# JOSEPHSON D/A CONVERTER WITH FUNDAMENTAL ACCURACY*

C.A. Hamilton, C.J. Burroughs, and R.L. Kautz
National Institute of Standards and Technology
Boulder, CO 80303

## Abstract

A binary sequence of series arrays of shunted Josephson junctions is used to make a 14-bit D/A converter. With thirteen bias lines any step number in the range -8192 to +8192 (1.2 to -1.2 V) can be selected in the time required to stabilize the bias current (a few microseconds). The circuit makes possible the digital synthesis of very-accurate ac waveforms whose amplitude derives directly from the internationally accepted definition of the volt.

## Introduction

A typical Josephson array voltage standard uses 20 000 or more junctions driven at 75 GHz to generate about 200 000 voltage steps that span the range from −14 to +14 V [1]. Although an array can be set to any step, the procedure to select a particular step is so slow that the standards are useful only for dc measurements. This paper describes a new Josephson circuit that allows the rapid selection of any step number. The new circuit has $N$ digital inputs which define any one of $2^N$ evenly spaced output voltages. The circuit is therefore a D/A converter whose output voltage has the full accuracy of the SI Volt Representation.

## Circuit Operation

The junctions and microwave drive used in the new standard are designed to generate a current-voltage ($I$–$V$) curve similar to that shown in Fig. 1a. This curve has three stable voltages: 0, $f/K_J$, and $-f/K_J$, where $f$ is the microwave drive frequency and $K_J$ is the Josephson constant. The three voltages are uniquely selected by the bias currents 0, $+I_s$, and $-I_s$. The output voltage is accurate for any input current within about ±20% of the nominal value. When $M$ junctions similar to that described in Fig. 1 are connected in series, the steps occur at the voltages 0 and $\pm Mf/K_J$. Figure 1b is an experimental result using a reference frequency of 75 GHz and shows the $I$–$V$ curve of 2048 junctions in series. The steps occur at 0 and $\pm 2\,048 \times (75\ \mathrm{GHz})/(483\,597.9\ \mathrm{GHz/V}) = \pm 0.317$ V.

As shown in Fig. 2, the Josephson D/A converter consists of a binary sequence (1, 2, 4, 8, ...) of independently biased arrays. Any given output voltage is generated by applying bias currents to the appropriate set of arrays. The binary sequence of array lengths makes it possible to choose bias currents to generate a voltage $\pm Mf/K_J$ where $M$ is any integer from 0 up to the total number of junctions in all arrays. The vertical steps in the junction $I$–$V$ curves ensure that the output voltage will be accurate over about a ±20% variation in $I_s$ from its nominal value.

## Junction Design

The ideal $I$–$V$ curve for the junctions used in the D/A converter has constant-voltage steps at $V = 0$ and $V = f/K_J$ that extend over the largest possible non-overlapping ranges of dc bias. Large-amplitude steps are obtained and chaotic behavior is avoided when the junction parameters meet the condition [2]

$$2\pi f^2 C/K_J I_c \gg 1, \qquad (1)$$

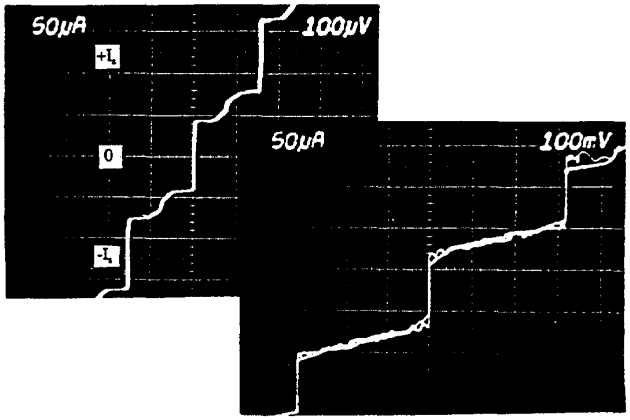

Fig. 1. (a) The $I$–$V$ curve of a single shunted junction driven at 75 GHz and (b) the $I$–$V$ curve for an array of 2048 junctions.

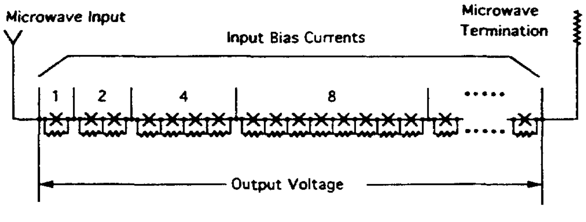

Fig. 2. A Josephson D/A converter based on a binary sequence of shunted junction arrays.

---

where $I_c$ is the junction critical current, $R$ is the shunt resistance, and $C$ is the shunt capacitance. In this case, the dc bias range of the $n$th step at voltage $V_n = nf/K_J$ is given by [2]

$$V_n/R - I_c|J_n(v_{rf})| < I < V_n/R + I_c|J_n(v_{rf})|, \quad (2)$$

where $J_n$ is the $n$th order Bessel function and $v_{rf} = V_{rf}/V_1$ is the amplitude of the applied microwave voltage $V_{rf}$ normalized to the voltage of the first step. According to Eq. (2), the largest possible $n = 0$ and $n = 1$ steps are obtained in the same $I$–$V$ characteristic when $v_{rf}$ is chosen to simultaneously maximize $|J_0(v_{rf})|$ and $|J_1(v_{rf})|$. The maximum results for $v^*_{rf} = 1.435$, for which argument $J_0 = J_1 = 0.5476$. Applying Eq. (2) to this case shows that the $n = 0$ and $n = 1$ steps will overlap unless

$$f/K_J I_c R > 2J_0(v^*_{rf}) = 1.095. \quad (3)$$

Ideally, the junctions used in the array should meet the conditions expressed by Eqs. (1) and (3).

The condition given by Eq. (1) requires using a junction with a critical current density small enough that its plasma frequency is much less than the microwave drive frequency. At typical operating frequencies, the condition given by Eq. (2) requires a junction with a subgap $I_c R$ product of about 0.1 mV. The 100 $\mu$A junctions are therefore shunted with an external resistor of 1 $\Omega$.

## Microwave Distribution

Even if all of the junctions in an array are nearly identical, their $I$–$V$ curves will be similar only if each receives roughly the same microwave power. As in zero-bias arrays [2], a uniform microwave distribution is obtained by designing the array to act as a low-loss transmission line terminated by a matched load. Because microwaves are not significantly attenuated between the beginning and end of the array, each junction receives nearly the same power.

## External Loading

The circuit described so far has an important limitation because any current drawn at the output will shift the bias points of the junctions. Even small load currents of a few tens of microamperes may shift one or more junctions to a non-quantized voltage. However, if the load impedance is known, most of the load current can be supplied by a semiconductor D/A converter which is programmed to deliver the predicted load current ($V_0/R_L$). The Josephson array then needs to supply only the difference between the predicted and actual load currents. This addition increases the output current capability of the circuit by a factor of 10–100 with no loss in accuracy.

## Experimental Realization

A 14-bit version of the circuit shown in Fig. 2 has been fabricated and tested. Although fabrication defects and trapped magnetic flux prevented operation of some bits, the least significant 9 bits are fully functional leading to a maximum output voltage of ±77 mV with 0.15 mV resolution. The accuracy of the output has been confirmed to ±1 $\mu$V. Figure 3a shows a synthesized ±77 mV triangle wave using the most significant 4 bits of the 9 bit converter. Figures 3b, and 3c show the result with the 5 and 6 most significant bits in operation. Load compensation was not required for this data because the D/A output was connected only to the 1 M$\Omega$ oscilloscope input. The triangle wave frequency in the data of Fig. 3 is entirely limited by the automated test system used to drive the input bias currents. The Josephson D/A converter should be capable of input sample rates greater than 1 MHz. This will make possible the synthesis of ac waveforms with a calculable RMS value.

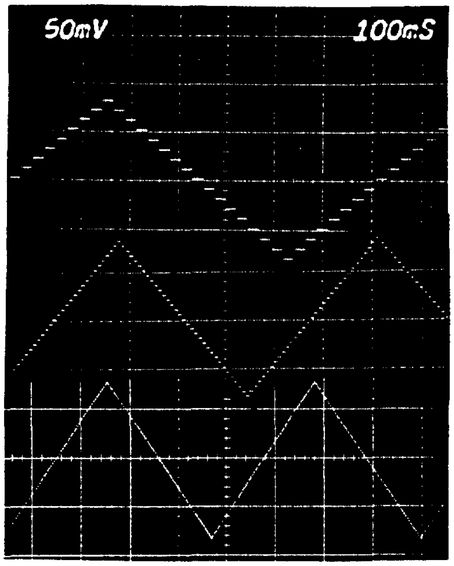

Fig. 3 Synthesized triangle waves using the 4, 5, and 6 most significant bits of the Josephson D/A converter.

## References

[1] C. A. Hamilton, Charles Burroughs, and Kao Chieh, "Operation of NIST Josephson Array Voltage Standards," J. Res. Natl. Inst. Stand. Technol. vol. 95, pp. 219-235, May 1990.

[2] R. L. Kautz, "Design and operation of series-array Josephson voltage standards," in Metrology at the Frontiers of Physics and Technology, edited by L. Crovini and T. J. Quinn, Amsterdam: North-Holland, pp. 259–296, 1992.