplifiers, as in an intermediate-frequency amplifier, the whole system is linear and its over-all performance is calculable. While the information obtainable from such a calculation does not provide all that is necessary to lay out and build a successful intermediate-frequency amplifier, it does provide a criterion of performance. The selected values of coil Q for the design may be measured in the laboratory, the coils set up on a coil form and adjusted for the desired proportion of critical coupling in a single-amplifier stage and the experimental results matched with the calculated design for these elements. From this basis the amplifier is then constructed. Usually the over-all amplifier will not exhibit the calculated characteristic because of unpredicted regeneration or circuit loading. Then the sources of difficulty are traced down and finally the over-all amplifier characteristic is brought into agreement with the calculated design.

Design Requirements

This paper will be limited to the design calculation of the selectivity characteristic of a coupled circuit amplifier. The first step would be to set up the requirements which must be met. Suppose we have a communication system to provide, in which a transmitter and a receiver are to maintain communication after being tuned to a predetermined frequency by their calibrated charts or dials. In addition we shall assume a frequency spacing for adjacent-channel operation. The problem is, then, to find the intermediate-frequency characteristic which is required to hold the desired signal and reject an adjacent undesired signal. These requirements specify two features of the characteristic. First, to hold the desired signal requires a portion of the characteristic to pass a range of frequencies with only slight variations in output. Second, after sufficient width of the frequency characteristic has been provided for the desired signal, a certain sharpness is required on the sides of the characteristic to reduce the output from an undesired adjacent signal to a level assumed to be sufficiently small to avoid interference.

General practice has established the level of variations for satisfactory reception to lie within a 6-decibel range. While the amount of attenuation required on an undesired signal (which may be much stronger than the desired signal) is different for each interfering situation, a general accepted amount of attenuation for this purpose is 60 decibels. Both the acceptance width of the frequency characteristic and the attenuation at the adjacent channel for our hypothetical communication system depend upon several factors. The acceptance band must encompass modulation sidebands (frequency or amplitude modulation) and it must allow for errors in chart or dial reading and calibration inaccuracies. Further, it must allow for frequency variations during unattended operation which implies accommodation

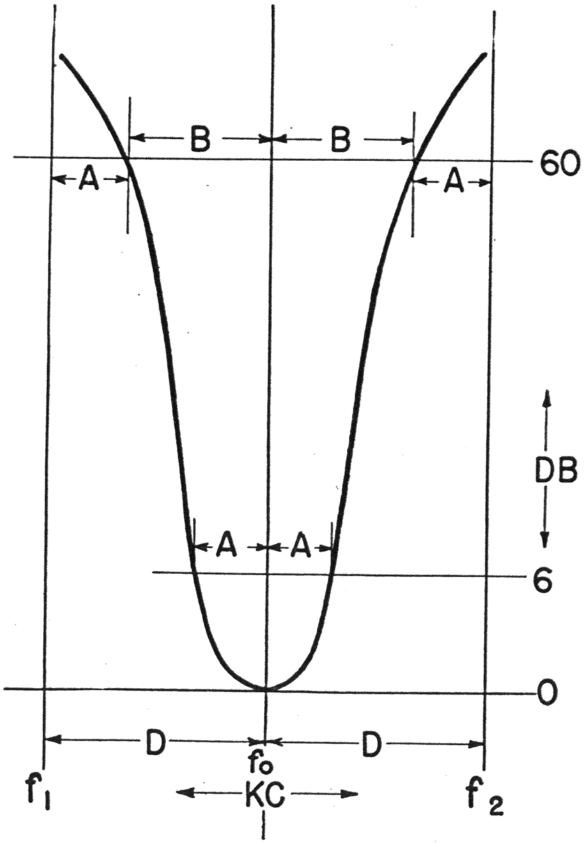

Fig. 1—Communication setup.

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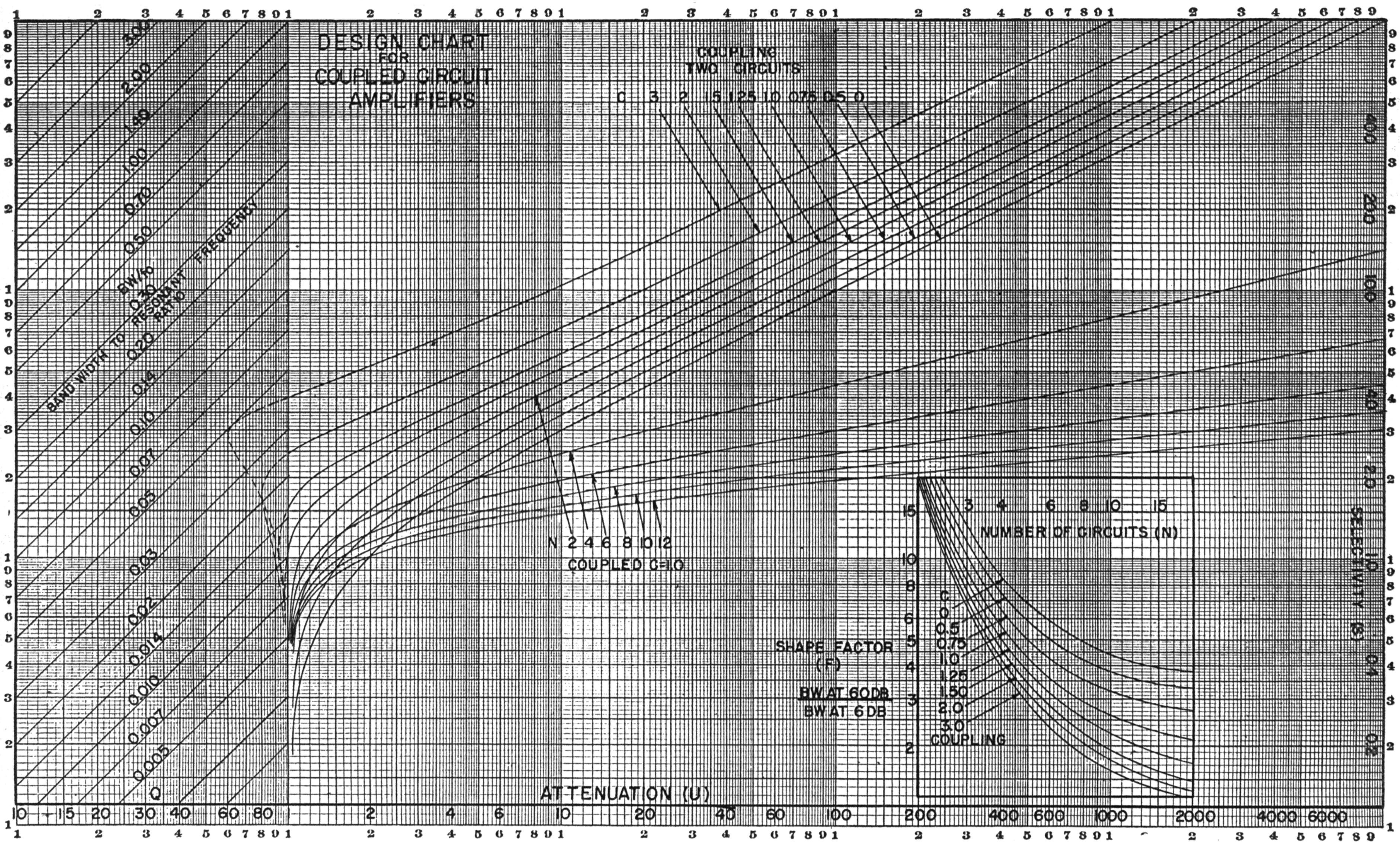

Fig. 2—Design chart for coupled-circuit amplifiers.

of frequency drifts due to climatic or power-supply changes. Since tuning and modulation excursion may not only cause deviations from the desired-signal frequency at receiver and transmitter, but also may cause approaches to and of the undesired signal, these frequency deviations must be considered to occur equally at each attenuation level. The situation is illustrated graphically in Fig. 1. A represents the total frequency deviation allowable to maintain the desired signal (f_0) within a 6-decibel range on the selectivity curve; it also represents the frequency deviation allowable to maintain adjacent signals (f_1, f_2) outside the interference level of 60 decibels below the desired signal. The selectivity curve required to provide a channel spacing D will therefore be specified by the "shape factor" F or ratio of frequency deviations B/A. The shape factor required of the intermediate-frequency amplifier, therefore, is

$$F = D/A - 1. \tag{1}$$

As a concrete illustration we shall use a fictitious set of figures. Assume that a transmitter operates at a frequency of 10 megacycles and that the maximum frequency drifts of transmitter and receiver add up to not more than ± 0.1 per cent of the operating frequency, that errors in setting both transmitter and receiver add up to 0.05 per cent or less of the operating frequency and that the maximum modulation sidebands require ± 5 kilocycles. In addition assume that the channel spacing desired between adjacent signals is 80 kilocycles. The acceptance band will be

$$A = 10 + 5 + 5 = 20 \text{ kilocycles} \tag{2}$$

and the shape factor of the required selectivity curve will be

$$F = 80/20 - 1 = 3. \tag{3}$$

Design Chart

A chart, Fig. 2, has been prepared by means of which the required shape factor may be translated into a definite answer in terms of coil Q, number of circuits and coupling between circuits. The basis of the chart is a set of universal selectivity curves. It may be shown[1] that any pair of coupled resonant circuits may be represented by

$$U = \frac{\sqrt{(1 + C^2 - S^2)^2 + 4S^2}}{1 + C^2} \tag{4}$$

where U is the attenuation through the circuits (unity at resonant frequency), S is a selective variable proportional to frequency deviation from resonance, and C is a coupling parameter expressing proportionality to critical coupling ($C=1$). Critical is used here to define the transition point between single- and double-peaked resonant curves. These terms are specifically defined as follows:

$$U = G_0/G_f \tag{5}$$

$$S = (BW/f_0)Q \tag{6}$$

$$1 + C^2 = (Q_g/Q_a)^2(1 + K^2Q_g^2) \tag{7}$$

$$Q = (Q_g/Q_a)Q_g \tag{8}$$

[1] J. E. Maynard, "Tuned transformers," *Gen. Elec. Rev.*, vol. 46, pp. 559–561, October; and pp. 606–609; November, 1943.

where G_0 is amplifier gain at resonant frequency f_0, and G_f is amplifier gain at the frequency under consideration, BW is the bandwidth of the selectivity curve (twice frequency deviation from resonance), K is the coefficient of coupling between the two circuits, Q_g is the geometric mean, and Q_a the arithmetic mean of the primary and secondary Q's. The approximations used in the development of (4) result in selectivity curves which are symmetrical about resonant frequency; lack of agreement with these curves in practice will be found to be largely in this lack of symmetry which is usually, except for very low values of Q, negligible. Since the results are symmetrical, the curves may be plotted on one side of resonance only, as in the chart.

By means of the ratio lines at the left of the chart the attenuation to be obtained for any ratio of bandwidth to resonant frequency may be determined for a given circuit Q by following the Q ordinate to the ratio line, crossing over to the curve for the coupling to be used along the value of S so determined, and then down from the intersection with the selectivity curve to the corresponding value of attenuation U. For instance, using $Q=50$, $BW/f_0=0.10$, we find $S=5.0$ and for critical coupling ($C=1.0$), the attenuation will be $U=12.5$.

In order to extend the use of the curves to an amplifier containing several such pairs of tuned coupled circuits, a set of curves based on the curve for two coupled circuits at $C=1$ have been drawn for cascaded pairs of circuits. If the value of $C=1.0$ is substituted in (4) the formula for a critically coupled pair of circuits is obtained:

$$U = \sqrt{1 + S^4/4}. \tag{9}$$

For four circuits cascaded in critically coupled pairs the attenuation is the square of (9), for six circuits the attenuation is the cube of (9), etc. This implies identical values of Q and C in the coupled circuits. Since for any value of S we move up in attenuation exponentially in the same degree for each pair of cascaded circuits, the whole set of curves for two circuits can be extended graphically to any number of cascaded pairs for which we have the basic critically coupled curve. A numerical example will best illustrate this point. Suppose we wish an attenuation of 100 from eight circuits cascaded in pairs for $BW/f_0=0.10$ and wish to use 25 per cent overcoupling ($C=1.25$). Following $U=100$ to the curve for eight circuits we find $S=2.44$; following this back to the two circuit curve for $C=1$ we arrive at a point corresponding to $U=3.16$; following this up to the two circuit for $C=1.25$ we find a value of $S=2.9$. Follow this value of S to the ratio line $BW/f_0=0.10$ and we have the required $Q=29$.

The shape-factor chart in the lower right corner is a plot of the ratio of bandwidth obtained at 60 to that at 6 decibels attenuation, for various amounts of coupling, against the number of circuits used in the amplifier. These curves are obtainable from the other curves in the chart by taking ratios of S at $U=1000$ to those at $U=2$.

Application

Returning to (3), we wish to design an amplifier to have a shape factor $F=3.0$. First it is necessary to choose a resonant frequency. This involves several interference considerations such as images and harmonic beats; we will assume, however, that 2.0 megacycles is a satisfactory choice. From the shape-factor chart $F=3$ might be obtained with a slight margin from eight circuits cascaded in critically coupled pairs. This would require three amplifier stages containing four intermediate-frequency transformers. The ratio BW/f_0 required is $2A/f_0$ at 6 decibels, or from (2) $BW/f_0 = 40/2000 = 0.02$. Follow $U=2$ to the curve for eight circuits to find $S=1.14$ and following S to $BW/f_0=0.02$, we find the Q required to be 57.

Ordinarily, a radio-frequency amplifier would precede the intermediate-frequency system. Suppose this consisted of two cascaded single circuits. The attenuation would then be found on the two-circuit curve for $C=0$. The resonant frequency for these circuits is 10,000 kilocycles and at the bandwidth for 6 decibels in the intermediate-frequency system, BW/f_0 for the radio-frequency system is $40/10{,}000=0.004$. Assume a Q of 50 in the radio-frequency circuits, then $U=1.05$ so that the over-all attenuation at 40 kilocycles bandwidth will be slightly in excess of 6 decibels. At the intermediate-frequency bandwidth of 120 kilocycles (60 decibels), the radio-frequency attenuation for $BW/f_0=120/10{,}000 = 0.012$ will be $U=1.35$ so that the over-all attenuation at 120 kilocycles bandwidth will be well over 60 decibels. It will be somewhat more than 1350 since the shape factor F is actually 2.9. This might suggest the possibility of using fewer intermediate-frequency circuits and utilizing the additional attenuation in the radio-frequency amplifier to meet the requirements. Going back to the shape factor curves, it will be noticed that we are just slightly short of $F=3$ if we use six circuits at 25 per cent overcoupling ($C=1.25$). The question then will occur: Is this amount of overcoupling desirable? For two circuits the attenuation minimum occurs at $U=0.975$, cubing this for six circuits the over-all intermediate-frequency minimum will be $U=0.925$. Since this will be somewhat alleviated by the radio-frequency characteristic we shall assume it is not objectionable. For six circuits at $U=2$, $S=1.23$; crossing to two circuits at $C=1$ we find $U=1.25$; and up to the curve for $C=1.25$, $S=1.61$; over to $BW/f_0=0.02$ we find that a Q of 80 is required in the intermediate-frequency circuits. The attenuation at $BW=120$ kilocycles would be traced from the intersection of $Q=80$ and $BW/f_0=0.06$ which occurs at $S=4.8$, following to the two-circuit curve for $C=1.25$ we arrive at $U=8.8$, coming down to the two-circuit curve for $C=1.0$ and across to the six-circuit curve along $S=4.2$, the attenuation is $U=700$. Applying the radio-frequency attenuation of 1.35, the over-all attenuation 60 kilocycles from resonance will be (1.35) (700) or 950.

General Observations

Several pertinent observations[1] on the character of these curves will aid in their use. By differentiating (4) with respect to S and solving for zero slope, the minimum points are found to occur at

$$S_1 = \pm \sqrt{C^2 - 1} \tag{10}$$

$$U_1 = 2C/(C^2 + 1). \tag{11}$$

It will be observed that, on a log-log plot, all the curves for two circuits approach asymptotically a line of the same slope. By taking the limit of (4) as S becomes large, it can be seen that this is a line of slope 2/1 on the chart. Similarly, for any number of circuits N the asymptotic line has a slope N regardless of Q or coupling. This is obviously also true if the curves are plotted against frequency deviation from resonance on a log-log scale. The general outline of any selectivity curve is, therefore, a line of slope N intersecting the line $U=1$ at some value of S. By solving the equation for the asymptotic line (two circuits) for S at $U=1$, the intercept is found to be

$$S_0 = \pm \sqrt{C^2 + 1}. \tag{12}$$

At this value of S the actual attenuation on the curve will be

$$U_0 = 2/S_0. \tag{13}$$

The actual curve intersects $U=1$ at a value of S

$$S_0' = \pm \sqrt{2}\, S_1. \tag{14}$$

In those cases in which the selectivity curve approaches its asymptote from the outside, it crosses to the inside of the asymptote in the lower part of the curve; this intersection with the asymptote occurs at

$$S_a = \pm S_0^2/S_0' \tag{15}$$

$$U_a = S_a/S_0'. \tag{16}$$

All asymptotes, regardless of slope N, pass through $U=1$ at the same point $S=S_0$, for a given coupling.

We have been considering, up to this point, designs in which the coupled pairs of circuits could be represented by equivalent identical coupled pairs. Although this is usually quite practicable, and in fact usually desirable so that intermediate-frequency transformers are alike, there may be occasions in which it is desired to make succeeding transformers appreciably different. In such cases it is necessary to calculate the characteristic against frequency of each transformer or identical group of transformers and combine the curves so obtained by taking the product of attenuations, in order to obtain an accurate characteristic. It is possible, however, to approximate the over-all characteristic in one operation. The asymptote to any pair of coupled circuits from (4) will be

$$U = (S/S_0)^2. \tag{17}$$

It is apparent that for N circuits cascaded the over-all or product attenuation will be

$$U = (S/S_{0m})^N \tag{18}$$

along the asymptote, where S_{0m} is the geometric mean of all the values of S_0 for the individual pairs of circuits.

Taking logarithms of each side of this equation it can be seen that the plot on log-log paper will be a line of slope N starting from S_{0m} at $U=1$. All factors or products of S or S_{0m} cause a lateral shift in the curve (Fig. 3)

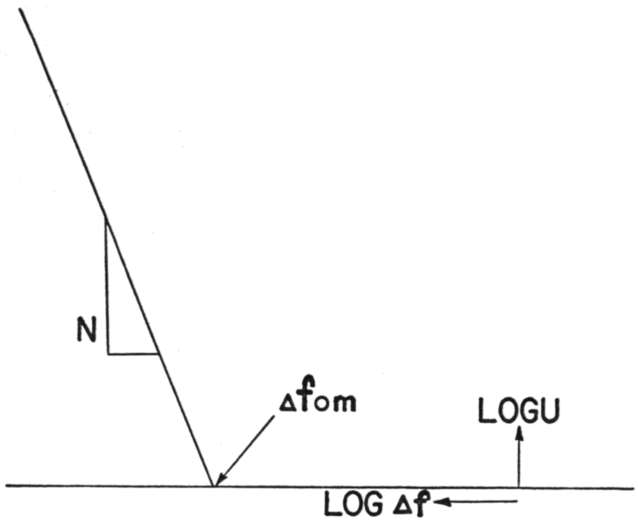

Fig. 3—Approximate boundary of selectivity curve for N circuits.

and can be absorbed in the mean value of S_{0m}. This means that (18) expresses the asymptote for N circuits regardless of how they are coupled or of what Q they have. This may include single circuits if we think of S_0 as unity for a single circuit (zero coupling). Since each value of S_0 is proportional to the product of frequency deviation from resonance (Δf) and Q, the mean S_{0m} is proportional to the product of a mean deviation (Δf_{0m}) and a mean Q_m. Therefore when a plot of attenuation against frequency deviation is made on log-log paper the asymptote will start from Δf_{0m} and have a slope N (Fig. 3), where

$$\Delta f_{0m} = S_{0m} f_0 / 2Q_m \tag{19}$$

$$Q_m = \sqrt[N]{Q_1 Q_2 Q_3 \cdots Q_N} \tag{20}$$

$$S_{0m} = \sqrt[N]{S_{01} S_{02} S_{03} \cdots S_{0N}} \tag{21}$$

with the following interpretation. For a single circuit Q_n is the Q of the n^{th} circuit; for a coupled pair of circuits $(n, n+1)$ the product $Q_n Q_{n+1}$ is the square of the Q in (8). For a single circuit S_{0n} is unity; for a coupled pair of circuits $(n, n+1)$ the product $S_{0n} S_{0(n+1)}$ is $1+C^2$, as in (7). To obtain a closer approximation to the over-all curve an equivalent mean coupling from the value of S_{0m} may be used. This remains, however, an approximation which is particularly likely to be inaccurate where there is considerable curvature since the cascading of different characteristics will not necessarily give us an over-all shape identical with any power of a curve for a coupled pair. In such cases the use of the shape factor chart needs to be seasoned with good judgment since the curve shape at 6 decibels attenuation may depart appreciably from that used to develop the shape-factor chart.

It should be noted that nowhere has the impedance or the tuning capacity of the circuits entered into the discussion. This occurs because we have been limited to a discussion of selectivity which is a ratio of gains so that impedances cancel out in this ratio. Selectivity, furthermore, is a function of Q which includes the effects of all losses whether in coil, capacitor, or circuit and which is a ratio of kilovolts-amperes to kilowatts so that tuning capacity has no direct bearing on the characteristic except through this Q ratio. As long as Q is 10 or more, all losses may be lumped into an equivalent constant Q in this manner with negligible errors. The preceding definition of Q is somewhat more general than reactance-to-resistance ratio.

Conclusion

A general inspection of the chart will reveal some conclusions of interest. In most applications the shape factor will fall between $2\frac{1}{2}$ and 5. Further improvement in shape factor becomes increasingly more difficult of attainment as the number of circuits used increases. Beyond eight or ten circuits, improvement in shape factor is very expensive in terms of circuits required. Increasing coupling to more than 25 per cent over critical is not very effective in improving shape factor. Shape factors using a permissible amount of overcoupling attain a figure approximately half that obtained by cascading the same number of circuits singly ($C=0$). Overcoupling of an amount in the region of 125 per cent critical is usually quite acceptable; a pair of circuits coupled at $C=1.25$ will produce a response curve with 2.5 per cent rise at the double peaks ($U=0.975$).