Bilateral Comparison of 1.018 V and 10 V Standards between the KEBS (Kenya) and the BIPM January to March 2018 (part of the ongoing BIPM key comparison BIPM.EM-K11.a and b)

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Introduction

As part of the ongoing BIPM key comparison BIPM.EM-K11.a and b, a comparison of the 1.018 V and 10 V voltage reference standards of the BIPM and the Kenya Bureau of Standards, KEBS, was carried out from January to February 2018. Two BIPM Zener diode-based travelling standards (Fluke 732B), BIPM_8 (Z8) and BIPM_9 (Z9), were hand carried to KEBS and back to BIPM by M. Gibson Aguko. Since the total duration of the travels from one institute to the other was not exceeding the capabilities of the self-powering of the Zeners (36 hours), the transfer standards didn't need any additional battery connected in parallel to the internal battery. At KEBS, the reference standard for DC voltage is a 732B Zener standard traceable to a primary voltage by means of a calibration service requested once a year from a National Measurement Institute (UME, Turkey) operating a Josephson Voltage Standard (JVS) two times per year . The output EMF (Electromotive Force) of each travelling standard was measured against the Zener voltage standard by means of an accurate multimeter.

At the BIPM, the travelling standards were calibrated, before and after the measurements at KEBS, with the Josephson Voltage Standard. Results of all measurements were corrected for the dependence of the output voltages of the travelling Zener standards on atmospheric pressure. Since KEBS didn't record the temperature of the internal voltage reference during its measurement session, no correction for the Zener temperature dependence was applied. However, as a consequence the related uncertainty was adjusted.

Outline of the measuring method

KEBS 1.018 V and 10 V measurements

The objective for KEBS in participating in the BIPM.EM-K11.a and b comparison of 1.018 V and 10 V DC voltage references was to prove international equivalence of KEBS DC voltage measurement results and support future measurement capabilities to be published in the KCDB (Key Comparison Database of the BIPM) in the future.

On receipt, the comparison standards were immediately connected to mains power supply and left in the laboratory for 5 days before the first measurements were performed.

The following standards and equipment were used:

Transmille Precision Digital Multimeter series 8071* (serial number: N1542G16 to measure the KEBS Zener internal thermistor);

Fluke 8508A, Digital Multimeter (serial number 319169932);

Humidity recorder (serial number: B2B027);

Temperature recorder (serial number: B2B027);

Pressure recorder (serial number: 3722126)

Measurement procedure

Both the KEBS reference standard (Fluke 732B), the BIPM travelling standards and the reference multimeter Fluke 8508A as well as precision multimeter (8071) were powered on 240 V AC and allowed to stabilize in the laboratory undisturbed for 5 days (20th – 25th January 2018) before the actual measurements were done.

The mains AC was removed on a daily basis before the measurements were performed, after which, they were powered back.

The room was air conditioned to within 23±1 °C. Room temperature readings were taken three times for a single reading (initial, in-between and the final). The mean was then computed and considered as the measured temperature.

The atmospheric pressure as well was computed in the same manner.

The laboratory reference standard, the BIPM Units Under Test (UUTs), the reference multimeter and the precision multimeter were connected as shown on Figure 1.

* Certain commercial equipment, instruments, or materials are identified in this report to facilitate understanding. Such identification does not imply recommendation or endorsement by BIPM or KEBS, nor does it imply that the materials or equipment that are identified are necessarily the best available for the purpose.

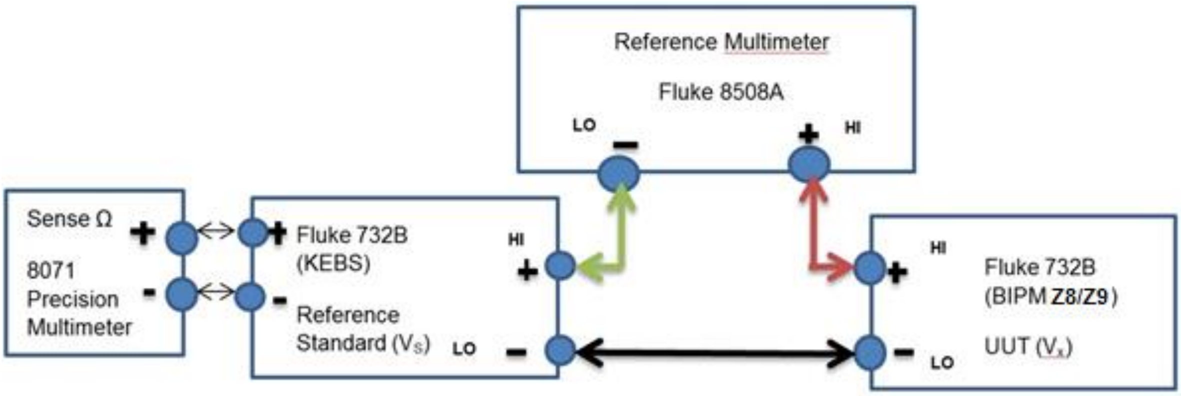

Figure 1: Schematic of the Measurement setup operated at KEBS where the Unit Under Test (UUT) is one of the BIPM Transfer Standards.

Since differential method was applied, the reference standard and one of the Units Under Test (UUT) were connected in series opposition. For the measurement of the positive polarity of the reference, using the connection as per Fig. 1, the low terminals (LO) of the KEBS reference standard (Fluke 732B) and the UUTs (BIPM - Fluke 732Bs- Z_8/Z_9) were connected together while the high terminals (HI) of the reference standard and the UUTs were connected to LO and HI terminals of the reference multimeter, respectively. The binding posts “GUARD” and “CHASSIS” for both the reference standard and the UUT were connected together while for the Precision Multimeter, the “GUARD” was connected to the “LOW” terminal of “Sense Ω”. The prevailing initial environmental conditions (room temperature, relative humidity and atmospheric pressure) were recorded once connections had been accomplished. The resistance of the oven temperature thermistor of the KEBS reference standard was measured via the connector at the rear panel of the standard using the precision multimeter 8071. The reference multimeter indicated reading was then recorded (ΔV_1).

Once this had been done, the connections were reversed. This was achieved by connecting the HI of Fluke 732B reference standard to the HI of the UUT (back to back connections) while connecting the LO of the reference to HI of the multimeter and LO of the UUT to the LO of the multimeter. Again the prevailing environment conditions were recorded and computed accordingly. The reversed reading of the reference multimeter was then recorded (ΔV_2). Also the thermistor reading was recorded as well and averaged.

In order to obtain the measured value of the UUTs (V_{ix}), the average sum of the absolute values recorded from the reference multimeter were worked out and algebraically added to the certified value of Fluke 732B DC Reference Standard by computing from 10 measurements, that is:

$$\Delta V = \frac{\Delta V_1 - \Delta V_2}{2} ------------------------(i)$$
$$V_{iX} = V_S + \Delta V -------------------------(ii)$$

where
ΔV – averaged multimeter reading for positive reference and reversed measurements
ΔV_1 – reference multimeter reading for positive reference connection
ΔV_2 – reference multimeter reading for reversed connection

V_S – 732 B reference standard certified value
V_{ix} – UUT measured value

To obtain the mean of measured value, $\bar{V}_{ix}$, the mean of the 10 readings were computed:

$$\bar{V}_{ix} = \frac{1}{10}\sum_{i=1}^{i=10} V_{ix} \text{-------} (iii)$$

BIPM measurements for 1.018 V and 10 V

The output voltage of the Zener standard to be measured is connected in series opposition to the BIPM Josephson Voltage Standard - Hypres 10 V SIS array (S/N: 2538E-6), through a low thermal Electromotive Forces (EMF) switch. The binding post terminals “GUARD” and “CHASSIS” of the Zener standard are connected together to a single point which is the grounding reference point of the measurement setup and which is a dedicated instrument Earth potential.

The measurements start after at least two hours since the mains plug at the rear of the Zeners has been disconnected in order for the Zener internal temperature to stabilize.

The BIPM detector consists of an EM model N1a analog nanovoltmeter whose output is connected, via an optically-coupled isolation amplifier, to a digital voltmeter (DVM) which is connected to a computer.

This computer is used to monitor measurements, acquire data and calculate results. Low thermal electromotive force switches are used for critical switching, such as polarity reversal of the detector input.

The BIPM array biasing frequency has been adjusted to a value where the voltage difference between the primary and the secondary voltage standards is below 1 µV for both nominal voltages. The nanovoltmeter is set to its 3 µV range for the measurements performed at the level of 1.018 V and on its 10 µV range for those carried out at the level of 10 V. The measurement sequence can then be carried out. One individual measurement point is acquired according to the following procedure:

1- Positive array polarity and reverse position of the detector;
2- Data acquisition;
3- Positive array polarity and normal position of the detector;
4- Data acquisition;
5- Negative array polarity and reverse position of the detector;
6- Data acquisition;
7- Negative array polarity and normal position of the detector;

8- Data acquisition;
9- Negative array polarity and reverse position of the detector;
10- Data acquisition;
11- Negative array polarity and normal position of the detector;
12- Data acquisition;
13- Positive array polarity and reverse position of the detector;
14- Data acquisition;
15- Positive array polarity and normal position of the detector;
16- Data acquisition.

The reversal of the array polarity (by inversing the bias current) is always accompanied by a reversal of the Zener voltage standard using a switch. The reversal of the detector polarity is done to cancel out any detector internal thermo-electromotive forces with linear time-dependence and to check that there is no AC voltage noise rectified at the input of the detector (this is the case if the reading is different in the positive and negative polarity of the analog detector by up to a few hundred microvolts).

Each “Data Acquisition” step consists of 30 preliminary points followed by 500 measurement points. Each of these should not differ from the mean of the preliminary points by more than twice their standard deviation, if so the software warns the operator with a beep. If too many beeps occur, the operator can reject the “Data Acquisition” sequence and start it again. The “Data Acquisition” sequence lasts 25 s and the array must remain on its quantum voltage step during this period of time. The total measurement time (including polarity reversals and data acquisition) is approximately 5 minutes.

This procedure is repeated three times and the mean value corresponds to one result on the graph (Cf. Fig. 2).

Results at 10 V

Figure 2 shows the measured values obtained for the two standards by the two laboratories at 10 V. Figure 3 presents the voltage evolution of the simple mean of the two standards which is used to compute the final result at 10 V.

A linear least squares fit is applied to the results of the BIPM to obtain the results for both standards and their uncertainties at the mean date of the KEBS measurements (2018/02/02).

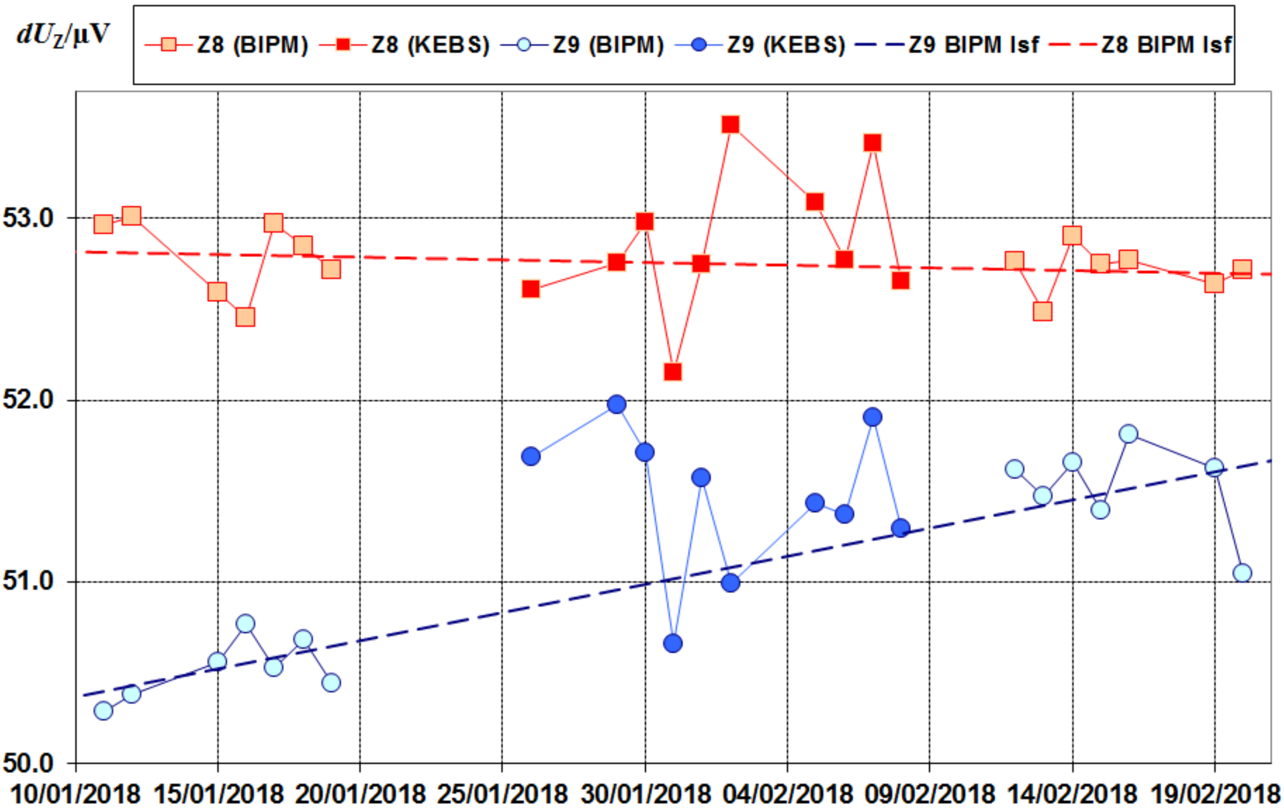

Figure 2. Voltage of Z8 (filled squares) and Z9 (disks) at 10 V measured at both institutes (light markers for BIPM and dark markers for KEBS), referred to an arbitrary origin as a function of time with a linear least-squares fit (lsf) to the BIPM measurements.

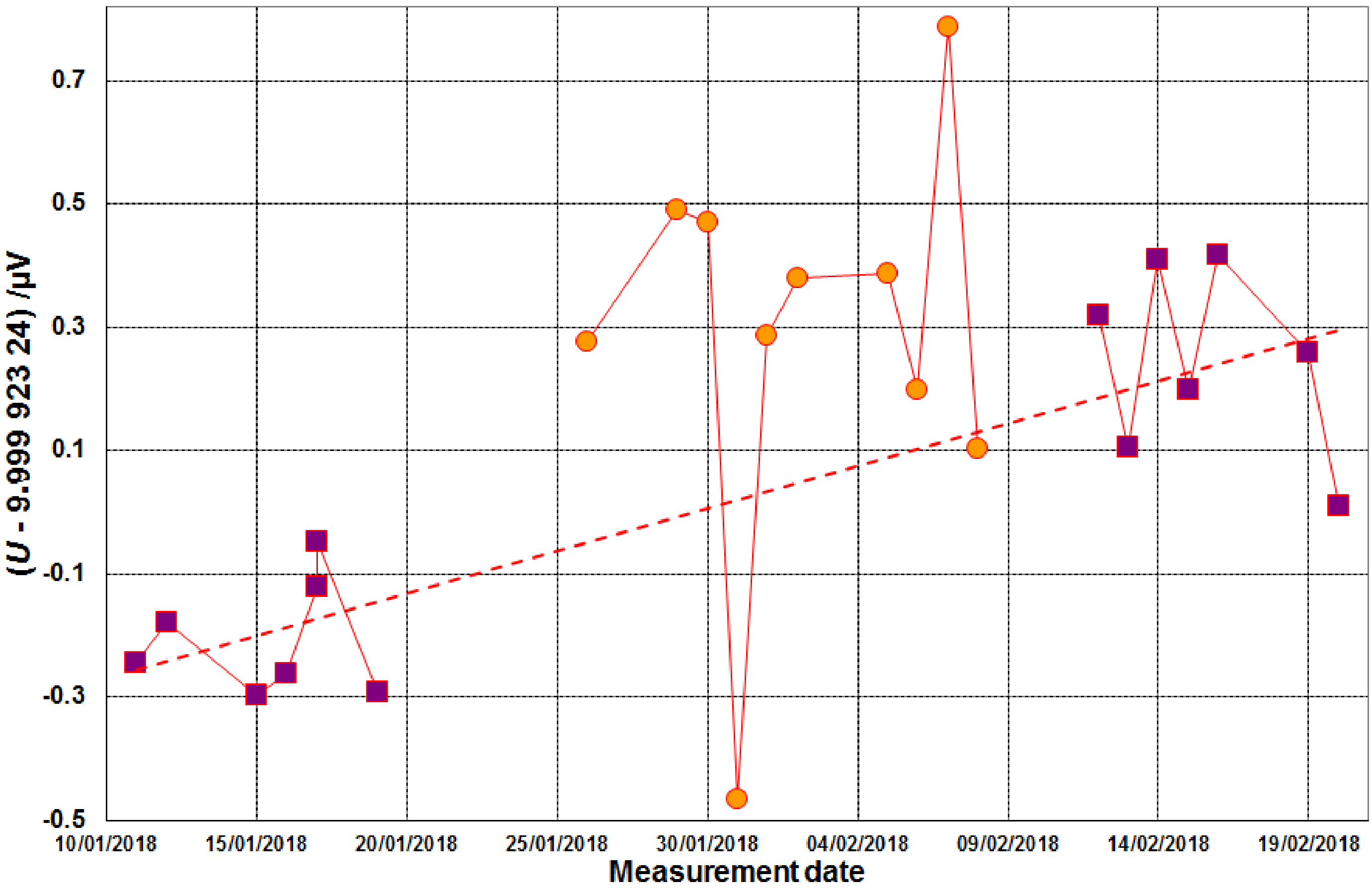

Figure 3. Voltage evolution of the simple mean of the two standards at 10 V.
KEBS measurements are represented by disks and BIPM measurements by squares.

Table 1 lists the results and the uncertainty contributions for the comparison KEBS/BIPM at 10 V. At BIPM, we consider that the relative value of the voltage noise floor due to flicker noise of the Zeners is about 1 part in 10^8 and that this represents the ultimate limit of the stability of Zener voltage standards [1].

Table 1. Results and uncertainties of the KEBS (Kenya)/BIPM bilateral comparison of 10 V standards using two Zener travelling standards: reference date 02 February 2018. Uncertainties are 1 σ estimates.

		BIPM_8	BIPM_9
1	*KEBS* (U_Z – 10 V)/µV	-67.93	-85.04
2	Type A uncertainty/µV	0.13	0.13
3	correlated (Type B) unc. /µV	1.53	
4	*BIPM* (U_Z – 10 V)/µV	-68.05	-85.42
5	Type A uncertainty/µV	0.1	0.1
6	correlated (Type B) unc./µV	0.001	
7	pressure and temperature correction uncertainty/µV	0.09	0.09
8	(U_{KEBS} – U_{BIPM})/µV	0.12	0.37
9	uncorrelated uncertainty/µV	0.19	0.19
10	< U_{KEBS} – U_{BIPM} >/µV	0.25	
11	*a priori* uncertainty/µV	0.13	
12	*a posteriori* uncertainty/µV	0.13	
13	correlated uncertainty/µV	1.54	
14	comparison total uncertainty/µV	1.55	

In Table 1, the following elements are listed:

(1) the value attributed by KEBS to each Zener U_{KEBS}, computed as the simple mean of all data from KEBS;

(2) the KEBS Type A uncertainty (Cf. Tables 3a and 3b),
The experimental standard deviation of the mean of the measurements are:
0.05 µV and 0.15 µV for Z8 and Z9, respectively, 0.13 µV for Z8 and Z9, once corrected for atmospheric pressure;

(3) the uncertainty component arising from the maintenance of the volt at KEBS: this uncertainty is completely correlated between the different Zeners used for the comparison;

(4-6) the corresponding quantities for the BIPM referenced to the mean date of KEBS measurements;

Note: at BIPM, the Type A uncertainty is considered as the larger of the experimental standard deviation of the mean of the measurements performed at BIPM, and the 1/*f* Zener noise floor which, according to the experience of the BIPM, in general limits the accuracy of Zener voltage standards and is equal in 10^8 in relative parts [1];

(7) Since there was no record of the temperature of the transfer standards at KEBS, the related uncertainty is assumed to be the mean value of the correction that would have been applied to BIPM measurements based on the internal thermistor records of the standards. A rectangular statistical distribution is applied to these corrections to give $u_{T,i}$ of Zener *i.* u_{T,Z_8}= 0.009 µV and u_{T,Z_9} = 0.002 µV.

The uncertainty due to the effects of the pressure coefficients[1] and to the differences of the mean pressures in the participating laboratories is calculated as follows:

$$u_{P,\mathrm{i}} = U \times u(c_{\mathrm{P,i}}) \times \Delta P_i$$

where U = 10 V, $u(c_{\mathrm{P,Z8}})$= 0.050×10^{-9} / hPa, $u(c_{\mathrm{P,Z9}})$ = 0.052×10^{-9} / hPa, ΔP_{Z8} = 171.3 hPa and ΔP_{Z9} = 171.0 hPa.

The significant difference in the mean value of the pressure between both laboratories is mainly due to the difference in elevation between the location on both laboratories.

The uncertainty on the measurement of the pressure is negligible.

(8) the difference (U_{KEBS} – U_{BIPM}) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7;

(10) the result of the comparison is the simple mean of the differences of the calibration results for the different standards;

(11 and 12) the uncertainty related to the transfer, estimated by the following two methods:

(11) the a priori uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners

(12) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results;

(13) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, and

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of the larger of (11) and (12).

To estimate the uncertainty related to the stability of the standards during transportation, we have calculated the "*a priori"* uncertainty of the mean of the results obtained for the two standards (also called statistical internal consistency). It consists of the quadratic combination

[1] A first evaluation of the correction coefficients was performed in 2000. New determinations of the temperature sensitivity coefficients and the pressure coefficients and were carried out at BIPM in 2016 [2] and 2017, respectively [3].

of the uncorrelated uncertainties of each result. We compared this component to the "*a posteriori"* uncertainty (also called statistical external consistency) which consists of the experimental standard deviation of the mean of the results from the two travelling standards[2]. If the "*a posteriori"* uncertainty is significantly larger than the "*a priori"* uncertainty, we assume that a standard has changed in an unusual way, probably during their transportation, and we use the larger of these two estimates in calculating the final uncertainty. In the present comparison, the "*a posteriori"* uncertainty is comparable to the "*a priori"* uncertainty at 10 V and is equal to 130 nV, and therefore we can assume that there was no change on the outputs of the travelling standards due to their shipment from one laboratory to the other.

The comparison result is presented as the difference between the value assigned to a 10 V standard by KEBS, at KEBS, U_{KEBS}, and that assigned by the BIPM, at the BIPM, U_{BIPM}, which for the reference date is

$$U_{\mathrm{KEBS}} - U_{\mathrm{BIPM}} = 0.25\ \mu\mathrm{V};\quad u_{\mathrm{c}} = 1.55\ \mu\mathrm{V} \qquad \textbf{on 2018/02/02},$$

where u_{c} is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at KEBS, at the BIPM (based on $K_{\mathrm{J-90}}$), and the uncertainty related to the comparison.

Uncertainty Budgets

Table 2 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM.

Tables 3a and 3b list the uncertainties related to the calibration of the Zeners at the KEBS for Z8 and Z9 respectively. Note that the uncertainty of the temperature and pressure corrections (last line in Italic) are given as an indication only and do not appear in the final uncertainty budget as they are included separately in the comparison uncertainty budget (Table 1).

Note: the uncertainty of the temperature, pressure corrections and the contribution of the Zener noise (in italic in the tables) are given for completeness only and are not included in the total uncertainty as they are included separately in the comparison uncertainty budget (Table 1).

[2] With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself.

Table 2: Estimated standard uncertainties arising from the JVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 10 V.

JVS & detector uncertainty components	Uncertainty/nV
Noise of the measurement loop that includes the residual thermal electromotive forces including the residual EMF of the reversing switch	0.86
Zener noise (Type A)	*Not lower than the 1/f noise estimated to 100 nV*
Detector gain	0.11
Leakage resistance	3×10^{-2}
Frequency	3×10^{-2}
Pressure and temperature correction	*included in the Zener uncertainty budget*
Total	0.87

The standard deviation of the mean of the KEBS measurement results are 130 nV for both BIPM_8 and BIPM_9 (once corrected for the dependence of the standards on pressure variations).

Table 3a: Estimated standard uncertainties for a Zener calibration with the KEBS equipment at the level of 10 V for Zener 8

Contribution	**Probability distribution**	**Standard uncertainty $u(x_i)$**	**Sensitivity coefficient c_i**	**Uncertainty contribution $U(y_i) = c_i u(x_i)$**
732B DC Standard calibration	Normal	0.013 µV/V	1	0.013 µV/V
Type A	Normal	0.013 µV/V	1	0.013 µV/V
Drift of the 732B DC Standard	Rectangular	0.153 µV/V	1	0.153 µV/V
732B Standard Short term stability	Rectangular	0.174 µV/V	1	0.174 µV/V
Output terminal Noise of 732B	Rectangular	0.038 µV/V	1	0.038 µV/V
Temperature coefficient (KEBS reference)	Rectangular	0.015 µV/V	1	0.015 µV/V
Reference multimeter resolution	Rectangular	0.001 µV/V	1	0.001 µV/V
Combined uncertainty, u_c				**0.236 µV/V**
Expanded Uncertainty, U_c (k = 2)				**0.5 µV/V**

Table 3b: Estimated standard uncertainties for a Zener calibration with the KEBS equipment at the level of 10 V for Zener 9

Contribution	**Probability distribution**	**Standard uncertainty $u(x_i)$**	**Sensitivity coefficient c_i**	**Uncertainty contribution $U(y_i) = c_i u(x_i)$**
732B DC Standard calibration	Normal	0.013 µV/V	1	0.013 µV/V
Type A	Normal	0.013 µV/V	1	0.013 µV/V
Drift of the 732B DC Standard	Rectangular	0.153 µV/V	1	0.153 µV/V
732B Standard Short term stability	Rectangular	0.174 µV/V	1	0.174 µV/V
Output terminal Noise of 732B	Rectangular	0.038 µV/V	1	0.038 µV/V
Temperature coefficient	Rectangular	0.006 µV/V	1	0.006 µV/V
Reference multimeter resolution	Rectangular	0.001 µV/V	1	0.001 µV/V
Combined uncertainty, u_c				**0.236 µV/V**
Expanded Uncertainty, U_c (k = 2)				**0.5 µV/V**

Results at 1.018 V

Figure 4 shows the measured values obtained for the two standards by the two laboratories at 1.018 V and figure 5 presents the voltage evolution of the simple mean of the two standards which is used to compute the final result at 1.018 V. A linear least squares fit is applied to the results of the BIPM to obtain the results for both standards and their uncertainties at a common reference date corresponding to the mean date of the KEBS measurements (2018/02/02).

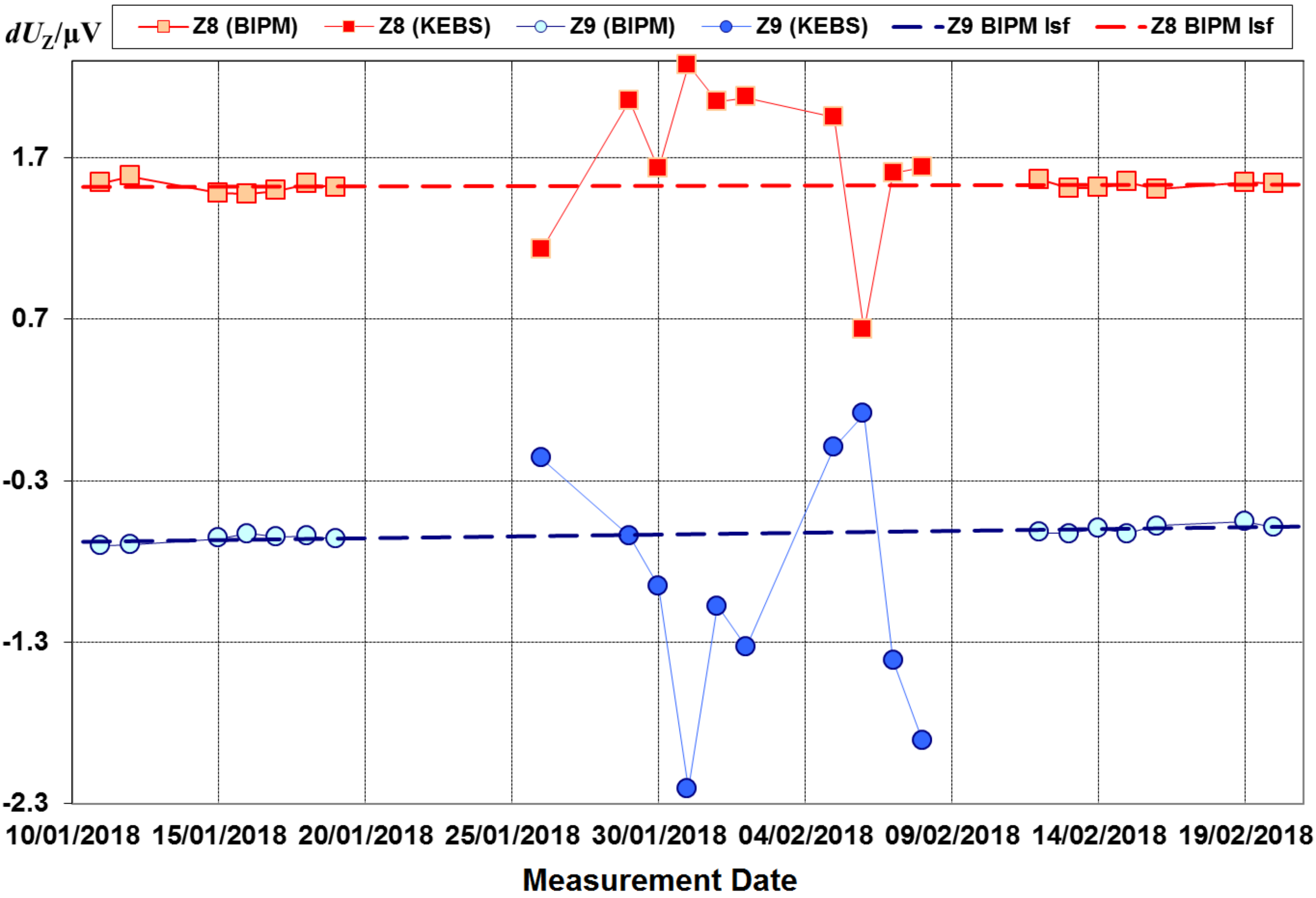

Figure 4. Voltage of BIPM_8 (squares) and BIPM_9 (disks) at 1.018 V measured at both institutes (light markers for BIPM and dark ones for KEBS), referred to an arbitrary origin, as a function of time, with a linear least-squares fit to the measurements of the BIPM.

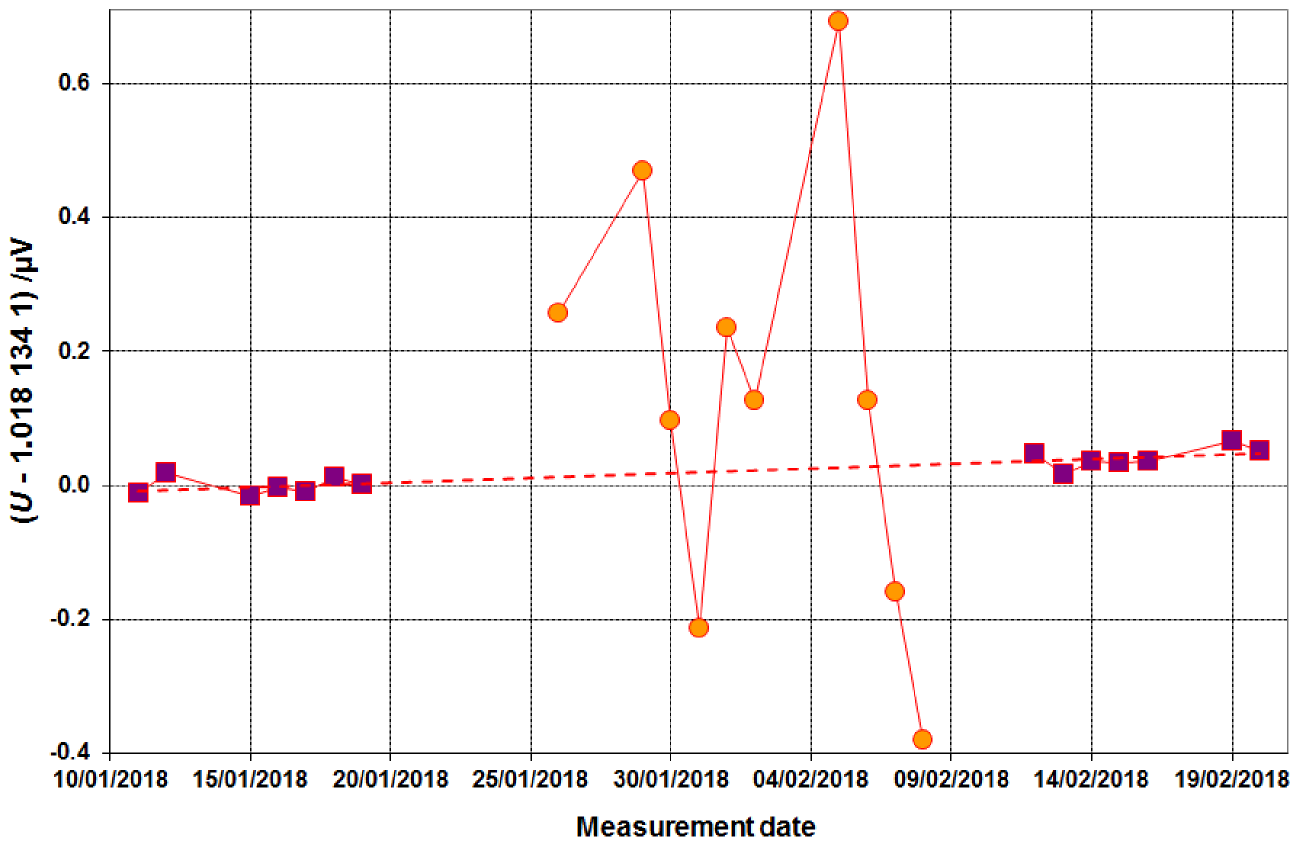

Figure 5. Voltage evolution of the simple mean of the two standards at 1.018 V. KEBS measurements are represented by disks and BIPM measurements by squares.

Table 4 lists the results of the comparison and the uncertainty contributions for the comparison KEBS/BIPM at 1.018 V. Experience has shown that flicker or 1/*f* noise ultimately limits the stability characteristics of Zener diode standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the voltage noise floor due to flicker noise is about 1 part in 10^8.

In estimating the uncertainty related to the stability of the standards during transportation, we have calculated the "*a priori"* uncertainty of the mean of the results and the "*a posteriori"* uncertainty as described for the measurements at 10 V.

Table 5 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM and Table 6a and 6b list the uncertainties related to the calibration of Z8 and Z9 respectively.

Table 4. Results and uncertainties of the KEBS (Kenya)/BIPM bilateral comparison of 1.018 V standards using two Zener travelling standards: reference date 02 February 2018. Uncertainties are 1 σ estimates.

		BIPM_8	BIPM_9
1	*KEBS* (U_Z – 1.018 V)/µV	174.16	94.32
2	Type A uncertainty/µV	0.11	0.20
3	correlated unc. /µV	0.32	
4	*BIPM* (U_Z – 1.018 V)/µV	174.03	94.24
5	Type A uncertainty/µV	0.01	0.01
6	correlated unc./µV	0.001	
7	pressure and temperature correction uncertainty/µV	0.042	0.048
8	(U_{KEBS} – U_{BIPM})/µV	0.13	0.08
9	uncorrelated uncertainty/µV	0.12	0.21
10	< U_{KEBS} – U_{BIPM} >/µV	0.106	
11	*a priori* uncertainty/µV	0.120	
12	*a posteriori* uncertainty/µV	0.027	
13	correlated uncertainty/µV	0.32	
14	comparison total uncertainty/µV	0.34	

In Table 4, the following elements are listed:

(1) the value attributed by KEBS to each Zener U_{KEBS}, computed as the simple mean of all data from KEBS;

(2) the Type A uncertainty claimed by KEBS (Cf. Tables 6.a and 6.b),

(3) the uncertainty component arising from the realization and maintenance of the volt at KEBS: this uncertainty is completely correlated between the different Zeners used for a comparison;

(4-6) the corresponding quantities for the BIPM referenced to the mean date of the KEBS measurements;

(5) see text of Table 1. The standard deviation of the mean of the BIPM measurement results, at the mean date of KEBS measurements is in the interval from 8 nV to 12 nV for Z8 and Z9 respectively, once corrected for the dependence of the standards to temperature and pressure variations

(7) Since there was no record of the temperature of the transfer standards at KEBS, the uncertainty on the temperature correction is the mean value of the correction that would have been applied to BIPM measurements based on the internal thermistor records of the standards. A rectangular statistical distribution is applied to these corrections to give $u_{T,i}$ of Zener *i*. u_{T,Z_8}=0.072/√3= 0.041 µV , u_{T,Z_9} = 0.082/√3= 0.047 µV.

The following procedure is applied for the uncertainty $u_{P\mathrm{i}}$ on the pressure correction coefficient[3] applied to the difference *Pi* between the mean values of the pressure measured at both institutes:

$$u_{P\mathrm{i}} = U \times u(c_{P\mathrm{i}}) \times \Delta P_{\mathrm{i}}$$

where U = 1 V, $u(cP_{\mathrm{Z8}})$= 0.04×10^{-9} / hPa, $u(cP_{\mathrm{Z9}})$ = 0.048×10^{-9} / hPa,
ΔP_{Z8} = 170.9 hPa and ΔP_{Z9} = 171.1 hPa.

The uncertainties on the measurement of the temperature and the pressure are negligible.
(8) the difference (U_{KEBS} – U_{BIPM}) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7;
(10) the result of the comparison is the simple mean of the differences of the calibration results for the different standards;
(11 and 12) the uncertainty related to the transfer, estimated by the following two methods:

(11) the *a priori* uncertainty,

(12) the *a posteriori* uncertainty;

(13) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, and
(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of the larger of (11) and (12).

In this case the *a posteriori* uncertainty is more than half the *a priori* uncertainty. We assume that the dispersion of the KEBS measurements are mostly responsible for the difference between the two approaches. We reject any possible effect of the transport of the travelling standards on the dispersion of the results as the BIPM measurement exhibit a good agreement between the preliminary and return measurements.

[3] The first evaluation of the correction coefficients was performed in 2000. A new determination of the temperature and pressure sensitivity coefficients of the BIPM secondary voltage standards has been carried out at in 2016 [2] and 2017 [3].

The result of the comparison is presented as the difference between the value assigned to a 1.018 V standard by KEBS, at KEBS, U_{KEBS}, and that assigned by the BIPM, at the BIPM, U_{BIPM}, which for the reference date is:

$$U_{KEBS} - U_{BIPM} = 0.11\ \mu V; \quad u_c = 0.34\ \mu V \quad \text{on 2018/02/02},$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at the BIPM, (based on K_{J-90}) and at KEBS and the uncertainty related to the comparison.

Table 5. Estimated standard uncertainties for Zener calibrations with the BIPM equipment at the level of 1.018 V. The uncertainty of the temperature, pressure corrections and the contribution of the Zener noise (in italic in the tables) are given for completeness only and are not included in the total uncertainty as they are included separately in the comparison uncertainty budget (table 4).

JVS & detector uncertainty components	Uncertainty/nV
Residual thermal electromotive forces	included in the Type A uncertainty
Noise of the measurement loop that includes the residual thermal electromotive forces including the residual EMF of the reversing switch	0.34
Zener noise (Type A)	*Not lower than the 1/f noise estimated to 10 nV*
Detector gain	0.11
Leakage resistance	3×10^{-3}
Frequency	3×10^{-3}
Pressure and temperature correction	*included in the Zener unc. budget*
Total	0.36

The standard deviation of the mean of the KEBS measurement results is in the interval from 160 nV to 250 nV for BIPM_8 and BIPM_9 respectively (once corrected for the dependence of the standards to pressure variations).

Table 6a: Estimated standard uncertainties for Zener calibrations with the KEBS equipment at the level of 1.018 V for Zener 8

Contribution	Probability distribution	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Uncertainty contribution $U(y_i) = c_i\, u(x_i)$
732B DC Standard calibration	Normal	0.075 µV/V	1	0.075 µV/V
Type A	Normal	0.156 µV/V	1	0.156 µV/V
Drift of the 732B DC Standard	Rectangular	0.313 µV/V	1	0.313 µV/V
732B Standard Short term stability	Rectangular	0.462 µV/V	1	0.462 µV/V
Output terminal Noise of 732B	Rectangular	0.058 µV/V	1	0.058 µV/V
Temperature coefficient	Rectangular	0.044 µV/V	1	0.044 µV/V
Reference multimeter resolution	Rectangular	0.003 µV/V	1	0.003 µV/V
Combined uncertainty, u_c				**0.589 µV/V**
Expanded Uncertainty, U_c (k = 2)				**1.2 µV/V**

Table 6b: Estimated standard uncertainties for Zener calibrations with the KEBS equipment at the level of 1.018 V for Zener 9

Contribution	Probability distribution	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Uncertainty contribution $U(y_i) = c_i\, u(x_i)$
732B DC Standard calibration	Normal	0.075 µV/V	1	0.075 µV/V
Type A	Normal	0.245 µV/V	1	0.245 µV/V
Drift of the 732B DC Standard	Rectangular	0.313 µV/V	1	0.313 µV/V
732B Standard Short term stability	Rectangular	0.462 µV/V	1	0.462 µV/V
Output terminal Noise of 732B	Rectangular	0.058 µV/V	1	0.058 µV/V
Temperature coefficient	Rectangular	0.029 µV/V	1	0.029 µV/V
Reference multimeter resolution	Rectangular	0.003 µV/V	1	0.003 µV/V
Combined uncertainty, u_c				**0.617 µV/V**
Expanded Uncertainty, U_c (k = 2)				**1.3 µV/V**

Conclusion

The final result of the comparison is presented as the difference between the values assigned to DC voltage standards by KEBS, at the level of 1.018 V and 10 V, at KEBS, U_{KEBS}, and those assigned by the BIPM, at the BIPM, U_{BIPM}, at the reference dates of the 2nd of February 2018.

$$\boldsymbol{U_{\mathrm{KEBS}} - U_{\mathrm{BIPM}} = +\ 0.11\ \mu\mathrm{V};\ u_{\mathrm{c}} = 0.34\ \mu\mathrm{V},\ \text{at } 1.018\ \mathrm{V}}$$

$$\boldsymbol{U_{\mathrm{KEBS}} - U_{\mathrm{BIPM}} = +\ 0.25\ \mu\mathrm{V};\ u_{\mathrm{c}} = 1.55\ \mu\mathrm{V},\ \text{at } 10\ \mathrm{V}}$$

where u_{c} is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at the BIPM and at KEBS, based on $K_{\mathrm{J-90}}$, and the uncertainty related to the comparison. KEBS's uncertainties appear to be overestimated and would require to be revised following *Hamilton* procedure and recommendations [4].

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