

BUREAU INTERNATIONAL DES POIDS ET MESURES

Bilateral comparison of 1 Ω and 10 k Ω standards (ongoing BIPM key comparisons BIPM.EM-K13.a and 13.b) between the CMI (Czech Republic) and the BIPM

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Final Report

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1 Introduction

A comparison of values assigned to 1 Ω and 10 k Ω resistance standards was carried out between the BIPM and the CMI (Czech Republic) in the period May to December 2015.

Two 1 Ω and two 10 k Ω BIPM travelling standards were calibrated first at the BIPM, then at the CMI and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: May - June 2015

CMI measurements: October - November 2015

'After' measurements at the BIPM: November - December 2015

This report is organised as follows: details of the travelling standards used are listed in section 2. The results of the BIPM measurements are given in section 3, and the calibration reports provided by the CMI are summarized in section 4; these two sections include the uncertainty budgets for each laboratory. Finally, the two sets of measurements are compared and analysed in section 5. The uncertainties arising from the transfer of the standards between the two laboratories are estimated and included at this point. The final results of the comparisons are given, in the form of the degrees of equivalence (deviations from the KCRV and associated uncertainties) between the CMI and the BIPM for measurements of 1 Ω and 10 k Ω resistance standards.

This report covers the comparison of both 1 Ω standards (BIPM.EM-K13.a) and 10 k Ω standards (BIPM.EM-K13.b). The measurements of these two different resistance values are analysed separately, but are reported together here as the two comparisons were carried out simultaneously.

2 Travelling Standards

Four travelling standards provided by the BIPM were used for this comparison. The two 1 Ω standards are of CSIRO type, with working labels BIV203 (manufacturer's serial number S-64203) and BIV207 (manufacturer's serial number S-64207). The two 10 k Ω standards are TEGAM S104 type, and have the working labels B10k08 (manufacturer's serial number K201039730104) and B10k12 (serial number K201089830104). The standards were shipped by regular air freight between the laboratories.

All measurements are corrected to a reference temperature of 23.000 $^{\circ}\text{C}$ and reference pressure 1013.25 hPa using the known coefficients of the standards, given in table 1.

According to the protocol, the CMI did not apply pressure and temperature corrections to its results, but supplied the raw values and the measured temperature and pressure. The corrections were applied in the analysis made by the BIPM.

Standard #	Relative temperature coefficients		Relative pressure coefficients.
	$\alpha_{23} / (10^{-6}/\text{K})$	$\beta / (10^{-6}/\text{K}^2)$	$\gamma / (10^{-9}/\text{hPa})$
BIV203	- 0.010	- 0.0016	- 0.20
BIV207	- 0.009	0.000	- 0.25
B10k08	- 0.010	- 0.023	- 0.16
B10k12	+ 0.010	- 0.023	- 0.23

Table 1: Temperature and pressure coefficients of the travelling standards.

3 Measurements at the BIPM

The BIPM measurements are traceable to the quantum Hall resistance (QHR) standard via different measurement bridges and working standards for the two nominal values. In all cases, values are based on the conventional value of the von Klitzing constant, $R_{K-90} = 25812.807 \Omega$. (The standard uncertainty associated with the use of R_{K-90} , which has a relative value of 1×10^{-7} , has not been included.)

The 1Ω measurements were carried out by comparison with a 100Ω reference resistor (identifier BI100-3) whose value is calibrated against the BIPM QHR standard regularly (at least once every 6 months). The comparison was performed using a DC cryogenic current comparator operating with 50 mA current in the 1Ω resistors.

The 1Ω travelling standards were kept in a temperature controlled oil bath at a temperature which is close (within a few mK) to the reference temperature of $23 \text{ }^\circ\text{C}$. The oil temperature close to each standard was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT), in conjunction with thermocouples. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement.

The BIPM CCC bridge operates with a relatively slow cycle time for current reversals, which we note here: 340 s total cycle time, ramping time between positive and negative 50 s. Due to the effects described in [1], variations from this timing may produce significant shifts in the apparent values of 1Ω standards. The CMI investigated this dependence for the two transfer standards in this comparison, and we use those results to apply a correction for this effect, as described in the following section.

The travelling standards were measured 11 times during the period labelled ‘before’ (May - June 2015) and 9 times during the period labelled ‘after’ (December 2015). The individual BIPM measurement data are plotted in figures 1 and 2 of section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 2 and the uncertainty budget in Table 3. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	Relative difference from nominal 1Ω value / 10^{-6}				
	Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}
BIV203		+ 0.516	0.010	+ 0.543	0.008
BIV207		- 0.505	0.009	- 0.440	0.006

Table 2: Summary of BIPM calibrations of the 1Ω standards.

Source of uncertainty	relative standard uncertainty /10 ⁻⁹
Imperfect realisation of R_H	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_H	3
Interpolation / extrapolation of the value of BI100-3	13
Measurement of the (1 Ω / BI100-3) ratio	8
Temperature correction for the 1 Ω standard	2
Pressure correction for the 1 Ω standard	3
Combined uncertainty u_2	16

Table 3: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

The 10 k Ω measurements were carried out by comparison with a set of two 10 k Ω reference resistors (identifiers B10K1 and B10K2) which are calibrated regularly (at least once every 6 months) against the BIPM QHR standard. The comparison was performed using a Warshawsky bridge operating with a 0.1 mA DC current (i.e. at a measurement voltage of 1 V).

The 10 k Ω travelling standards were kept in a temperature-controlled air bath at a temperature which is close to the reference temperature of 23 °C (within 0.05 °C). The temperature of the standards was determined by means of a calibrated platinum resistance thermometer, in conjunction with thermocouples. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The relative humidity in the air bath was not monitored, but the laboratory air conditioning system controls the relative humidity to 50 % (\pm 10 %).

The travelling standards were measured 17 times during the period labelled ‘before’ (May - June 2015) and 9 times during the period labelled ‘after’ (December 2015). The individual BIPM measurement data are plotted in figures 3 and 4 of section 5 (after application of the temperature and pressure correction). The mean results are summarized in Table 4 and the uncertainty budget in Table 5. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	Relative difference from nominal 10 k Ω value / 10 ⁻⁶				
	Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}
B10k08		+ 0.828	0.005	+ 0.865	0.003
B10k12		+ 0.818	0.005	+ 0.783	0.001

Table 4: Summary of BIPM calibrations of the 10 k Ω standards.

Source of uncertainty	relative standard uncertainty / 10^{-9}
Imperfect realization of $R_H(2)$	2
Link $R_H(2)$ / 100 Ω	3
Link 100 Ω / 10 000 Ω	5
Link 10 000 Ω / (mean reference B10K1-B10K2)	7
Extrapolation of mean value of 10 k Ω reference	8
Measurement of the voltage applied to the bridge	5
Leakage resistances	5
Temperature correction for travelling standard	3
Pressure correction for travelling standard	2
Combined uncertainty u_2	15

Table 5: BIPM uncertainty budget for the calibration of the 10 k Ω travelling standards.

4 Measurements at the CMI

4.1 Method of calibration:

The CMI primary reference of dc resistance is based on the Quantized Hall Resistance (QHR). A 100 Ω reference standard is calibrated against the QHR using a cryogenic current comparator (CCC) bridge. The QHR device is operated on the $i = 2$ plateau, obtained at a temperature of 0.3 K and a magnetic flux density of 9.5 T at several current levels. The BIPM transfer standards are then calibrated against the 100 Ω reference by means of the same CCC bridge.

4.2 Operating conditions:

The two 1 Ω travelling standards were placed in a temperature-stabilized oil bath, the two 10 k Ω travelling standard were placed in a temperature-stabilized air bath. Both baths were maintained at (23.000 ± 0.020) $^{\circ}\text{C}$ (uncertainty equals to 95 % probability) and all standards were allowed to stabilize three weeks prior to the measurements. Their temperature was recorded during each resistance measurement, using calibrated Pt resistance thermometers. This temperature did not deviate by more than ± 0.005 $^{\circ}\text{C}$ from 23.000 $^{\circ}\text{C}$ over the entire measurement period. The 1 Ω travelling standards had insulating plates above the level of the oil bath.

Operating current for 1 Ω standards: 50 mA.

Operating current for 10 k Ω standards: 0.1 mA.

Operating current cycle duration: corresponds to configuration A in Table 7.

Barometric pressure ranged from 981.5 hPa to 1005.0 hPa over the entire measurement period. The pressure was measured with uncertainty ± 2 hPa (uncertainty equals to 95 % probability).

Relative humidity ranged from 30 % to 50 % over the entire measurement period.

4.3 CMI results at 1 Ω :

The 1 Ω travelling standards were measured 18 times (BIV203) and 16 times (BIV207) in the period October – November 2015. Table 6 gives the mean values at the mean date of 26 October 2015, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 1 Ω value / 10^{-6}	Std dev. / 10^{-6}	Mean temperature / $^{\circ}\text{C}$	Mean atmospheric pressure / hPa
BIV203	+ 0.514	0.011	23.000	992.0
BIV207	- 0.548	0.009	23.000	990.2

Table 6: Summary of CMI 1 Ω calibrations.

Corrections for temperature and pressure and cycle time:

As noted above, the measurement cycle of current reversals in the CCC is significantly faster in the CMI instrument than in the one used at BIPM. From [1], we expect that this may affect the results at 1 Ω , and the CMI performed additional measurements to test for this, which are summarised in tables 7 and 8.

Property	Standard configuration A	Experimental configuration B
Full cycle time	24 s	340 s
Ramping time	0.4 s	10 s
Start of data sampling (reference to start of cycle)	4 s	50 s

Table 7: CCC reversal cycle details

Configuration A is the timing usually used at the CMI for CCC measurements; configuration B is designed to match as closely as possible the BIPM CCC cycle. These two configurations give the following results for the four standards involved in this comparison.

Travelling Standard	Configuration A (fast)		Configuration B (slow)		Rel. Difference, $(R_B/R_A - 1) \times 10^6$
	Measured resistance, R_A / Ω	Rel. Type A uncertainty $\times 10^6$	Measured resistance, R_B / Ω	Rel. Type A uncertainty $\times 10^6$	
1 Ω BIV203	1.000 000 499 8	0.0017	1.000 000 542 9	0.0035	+ 0.043
1 Ω BIV207	0.999 999 449 4	0.0010	0.999 999 472 3	0.0062	+ 0.023
10 k Ω B10k08	10 000.008 305	0.0004	10 000.008 323	0.0016	+ 0.002
10 k Ω B10k12	10 000.007 908	0.0008	10 000.007 915	0.0023	+ 0.001

Table 8: results of varying the cycle time

These results confirm that there is a significant effect for the 1 Ω standards, but not for the 10 k Ω standards. We apply the measured difference as a correction to the CMI 1 Ω results, to make them comparable to the BIPM measurements (the main CMI results having been obtained with the standard configuration A of table 7). We estimate the relative standard uncertainty on this correction as 0.01×10^{-6} . This is larger than the experimental uncertainties of table 8, and somewhat conservative, as the CMI ‘configuration B’ does not exactly match the current ramping of the BIPM CCC.

The standard uncertainties on temperature and pressure measurements at the CMI are 0.01 $^{\circ}\text{C}$ and 1 hPa respectively. Taking into account the differences from the reference temperature and reference pressure, and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\text{Temp}} < 0.001 \times 10^{-6}$ and $u_{\text{Press}} < 0.001 \times 10^{-6}$.

The total combined relative standard uncertainty for the temperature, pressure and cycle time corrections is thus $u_3 = 0.01 \times 10^{-6}$.

Reference temperature = 23.000°C Reference pressure = 1013.25 hPa			
	Relative corrections /10 ⁻⁶		
Standard #	For temperature	For pressure	For cycle time
BIV203	0.000	- 0.004	+ 0.043
BIV207	0.000	- 0.006	+0.023

Table 9: Corrections applied to the CMI 1 Ω results.

Uncertainty Budget Provided by the CMI

Part one: uncertainties associated with the value of the 100 Ω reference standard

Source of uncertainty	Type	Relative standard uncertainty/ 10 ⁻⁶
Short term stability of the standard	B	0.0020
CCC winding ratio	B	0.0010
CCC electronics and SQUID	B	0.0029
Compensation ratio k	B	0.0005
Bridge voltage measurement	B	<0.0001
Measurement of the DUT voltage	B	<0.0001
Leakage resistance	B	0.0040
Imperfect realization of $R_H(2)$	B	0.0020
Rel. Std. deviation of the value	A	0.0021
Combined relative standard uncertainty $u_{100\Omega}$		0.0062

Table 10: Summary of the CMI uncertainty budget for 100 Ω reference.

Part two: transfer from the 100 Ω reference to 1 Ω (corrections for temperature, pressure and cycle time effect not included)

Source of uncertainty	Type	Relative standard uncertainty/ 10 ⁻⁶
Ref. standard drift correction	B	0.0053
Ref. standard pressure correction	B	0.0010
Ref. standard temperature correction	B	0.0020
Ref. standard Joule heating effects	B	0.0010
CCC winding ratio	B	0.0010
CCC electronics and SQUID	B	0.0029
Compensation ratio k	B	0.0002
Bridge voltage measurement	B	<0.0001
Measurement of the DUT voltage	B	<0.0001
Leakage resistance	B	<0.0001
Rel. Std. deviation of one transfer	A	0.0018
Combined relative standard uncertainty $u_{100:1}$		0.0069

Table 11: Summary of the CMI uncertainty budget for 1 Ω .

CMI After corrections	Relative difference from nominal value / 10^{-6}	Relative standard uncertainties		
		Repeatability $u_1 / 10^{-6}$	Systematic $u_2 / 10^{-6}$	Corrections $u_3 / 10^{-6}$
BIV203	+ 0.5527	0.0123	0.0093	0.01
BIV207	- 0.5308	0.0081	0.0093	0.01

Table 12: Summary of the CMI results at 1 Ω , after corrections.

Note: The distinction between ‘systematic’ and ‘repeatability’ is made in tables 12 and 16 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. (This does not correspond exactly to the more usual division into Type A and Type B components.)

4.3 CMI results at 10 k Ω :

The 10 k Ω travelling standards were measured 11 times in the period October – November 2015. Table 13 gives the mean values at the mean date of 22 October 2015, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 10 k Ω value / 10^{-6}	Std dev./ 10^{-6}	Mean temperature / $^{\circ}\text{C}$	Mean atmospheric pressure / hPa
B10k08	+ 0.8308	0.0011	23.000	988.9
B10k12	+ 0.7912	0.0012	23.000	988.8

Table 13: Summary of CMI 10 k Ω calibrations.

Corrections for temperature and pressure:

Reference temperature = 23.000 $^{\circ}\text{C}$ Reference pressure = 1013.25 hPa		
	Relative corrections / 10^{-6}	
Standard #	For temperature	For pressure
B10k08	0.000	- 0.004
B10k12	0.000	- 0.006

Table 14: Corrections applied to the CMI 10 k Ω results.

The standard uncertainties on temperature and pressure measurements at the CMI are 0.01 $^{\circ}\text{C}$ and 1 hPa respectively. Taking into account the differences from the reference temperature and reference pressure, and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\text{Temp}} < 0.001 \times 10^{-6}$ and $u_{\text{Press}} < 0.001 \times 10^{-6}$, leading to a combined relative standard uncertainty $u_3 < 0.001 \times 10^{-6}$.

Uncertainty Budget Provided by CMI

Realization of 100 Ω same as table 10 above, followed by transfer from 100 Ω to 10 k Ω (not including temperature and pressure corrections).

Source of uncertainty	Type	Relative standard uncertainty/ 10^{-6}
Ref. standard drift correction	B	0.0053
Ref. standard pressure correction	B	0.0010
Ref. standard temperature correction	B	0.0020
Ref. standard Joule heating effects	B	0.0010
CCC winding ratio	B	0.0010
CCC electronics and SQUID	B	0.0029
Compensation ratio k	B	0.0003
Bridge voltage measurement	B	<0.0001
Measurement of the DUT voltage	B	<0.0001
Leakage resistance	B	0.0028
Rel. Std. deviation of one transfer	A	0.0008
Combined relative standard uncertainty $u_{100:1}$		0.0073

Table15: Summary of the CMI uncertainty budget for 10 k Ω .

CMI After corrections	Relative difference from nominal value / 10^{-6}	Relative standard uncertainties		
		Repeatability $u_1 / 10^{-6}$	Systematic $u_2 / 10^{-6}$	Corrections $u_3 / 10^{-6}$
B10k08	+ 0.8270	0.0013	0.0096	0.001
B10k12	+ 0.7855	0.0019	0.0096	0.001

Table 16: Summary of the CMI results at 10 k Ω , after corrections.

5 Comparison CMI – BIPM

The individual measurements results for each of the four standards are shown in figures 1 to 4 below. The plots also show the mean value of the CMI measurements with an uncertainty bar corresponding to the combined standard uncertainty provided in tables 12 and 16, and a linear fit to the BIPM before and after measurements. We assume that the value of each standard is subject to a simple linear drift during the period of the comparison. Inspection of figures 1 to 4 indicates that this is an appropriate model. All four standards seem to fit this model reasonably well within the period of the BIPM before and after measurements. There is not sufficient data to try to produce a more complicated model (although in the next section we include some information on the longer term behaviour of the standards to weight the results). We treat the 1 Ω and 10 k Ω results as two separate cases.

Within this model, the result of the comparison for a given standard is the difference between the mean of the CMI measurements and the interpolated value of the linear fit to the BIPM measurements on the mean date of the CMI measurements.

The difference between the CMI and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{CMI,i} - R_{BIPM,i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{CMI-BIPM} = \frac{1}{2} \sum_{i=1}^2 (R_{CMI,i} - R_{BIPM,i})$$

For each standard, the uncertainty u_1 associated with the dispersion for the interpolated BIPM value is calculated from the linear fit; u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in table 3 or 5. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard, the BIPM uncertainty $u_{BIPM,i}$ is obtained from: $u_{BIPM,i}^2 = u_{1,i}^2 + u_{2,i}^2$

When the mean (for two standards) of the CMI -BIPM relative difference is calculated, the BIPM contribution to the uncertainty is:

$$u_{BIPM}^2 = \sum_{i=1}^2 \frac{u_{1,i}^2}{2^2} + u_2^2$$

Similarly, for the CMI measurements, we expect the uncertainty components u_2 and u_3 of tables 12 and 16 to be correlated between standards, and u_1 to be uncorrelated. We therefore calculate the total uncertainty as

$$u_{CMI}^2 = \sum_{i=1}^2 \frac{u_{1,i}^2}{2^2} + u_2^2 + u_3^2$$

(where $u_2^2 = u_{100\Omega}^2 + u_{100:1}^2$ excluding the type A components in tables 10, 11 and 15).

Uncertainty associated with the transfer

Changes in the values of the standards due to the effects of transport can add an extra uncertainty component to a comparison. In this case, from inspection of the BIPM ‘before’ and ‘after’ measurements in figures 1 to 4, we do not see strong evidence of unwanted transport effects in the short term. We do not include any extra uncertainty components for the 10 k Ω results, but we consider the 1 Ω case further below.

Results at 1 Ω

The differences between the values assigned by the CMI, R_{CMI} , and those assigned by the BIPM, R_{BIPM} , to each of the two travelling standards on the mean date of the CMI measurements are shown in Table 17.

CMI - BIPM		
Standard #	$10^6 \times (R_{\text{CMI}} - R_{\text{BIPM}}) / (1 \Omega)$	$u_{\text{diff}} / 10^{-6}$
BIV203	+ 0.016	0.008
BIV207	- 0.075	0.019

Table 17: Differences for the two 1 Ω travelling standards.

We note that the results from the individual standards are discrepant within the uncertainties of the measurements, and we postulate that the discrepancy may be due to non-ideal transport behavior of one (or both) of the standards. To investigate this, we include in figures 5 and 6 some previous measurements at BIPM of these standards. This extra information suggests that standard BIV207 has likely undergone a larger change on transport than BIV203.

To include this extra information, we take a transport uncertainty for each standard estimated as $u_{\text{diff}} = (\Delta/2\sqrt{3})$, where Δ is the difference between the BIPM ‘before’ and ‘after’ measurements, and we assume a rectangular distribution for value of the standard in-between. We then use these u_{diff} as the weights for a weighted mean of the two differences in table 17, giving:

$$(R_{\text{CMI}} - R_{\text{BIPM}}) / (1 \Omega) = + 0.003 \times 10^{-6}$$

The effect of this weighted mean is to favour the result from BIV203, which is reasonable given the information presented in figures 5 and 6.

The relative combined standard uncertainty of the comparison, u_{C} , is:

$$u_{\text{C}}^2 = u_{\text{BIPM}}^2 + u_{\text{CMI}}^2$$

where

$$u_{\text{BIPM}} = 0.017 \times 10^{-6},$$

$$u_{\text{CMI}} = 0.015 \times 10^{-6},$$

Giving:

$$u_{\text{C}} = 0.023 \times 10^{-6}$$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D , between the CMI and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_{C}

$$D = (R_{\text{CMI}} - R_{\text{BIPM}}) / 1 \Omega = + 0.003 \times 10^{-6}$$

$$U_{\text{C}} = 0.046 \times 10^{-6}$$

The difference between the CMI and the BIPM calibration results is within the expanded uncertainty.

The agreement and uncertainty is limited by two factors: the transport behavior of the resistors and the uncertainty due to the variability of the measured resistance with cycle time.

We have chosen to include the corrections measured by the CMI, and hence to make the slow (BIPM) measurement cycle the reference condition for this comparison. This does not imply that this is necessarily the best measurement timing. The faster measurement cycle usually used at the CMI is probably more representative of typical measurements in calibration labs. It is not helpful to try and state which is the ‘true’ value of the 1 Ω standards; we must simply be aware of this effect and try to take it into account when comparing or using 1 Ω standards at the best uncertainty levels.

Results at 10 kΩ

The differences between the values assigned by the CMI, R_{CMI} , and those assigned by the BIPM, R_{BIPM} , to each of the two travelling standards on the mean date of the CMI measurements are shown in Table 18.

CMI - BIPM	
Standard #	$10^6 \times (R_{\text{CMI}} - R_{\text{BIPM}}) / (10 \text{ k}\Omega)$
B10k08	- 0.028
B10k12	- 0.007
mean	- 0.018

Table 18: Differences for the two 10 kΩ travelling standards.

The mean difference between the CMI and the BIPM calibrations is:

$$(R_{\text{CMI}} - R_{\text{BIPM}}) / (10 \text{ k}\Omega) = - 0.018 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_c , is:

$$u_c^2 = u_{\text{BIPM}}^2 + u_{\text{CMI}}^2$$

where

$$u_{\text{BIPM}} = 0.015 \times 10^{-6},$$

$$u_{\text{CMI}} = 0.009 \times 10^{-6},$$

Giving:

$$u_c = 0.017 \times 10^{-6}$$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D , between the CMI and the BIPM for values assigned to 10 kΩ resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_c

$$D = (R_{\text{CMI}} - R_{\text{BIPM}}) / 10 \text{ k}\Omega = - 0.018 \times 10^{-6}$$

$$U_c = 0.034 \times 10^{-6}$$

The difference between the CMI and the BIPM calibration results is within the expanded uncertainty.

Reference

- [1] Fletcher, N., Götz, M., Rolland, B., & Pesel, E. (2015). Behavior of 1 Ω resistors at frequencies below 1 Hz and the problem of assigning a dc value. *Metrologia*, 52(4), 509–513. <http://doi.org/10.1088/0026-1394/52/4/509>

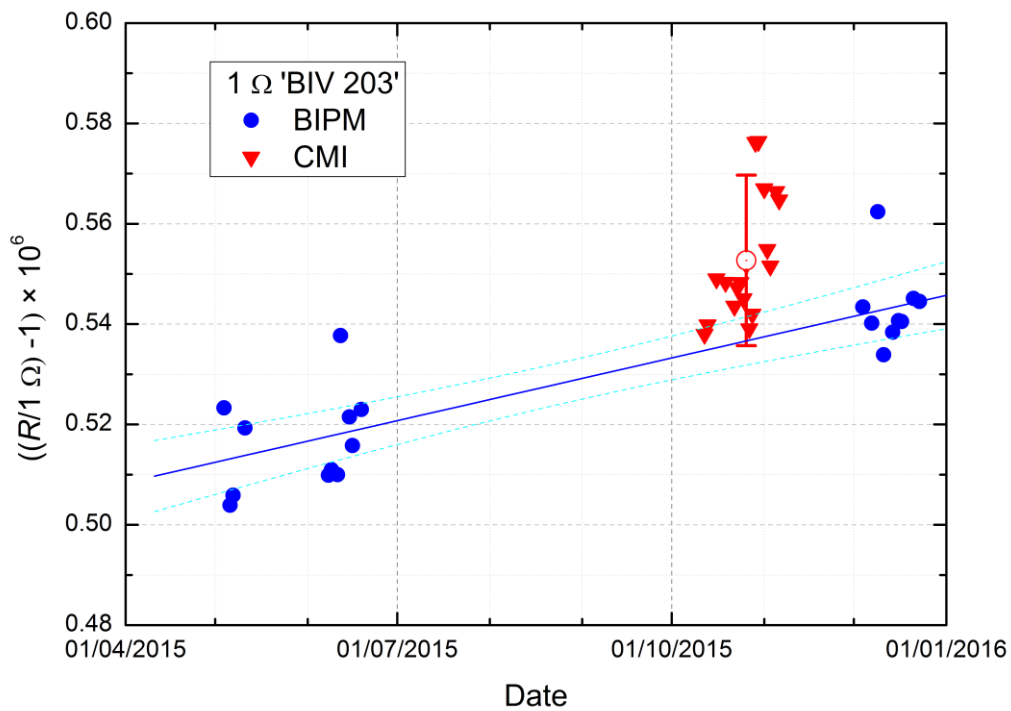


Figure 1: results for 1 Ω standard BIV203; uncertainty bar shows the combined standard uncertainty on the mean CMI results (including corrections)

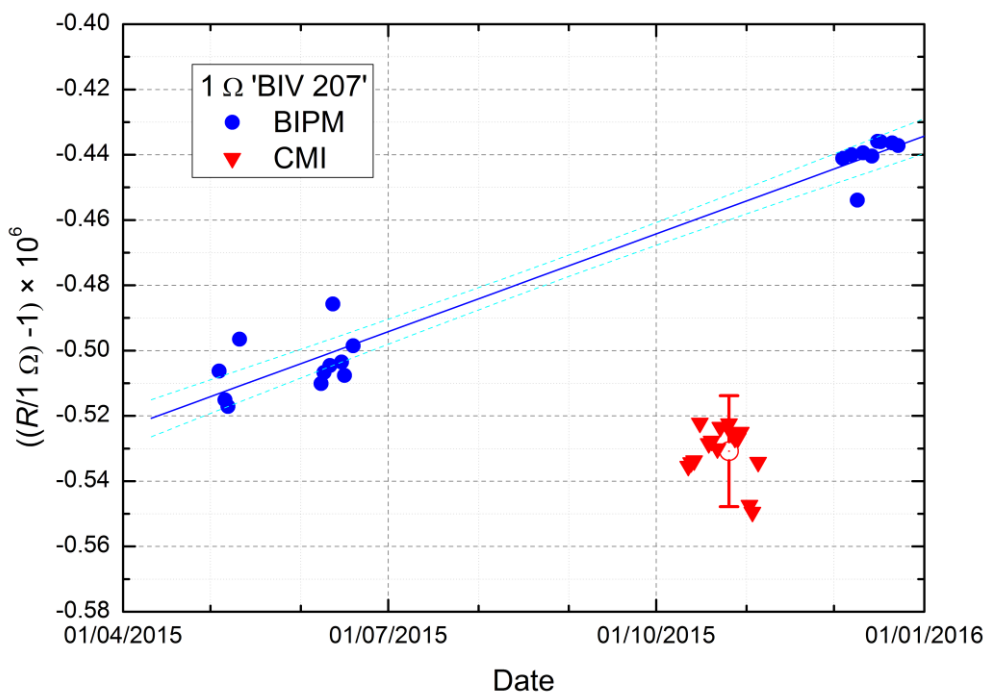


Figure 2: results for 1 Ω standard BIV207

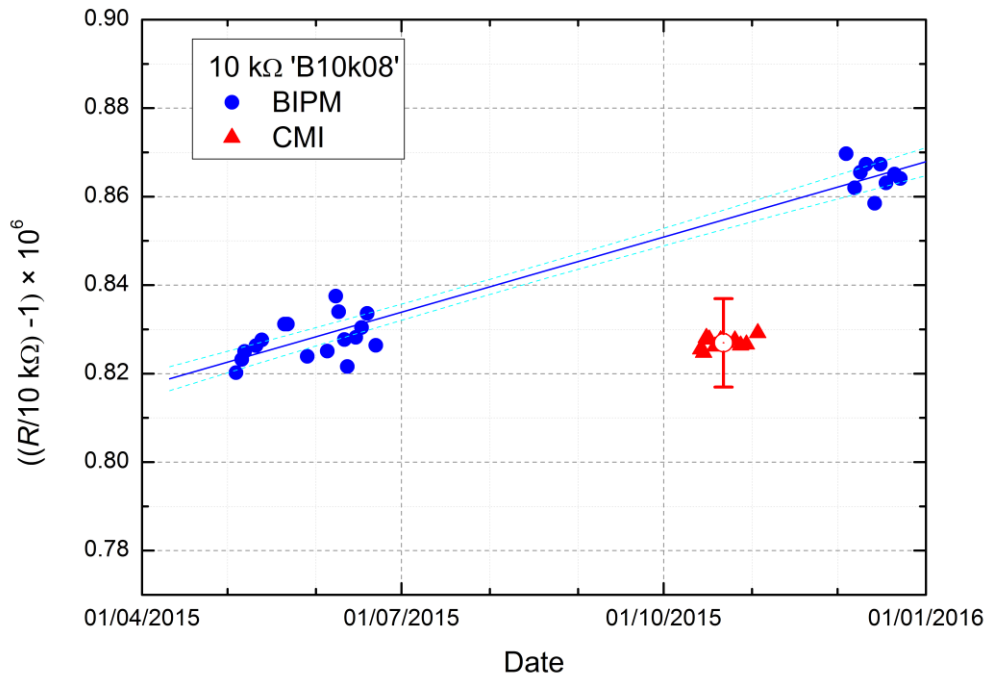


Figure 3: results for 10 kΩ standard B10k08; uncertainty bar shows the combined standard uncertainty on the mean CMI results (including corrections)

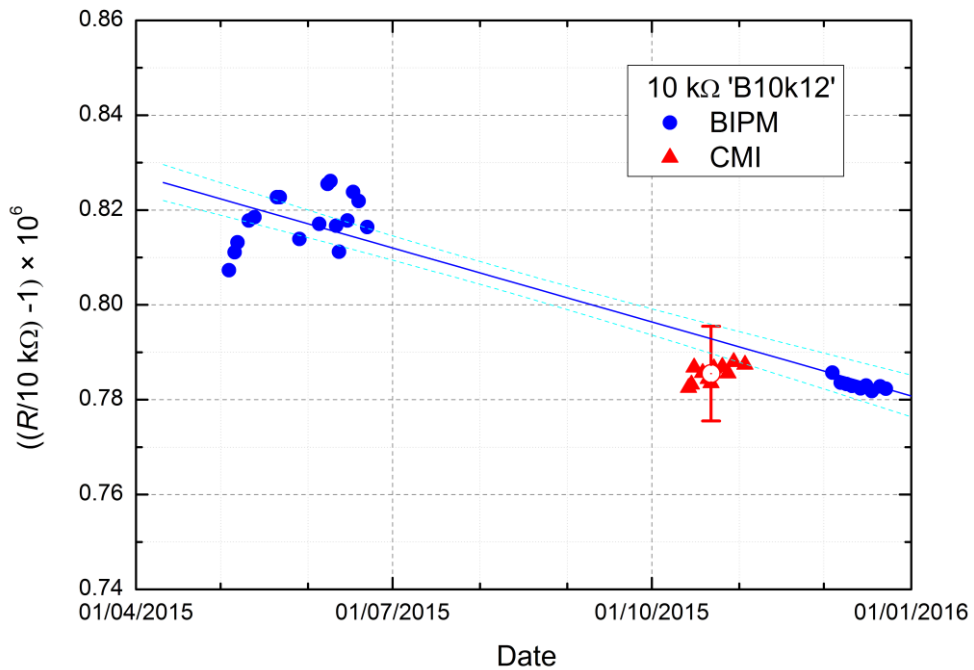


Figure 4: results for 10 kΩ standard B10k12

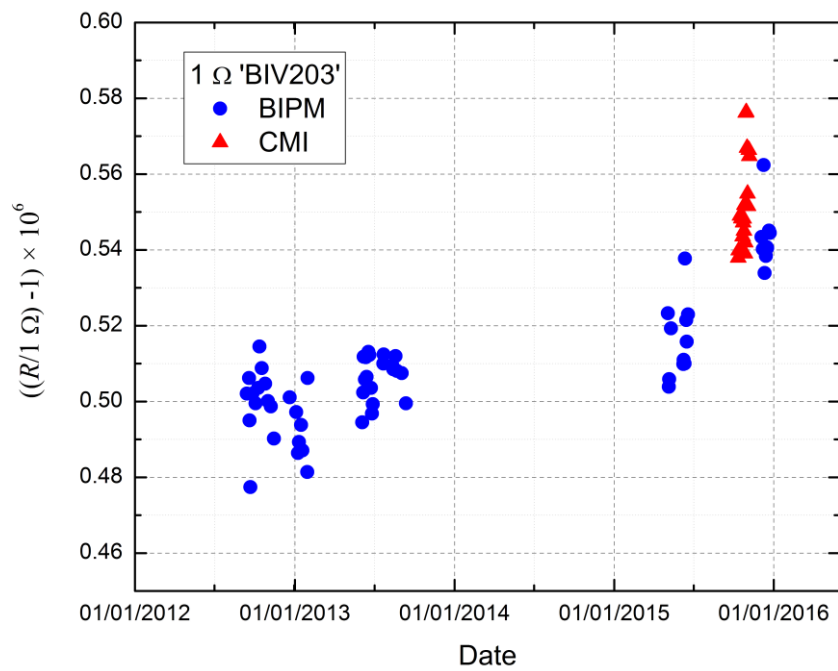


Figure 5: extended results for 1 Ω standard BIV203

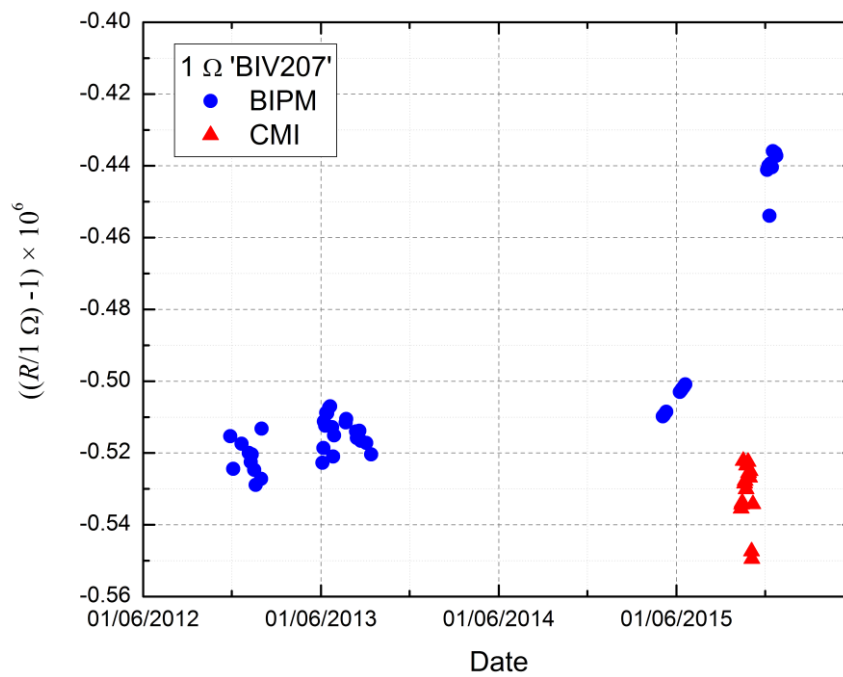


Figure 6: extended results for 1 Ω standard BIV207