BUREAU INTERNATIONAL
DES POIDS ET MESURES

Bilateral comparison of 1 Ω and 10 kΩ standards (ongoing BIPM key comparisons BIPM.EM-K13.a and 13.b) between the EMI (Emirates Metrology Institute, Abu Dhabi Quality and Conformity Council) and the BIPM

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Final report

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1 Introduction

A comparison of values assigned to 1 Ω and 10 kΩ resistance standards was carried out between the BIPM (Bureau International des Poids et Mesures) and the EMI (Emirates Metrology Institute, Abu Dhabi Quality and Conformity Council, Abu Dhabi, United Arab Emirates) in the period February 2020 to July 2020. Two 1 Ω and two 10 kΩ BIPM travelling standards were calibrated first at the BIPM, then at the EMI and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: February 2020 – March 2020
EMI measurements: May 2020
'After' measurements at the BIPM: June 2020 – July 2020

This report is organised as follows: details of the travelling standards used are listed in Section 2; the results of the BIPM measurements are given in Section 3 and the calibration report provided by the EMI is summarized in Section 4; these two sections include the uncertainty budgets for each laboratory. Finally, the two sets of measurements are compared and analysed in Section 5. The uncertainties arising from the transfer of the standards between the two laboratories are estimated and included at this point. The final results of the comparisons are given in the form of the degrees of equivalence (deviations from the KCRV and associated uncertainties) between the EMI and the BIPM for measurements of 1 Ω and 10 kΩ resistance standards.

This report covers the comparison of both 1 Ω standards (BIPM.EM-K13.a) and 10 kΩ standards (BIPM.EM-K13.b). The measurements of these two different resistance values are analysed separately, but are reported together here as the two comparisons were carried out simultaneously.

2 Travelling Standards

Two 1 Ω and two 10 kΩ travelling standards provided by the BIPM were used for this comparison. The two 1 Ω standards are of CSIRO type, with working labels BIV200 (manufacturer's serial number S-64200) and BIV207 (manufacturer's serial number S-64207). The two 10 kΩ standards are TEGAM S104 type and have the working labels B10K11 (manufacturer's serial number K205039730104) and B10K12 (serial number K201089830104). The standards were shipped by regular air freight between the laboratories.

All measurements are corrected to a reference temperature of 23.000 °C and reference pressure 1013.25 hPa using the known coefficients of the standards, given in Table 1. According to the protocol, the EMI did not apply pressure and temperature corrections to its results, but supplied the raw values and the measured temperature and pressure. The corrections were applied in the analysis made by the BIPM.

	Relative temperature coefficients		Relative pressure coefficient
Standard #	α_{23} / (10^{-6}/K)	β / (10^{-6}/K²)	γ / (10^{-9}/hPa)
BIV200	− 0.0074	+ 0.0004	− 0.100
BIV207	− 0.0094	+ 0.0001	− 0.250
B10K11	− 0.0700	− 0.0270	− 0.350
B10K12	+ 0.0100	− 0.0230	− 0.226

Table 1: Temperature and pressure coefficients of the travelling standards.

3 Measurements at the BIPM

3.1 Measurement of the 1 Ω standards at the BIPM:

The BIPM measurements are traceable to the quantum Hall resistance (QHR) standard via different measurement bridges and working standards for the two nominal values. In all cases, values are based on the revised SI value of the von Klitzing constant,
$R_K = h/e^2 = 25\,812.807\,46\ \Omega$, using the fixed numerical values for the Planck constant h and the elementary charge e.

The 1 Ω measurements were carried out by comparison with a 100 Ω reference resistor (identifier BI100-3) whose value is calibrated against the BIPM QHR standard regularly (at least once every 6 months). The comparison was performed using a DC cryogenic current comparator operating with 50 mA current in the 1 Ω resistors.

The 1 Ω travelling standards were kept in a temperature-controlled oil bath at a temperature which is close (within a few mK) to the reference temperature of 23 °C. The oil temperature close to each standard was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT) in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The additional pressure P_h exerted by the volume of the mineral oil above the resistors (reference plane corresponding to the plane containing the resistor terminals) has been considered for every measurement. P_h, is calculated using the following equation:

$$P_h = RD \times \rho \times g \times H$$

With RD the relative density of the oil Marcol 52 type = 0.83; ρ the density of the pure water = 1000 kg m^{-3} at 4°C; g the local gravity = 9.807 $m\ s^{-2}$ and H the height of the oil above the reference plane in m.
The height of the oil above the reference plane is recorded in the software of the measurement bridge and the additional pressure is calculated automatically at every measurement.

The 'dc' resistance value (or ratio) measured with the BIPM CCC-bridge results from a current signal passing through the resistors having polarity reversals with a waiting time between polarity inversions, cf. Figure 1. The polarity reversal frequency is of the order of 3 mHz (340 s cycle period) and the measurements are sampled only during 100 s before the change of polarity.

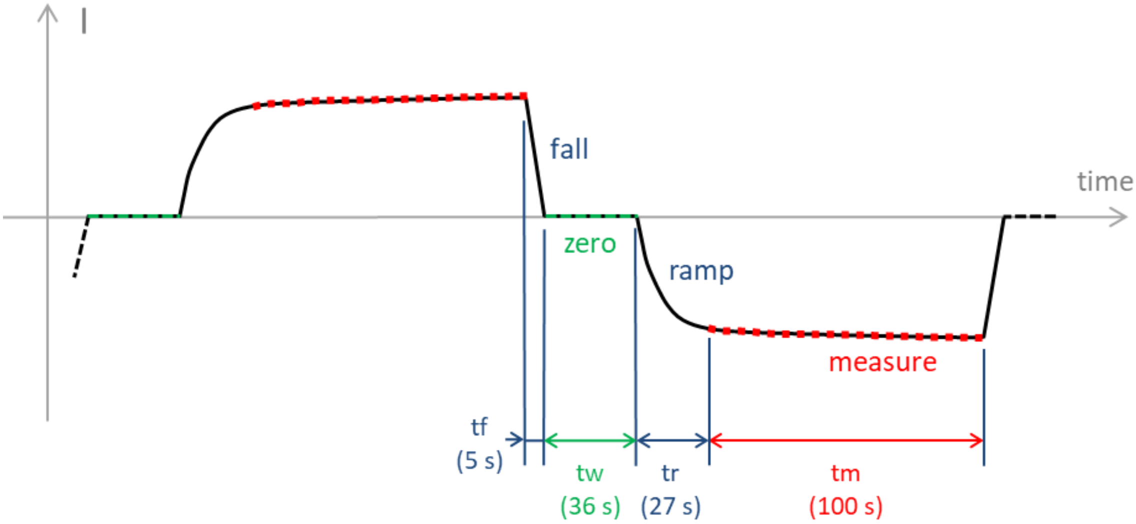

Figure 1: Schematic representation of the reference current signal with polarity reversals used in the BIPM CCC-bridge. The reversal cycle comprises a waiting time of about 36 s at zero current (green dotted line). The red dotted line corresponds to the sampling time period.

The travelling standards were measured 10 times during the period labelled ‘before’ (February 2020 – March 2020) and 12 times during the period labelled ‘after’ (June 2020 – July 2020).

The individual BIPM measurement data are plotted in Figures 2 and 3 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 2 and the uncertainty budget in Table 3. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	**Relative difference from nominal 1 Ω value / 10^{-6}**					
Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}	INTERPOLATED ON 15-05-2020	Std dev. u_1
BIV200	− 0.685	0.004	− 0.701	0.008	− 0.695	0.004
BIV207	− 0.526	0.006	− 0.505	0.009	− 0.514	0.005

Table 2: Summary of BIPM calibrations of the 1 Ω standards.

Source of uncertainty	**relative standard uncertainty /10^{-9}**
Imperfect realisation of R_K	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_K	3
Interpolation / extrapolation of the value of BI100-3	13
Measurement of the (1Ω / BI100-3) ratio	8
Temperature correction for the 1 Ω standard	2
Pressure correction for the 1 Ω standard	3
Combined uncertainty u_2	**16**

Table 3: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

3.2 Measurement of the 10 kΩ standards at the BIPM:

The 10 kΩ measurements were carried out by comparison with a set of two 10 kΩ reference resistors (identifiers B10K1 and B10K2) which are calibrated regularly (at least once every 6 months) against the BIPM QHR standard. The comparison was performed using a Warshawsky bridge operating with a 0.1 mA DC current (i.e. at a measurement voltage of 1 V).

The 10 kΩ travelling standards were kept in a temperature-controlled air bath at a temperature which is close to the reference temperature of 23 °C (within 0.05 °C). The temperature of the standards was determined by means of a calibrated platinum resistance thermometer, in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The relative humidity in the air bath was not monitored, but the laboratory air conditioning system controls the relative humidity to 50 % (± 10 %).

The travelling standards were measured 10 times during the period labelled ‘before’ (February 2020 – March 2020) and 12 times during the period labelled ‘after’ (June 2020 – July 2020). The individual BIPM measurement data are plotted in Figures 4 and 5 of Section 5 (after application of the temperature and pressure correction). The mean results are summarized in Table 4 and the uncertainty budget in Table 5. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	**Relative difference from nominal 10 kΩ value / 10^{-6}**					
Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}	INTERPOLATED ON 15-05-2020	Std dev. u_1
B10K11	+ 1.164	0.003	+ 1.236	0.004	+ 1.207	0.003
B10K12	+ 0.893	0.002	+ 1.022	0.003	+ 0.970	0.002

Table 4: Summary of BIPM calibrations of the 10 kΩ standards.

Source of uncertainty	**relative standard uncertainty / 10^{-9}**
Imperfect realization of R_K	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_K	3
Link 100 Ω / 10 000 Ω	5
Link 10 000 Ω / (mean reference B10K1-B10K2)	7
Extrapolation of mean value of 10 kΩ reference	8
Measurement of the voltage applied to the bridge	5
Measurements of the bridge unbalance voltage	5
Leakage resistances	1
Temperature correction for travelling standard	3
Pressure correction for travelling standard	2
Combined uncertainty u_2	**15**

Table 5: BIPM uncertainty budget for the calibration of the 10 kΩ travelling standards.

4 Measurements at the EMI

4.1 Preparation:

The standard resistors were received in the laboratory on 18 March 2020 and a visual check was made to ensure that the standards were in working condition. A check measurement was made to ensure that there was no unexpected deviation from the nominal value.

There were some unavoidable delays until 27 April 2020 when the 1 Ω standard resistors were placed in the oil bath and the 10 kΩ standard resistors were placed in an air bath. The resistors were then left in these controlled environments for more than 48 hours before measurements commenced.

4.2 Maintenance of DC Resistance at EMI:

Resistors are measured using the Measurements International 6010D Direct Current Comparator Bridge. This bridge measures the ratio of the resistance of the unknown resistor to the resistance of the standard resistor. The MI 6010D Direct Current Comparator Bridge is calibrated by the laboratory annually by use of Hamon resistors.

The bridge measures the ratio Rx:Rs. For greatest accuracy the bridge is used between 0.9:1 and 10:1. Usually the standard resistor is connected to Rs and the Unit Under Test (UUT) to Rx, but when measuring a resistor of lower value using a standard of higher value (e.g. "build-down") the standard is connected to Rx and the UUT to Rs. This ensures that the bridge ratio stays between 0.9:1 and 10:1.

Standard resistors at values 10 kΩ are calibrated externally every year to provide traceability. Standard resistors at values 1 Ω are calibrated externally every two years to provide a check on the performance of the build down and to provide traceability for values below 1 Ω. The intermediate standard resistors at values of 10 Ω, 100 Ω and 1 kΩ are calibrated by "build-down" from the 10 kΩ standard resistors. The 10 Ω, 100 Ω and 1 kΩ standard resistor are maintained in a temperature stabilized oil bath and the 10 kΩ is kept in a temperature stabilized air bath. The information related to the measurement standards used are summarized in Table 6.

Name	Manufacturer	Model No	Serial No	Traceability
10 kΩ Standard Resistor	IET Labs	SR-104	J2-1529649	NPL, UK
10 Ω Standard Resistor	Tinsley	5685A	17434/26	EMI, UAE*
Primary Resistance Bridge	Measurements International	6010D	1102774	EMI, UAE

Table 6: EMI measurement standard used.

*The 10 Ω Standard Resistor, Serial Number 17434/26, is calibrated by a "build-down" method from the 10 kΩ Standard Resistor, Serial Number J2-1529649.

A group of four resistors is maintained at both 1 Ω and 10 kΩ. These resistors are compared every six months. A "build-up" of resistors from 1 Ω to 10 kΩ is also performed every six months. The data from these measurements are monitored for any trends.

4.3 Measurement of the travelling standards:

The Measurements International 6010D Direct Current Comparator Bridge was used to measure the resistance of the travelling standards by comparison to calibrated standard resistors.

The 10 kΩ travelling standards were measured by comparison to calibrated IET SR104 10 kΩ standard resistor, serial number J2-1529649. As described in § 4.2, the 10 kΩ standard resistor was maintained in a temperature stabilized air bath throughout the measurements. The 10 kΩ

travelling standards were maintained in another temperature stabilized air bath throughout the measurements.
The 1 Ω travelling standards were measured by comparison to calibrated Tinsley 5685A 10 Ω standard resistor, serial number 17434/26, which had been calibrated at EMI by “build-down” from calibrated IET SR104 10 kΩ standard resistor, serial number J2-1529649 immediately before these measurements. The 10 Ω standard resistor was maintained in a temperature stabilized oil bath throughout the measurements. The 1 Ω travelling standards were maintained in the same temperature stabilized oil bath throughout the measurements.

Measurements were also made against other resistance standards maintained at EMI. The agreement between these measurements provided a confidence check, but the results of these additional measurements are not reported.

This process was repeated every 2 to 4 days. One set of 25 measurements was made on each travelling standard on each measurement day. The resistance values given in the measurement report are the mean of 25 individual measurements made with the primary resistance bridge on the given day.

The IET SR104 10 kΩ standard resistor, serial number J2-1529649, had been calibrated by NPL, UK in September 2019 with certificate reference 2019020044-1 (RM 34.69). The calibrated value has been adjusted, based on the expected drift from the history of external calibrations, to give a value on the mean date of the measurements for this resistor. This value has been checked for consistency by measurement of the EMI 10 kΩ group.

The nominal direct current was 50 mA for 1 Ω standard resistors and 100 µA for 10 kΩ standard resistors. The current supplied by the Measurements International 6010D Direct Current Comparator Bridge was measured as part of the annual calibration. The direction of the applied current was reversed every 6 s for 1 Ω standard resistors and every 12 s for 10 kΩ standard resistors.

The pressure given in the result tables is the mean value during the measurement period on the indicated day. The pressure for those resistors immersed in the oil bath includes the additional pressure for the head of oil above the reference plane, the insulating plate containing the potential and current terminals. This head was 65 mm for measurements before 13 May 2020 and 86 mm for measurements after 13 May 2020. The oil used is Drakeol 7 with a specific gravity of 0.833/0.861, the mean specific gravity of 0.847 was used and an allowance for the range was included in the uncertainty of the pressure measurement.

The additional pressure due to the head of oil, P_h, is calculated using the following equation

$$P_h = SG \times \rho \times g \times H$$

SG is the specific gravity of the oil $= 0.847$
ρ is the density of water at 4°C $= 1000$ kg m^{-3}
g is the local gravity $=$ 9.789 m s^{-2}
H is the height of the oil above the reference plane in m

The additional pressure due to the head of oil for measurements before 13 May 2020 is 5.4 hPa, and for measurements after 13 May 2020 is 7.1 hPa.

4.4 <u>EMI results at 1 Ω</u>:

The 1 Ω travelling standards were measured 11 times in the period 1st of May 2020 to 29th of May 2020. Table 7 gives the mean values at the mean date of 15 May 2020, before application of temperature and atmospheric pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements. The pressure of the mineral oil exerted on the resistors has been considered for every measurement and the mean atmospheric pressure from Table 7 is corrected from this effect.

Standard #	**Relative difference from nominal 1 Ω value /10^{-6}**	**Std dev. /10^{-6}**	**Mean temperature / °C**	**Mean pressure at the reference plane / hPa**
BIV200	− 0.752	0.031	23.000	1013.3
BIV207	− 0.571	0.030	23.000	1013.3

Table 7: Summary of EMI 1 Ω calibrations.

4.4.1 Corrections for temperature and pressure:

The value $R(23)$ of the resistance corrected for T_0 = 23 °C is:

$$\mathrm{R}(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$$

where $R(T)$ is the resistance of the standard at temperature T.

The value $R(1013.25)$ of the resistance corrected for P_0 = 1013.25 hPa is:

$$\mathrm{R}(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$$

where $R(P)$ is the resistance of the standard at pressure P.

The EMI results are corrected to the reference temperature and the reference pressure using the coefficients α_{23}, β and γ shown in Table 1.

Reference temperature = 23.000°C Reference pressure = 1013.25 hPa		
	Relative corrections /10^{-6}	
Standard #	For temperature	For pressure
BIV200	+ 0.000	+ 0.000
BIV207	+ 0.000	+ 0.000

Table 8: Corrections applied to the EMI 1 Ω results.

The uncertainties on temperature and pressure measurements at the EMI are 0.021 °C and 1.7 hPa respectively. Taking into account the differences from the reference temperature and reference pressure, and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\mathrm{Temp}} < 0.001 \times 10^{-6}$ and $u_{\mathrm{Press}} < 0.001 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.001 \times 10^{-6}$. u_3 is reported in Table 11.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

4.4.2 Uncertainty Budget Provided by the EMI:

CSIRO 1Ω, Serial number 64200						
Uncertainty Description	Relative Expanded Uncertainty $U(x_i) \times 10^6$	Reference	Probability Distribution	Divisor, k_i	Relative Uncertainty Contribution, $u_i(y) \times 10^6$	Degrees of Freedom, v_i
Repeatability of the measurements on 1 day		Maximum measured for this resistor	Normal		0.009	24
Uncertainty of Standard	0.295	EMI Certificate	Normal	2	0.147	inf
Uncertainty due to long term drift of standard	0.184	Difference between last calibration and prediction from history	Rectangular	$\sqrt{3}$	0.106	inf
Uncertainty due to temperature of standard	0.080	Standard certificate and oil bath certificate	Rectangular	$\sqrt{3}$	0.046	inf
Uncertainty due to power in standard	0.000	Specification and certificate	Rectangular	$\sqrt{3}$	0.000	inf
Drift of bridge ratios	0.040	Specification and History	Rectangular	$\sqrt{3}$	0.023	inf
6010D Error	0.110	EMI Certificate	Normal	2	0.055	inf

Combined Uncertainty Results	
Combined Relative Standard Uncertainty	0.197
ν_{eff}	5203619
Coverage Factor (k)	2.000
Expanded Relative Uncertainty	**0.394**
Expanded Absolute Uncertainty μΩ	**0.40**

Table 9: Summary of the EMI uncertainty budget for 1 Ω BIV200.

CSIRO 1Ω, Serial number 64207						
Uncertainty Description	Relative Expanded Uncertainty $U(x_i) \times 10^6$	Reference	Probability Distribution	Divisor, k_i	Relative Uncertainty Contribution, $u_i(y) \times 10^6$	Degrees of Freedom, v_i
Repeatability of the measurements on 1 day		Maximum measured for this resistor	Normal		0.009	24
Uncertainty of Standard	0.295	EMI Certificate	Normal	2	0.147	inf
Uncertainty due to long term drift of standard	0.184	Difference between last calibration and prediction from history	Rectangular	$\sqrt{3}$	0.106	inf
Uncertainty due to temperature of standard	0.080	Standard certificate and oil bath certificate	Rectangular	$\sqrt{3}$	0.046	inf
Uncertainty due to power in standard	0.000	Specification and certificate	Rectangular	$\sqrt{3}$	0.000	inf
Drift of bridge ratios	0.040	Specification and History	Rectangular	$\sqrt{3}$	0.023	inf
6010D Error	0.110	EMI Certificate	Normal	2	0.055	inf

Combined Uncertainty Results	
Combined Relative Standard Uncertainty	0.197
ν_{eff}	5449680
Coverage Factor (k)	2.000
Expanded Relative Uncertainty	**0.394**
Expanded Absolute Uncertainty μΩ	**0.40**

Table 10: Summary of the EMI uncertainty budget for 1 Ω BIV207.

Note: The expanded uncertainty of the measurement is calculated by multiplying the combined standard uncertainty of the measurement $u(y)$ by a coverage factor $k_{95.45}$ based on a t distribution for the effective degrees of freedom v_{eff} of the combined uncertainty $u(y)$ calculated from the degrees of freedom v_i of the individual standard uncertainties $U(x_i)$ using the Welch-Satterthwaite equation:

$$v_{eff} = \frac{u^4(y)}{\sum_i \left(\frac{u^4(x_i)}{v_i}\right)}$$

4.4.3 Uncertainties associated with the value of 1 Ω measurement

EMI After corrections	**Relative difference from nominal value / 10^{-6}**	**Relative standard uncertainties**		
		Repeatability u_1 / 10^{-6}	Systematic u_2 / 10^{-6}	Corrections u_3 / 10^{-6}
BIV200	− 0.752	0.031	0.197	0.001
BIV207	− 0.571	0.030	0.197	0.001

Table 11: Summary of the EMI results at 1 Ω, after corrections.

Note 1: The distinction between 'systematic' and 'repeatability' is made in Table 11 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into Type A and Type B components.
Note 2: The "Repeatability" line from the Table 9 and Table 10 corresponds to the standard deviation of the mean of the measurements.

4.5 EMI results at 10 kΩ:

The 10 kΩ travelling standards were measured 11 times in the period 1st of May 2020 to 29th of May 2020. Table 12 gives the mean values at the mean date of 15 May 2020, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 10 kΩ value /10^{-6}	Std dev. /10^{-6}	Mean temperature / °C	Mean atmospheric pressure / hPa
B10K11	+ 1.217	0.059	23.00	1006.9
B10K12	+ 0.996	0.051	23.00	1006.9

Table 12: Summary of EMI 10 kΩ calibrations.

4.5.1 Corrections for temperature and pressure:

The value $R(23)$ of the resistance corrected for T_0 = 23 °C is:

$$\mathrm{R}(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$$

where $R(T)$ is the resistance of the standard at temperature T.

The value $R(1013.25)$ of the resistance corrected for P_0 = 1013.25 hPa is:

$$\mathrm{R}(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$$

where $R(P)$ is the resistance of the standard at pressure P.

The EMI results are corrected to the reference temperature, the reference pressure and the power difference using the coefficients α_{23}, β and γ shown in Table 1.

Reference temperature = 23.000°C Reference pressure = 1013.25 hPa		
	Relative corrections /10^{-6}	
Standard #	For temperature	For pressure
B10K11	+ 0.000	− 0.002
B10K12	+ 0.000	− 0.001

Table 13: Corrections applied to the EMI 10 kΩ results.

The uncertainties on temperature and pressure measurements at the EMI are 0.30 °C and 1.7 hPa respectively. Taking into account the differences from the reference temperature and reference pressure, and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\mathrm{Temp}} = 0.021 \times 10^{-6}$ and $u_{\mathrm{Press}} = 0.001 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.021 \times 10^{-6}$. u_3 is reported in Table 16.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

4.5.2 Uncertainty Budget Provided by the EMI:

TEGAM SR104 10 kΩ, Reference B10K11, Serial number K205039730104						
Uncertainty Description	Relative Expanded Uncertainty $U(x_i) \times 10^6$	Reference	Probability Distribution	Divisor, k_i	Relative Uncertainty Contribution, $u_i(y) \times 10^6$	Degrees of Freedom, v_i
Repeatability of the measurements on 1 day		Maximum measured for this resistor	Normal		0.025	24
Uncertainty of Standard	0.060	NPL Certificate	Normal	2	0.030	inf
Uncertainty due to transportation of standard	0.037	Change in EMI measurement before and after last calibration	Rectangular	$\sqrt{3}$	0.022	inf
Uncertainty due to long term drift of standard	0.082	Difference between last calibration and prediction from history	Rectangular	$\sqrt{3}$	0.047	inf
Uncertainty due to temperature of standard	0.018	Standard certificate and air bath certificate	Rectangular	$\sqrt{3}$	0.010	inf
Uncertainty due to power in standard	0.000	Specification and certificate	Rectangular	$\sqrt{3}$	0.000	inf
Drift of bridge ratios	0.040	Specification and History	Rectangular	$\sqrt{3}$	0.023	inf
6010D Error	0.110	EMI Certificate	Normal	2	0.055	inf
Combined Uncertainty Results						
Combined Relative Standard Uncertainty	0.089					
ν_{eff}	3781					
Coverage Factor (k)	2.001					
Expanded Relative Uncertainty	**0.178**					
Expanded Absolute Uncertainty mΩ	**1.8**					

Table 14: Summary of the EMI uncertainty budget for 10 kΩ B10K11.

TEGAM SR104 10 kΩ, Reference B10K12, Serial number K201089830104						
Uncertainty Description	Relative Expanded Uncertainty $U(x_i) \times 10^6$	Reference	Probability Distribution	Divisor, k_i	Relative Uncertainty Contribution, $u_i(y) \times 10^6$	Degrees of Freedom, v_i
Repeatability of the measurements on 1 day		Maximum measured for this resistor	Normal		0.016	24
Uncertainty of Standard	0.060	NPL Certificate	Normal	2	0.030	inf
Uncertainty due to transportation of standard	0.037	Change in EMI measurement before and after last calibration	Rectangular	$\sqrt{3}$	0.022	inf
Uncertainty due to long term drift of standard	0.082	Difference between last calibration and prediction from history	Rectangular	$\sqrt{3}$	0.047	inf
Uncertainty due to temperature of standard	0.018	Standard certificate and air bath certificate	Rectangular	$\sqrt{3}$	0.010	inf
Uncertainty due to power in standard	0.000	Specification and certificate	Rectangular	$\sqrt{3}$	0.000	inf
Drift of bridge ratios	0.040	Specification and History	Rectangular	$\sqrt{3}$	0.023	inf
6010D Error	0.110	EMI Certificate	Normal	2	0.055	inf

Combined Uncertainty Results	
Combined Relative Standard Uncertainty	0.087
v_{eff}	22768
Coverage Factor (k)	2.000
Expanded Relative Uncertainty	**0.173**
Expanded Absolute Uncertainty mΩ	**1.8**

Table 15: Summary of the EMI uncertainty budget for 10 kΩ B10K12.

Note: The expanded uncertainty of the measurement is calculated by multiplying the combined standard uncertainty of the measurement $u(y)$ by a coverage factor $k_{95.45}$ based on a t distribution for the effective degrees of freedom v_{eff} of the combined uncertainty $u(y)$ calculated from the degrees of freedom v_i of the individual standard uncertainties $U(x_i)$ using the Welch-Satterthwaite equation:

$$v_{eff} = \frac{u^4(y)}{\sum_i \left(\frac{u^4(x_i)}{v_i}\right)}$$

4.5.3 Uncertainties associated with the value of 10 kΩ measurement

EMI After corrections	**Relative difference from nominal value / 10^{-6}**	**Relative standard uncertainties**		
		Repeatability u_1 / 10^{-6}	Systematic u_2 / 10^{-6}	Corrections u_3 / 10^{-6}
B10K11	+ 1.215	0.059	0.085	0.021
B10K12	+ 0.995	0.051	0.085	0.021

Table 16: Summary of the EMI results at 10 kΩ, after corrections.

Note 1: The distinction between ‘systematic’ and ‘repeatability’ is made in Table 16 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into Type A and Type B components.
Note 2: The “Repeatability” line from the Table 14 and Table 15 corresponds to the standard deviation of the mean of the measurements.

5 Comparison EMI – BIPM

The individual measurement results for each of the four standards are shown in Figures 2 to 5 below. The plots also show the mean value of the EMI measurements with the uncertainty bar corresponding to the expanded uncertainty (k = 2) of the comparison U_c provided below, and a linear fit to the BIPM before and after measurements. We assume that the value of each standard is subject to a simple linear drift during the period of the comparison. Inspection of Figures 2 to 5 indicates that this is an appropriate model. Both 1 Ω standards and 10 kΩ standards fit this model well. We treat the 1 Ω and 10 kΩ results as two separate cases.

Within this model, the result of the comparison for a given standard is the difference between the mean of the EMI measurements and the interpolated value of the linear fit to the BIPM measurements on the mean date of the EMI measurements.

The difference between the EMI and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{EMI},i} - R_{\mathrm{BIPM},i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\mathrm{EMI-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}\left(R_{\mathrm{EMI}\,,i} - R_{\mathrm{BIPM},i}\right)$$

For each standard, the uncertainty u_1 associated with the interpolated BIPM value is calculated from the linear fit; u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 3 or 5. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard R_i, the BIPM uncertainty $u_{\mathrm{BIPM},\,i}$ is obtained from: $u^2_{\mathrm{BIPM,i}} = u^2_{1,i} + u^2_{2,i}$. When the mean (for two standards) of the EMI − BIPM relative difference is calculated, the BIPM contribution to the uncertainty is

$$u^2_{\mathrm{BIPM}} = \sum_{i=1}^{2}\frac{u^2_{1,i}}{2^2} + u^2_2$$

Similarly, for the EMI measurements, we expect the uncertainty components u_2 and u_3 of Tables 11 and 16 to be correlated between standards, and u_1 to be uncorrelated. We therefore calculate the total uncertainty as:

$$u^2_{\mathrm{EMI}} = \sum_{i=1}^{2}\frac{u^2_{1,i}}{2^2} + u^2_2 + u^2_3$$

5.1 Uncertainty associated with the transfer

Changes in the values of the standards due to the effects of transport can add an extra uncertainty component to a comparison. In the present case, from inspection of the BIPM ‘before’ and ‘after’ measurements in Figures 2 to 5, we can see that any such effects are negligible compared to the overall uncertainty of the comparison. For simplicity, we do not include any extra uncertainty components.

5.2 Results at 1 Ω

The differences between the values assigned by the EMI, R_{EMI}, and those assigned by the BIPM, R_{BIPM}, to each of the two travelling standards on the mean date of the EMI measurements are shown in Table 17.

EMI − BIPM	
Standard #	$10^6 \times (R_{\mathrm{EMI}} - R_{\mathrm{BIPM}}) / (1\ \Omega)$
BIV200	− 0.057
BIV207	− 0.057
Mean	**− 0.057**

Table 17: EMI – BIPM differences for the two 1 Ω travelling standards.

The mean difference between the EMI and the BIPM calibrations is:

$$(R_{\mathrm{EMI}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = -0.057 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u_c^2 = u_{BIPM}^2 + u_{\mathrm{EMI}}^2$$

where

$$u_{\mathrm{BIPM}} = 0.017 \times 10^{-6},$$
$$u_{\mathrm{EMI}} = 0.199 \times 10^{-6},$$

Giving:

$$\boldsymbol{u_{\mathrm{C}} = 0.200 \times 10^{-6}}$$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the EMI and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_{C}:

$$D = (R_{\mathrm{EMI}} - R_{\mathrm{BIPM}}) / 1\ \Omega = -0.057 \times 10^{-6}$$

$$U_{\mathrm{C}} = 0.400 \times 10^{-6}$$

The difference between the EMI and the BIPM calibration results is within the expanded uncertainty.

5.3 Results at 10 kΩ

The difference between the value assigned by the EMI, R_{EMI}, and those assigned by the BIPM, R_{BIPM}, to each of the two travelling standards on the mean date of the EMI measurements are shown in Table 18.

EMI − BIPM	
Standard #	$10^6 \times (R_{\mathrm{EMI}} - R_{\mathrm{BIPM}})$ / (10 kΩ)
B10K11	+ 0.008
B10K12	+ 0.025
Mean	+ **0.017**

Table 18: EMI – BIPM differences for the two 10 kΩ travelling standards.

The mean difference between the EMI and the BIPM calibrations is:

$$\boldsymbol{(R_{\mathrm{EMI}} - R_{\mathrm{BIPM}}) / (10\ \mathrm{k\Omega}) = + 0.017 \times 10^{-6}}$$

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u_c^2 = u_{BIPM}^2 + u_{\mathrm{EMI}}^2$$

where $u_{\mathrm{BIPM}} = 0.015 \times 10^{-6}$,
$u_{\mathrm{EMI}} = 0.096 \times 10^{-6}$,

Giving: $\boldsymbol{u_{\mathrm{C}} = 0.097 \times 10^{-6}}$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the EMI and the BIPM for the value assigned to 10 kΩ resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_{C}:

$$\boldsymbol{D = (R_{\mathrm{EMI}} - R_{\mathrm{BIPM}}) / 10\ \mathrm{k\Omega} = + 0.017 \times 10^{-6}}$$

$$\boldsymbol{U_{\mathrm{C}} = 0.194 \times 10^{-6}}$$

The difference between the EMI and the BIPM calibration results is within the expanded uncertainty.

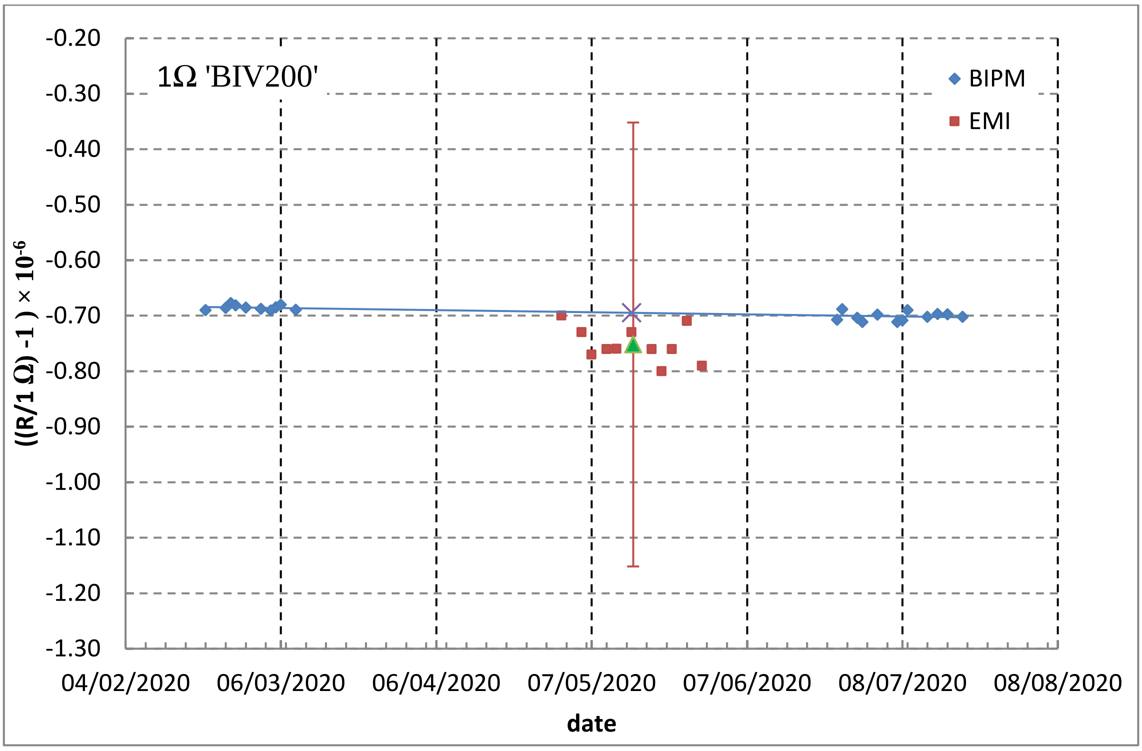

Figure 2: results for 1 Ω standard BIV200;
Uncertainty bar shows the combined expanded uncertainty of the comparison on the mean EMI results

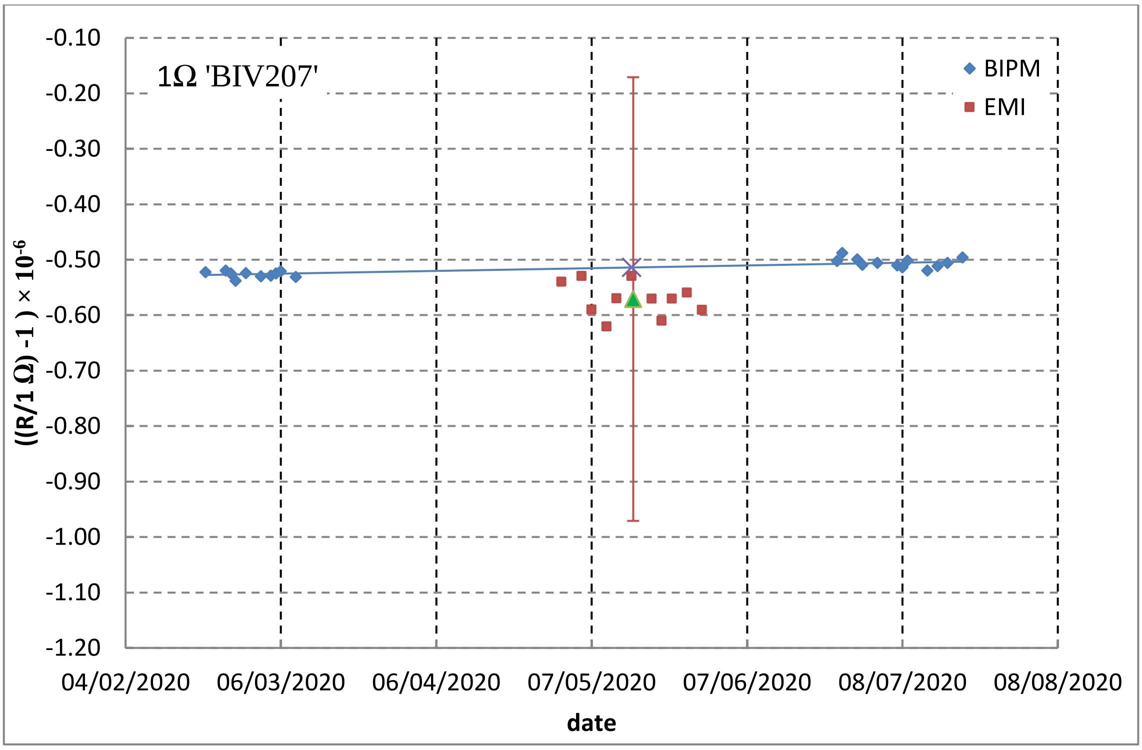

Figure 3: results for 1 Ω standard BIV207;
Uncertainty bar shows the combined expanded uncertainty of the comparison on the mean EMI results

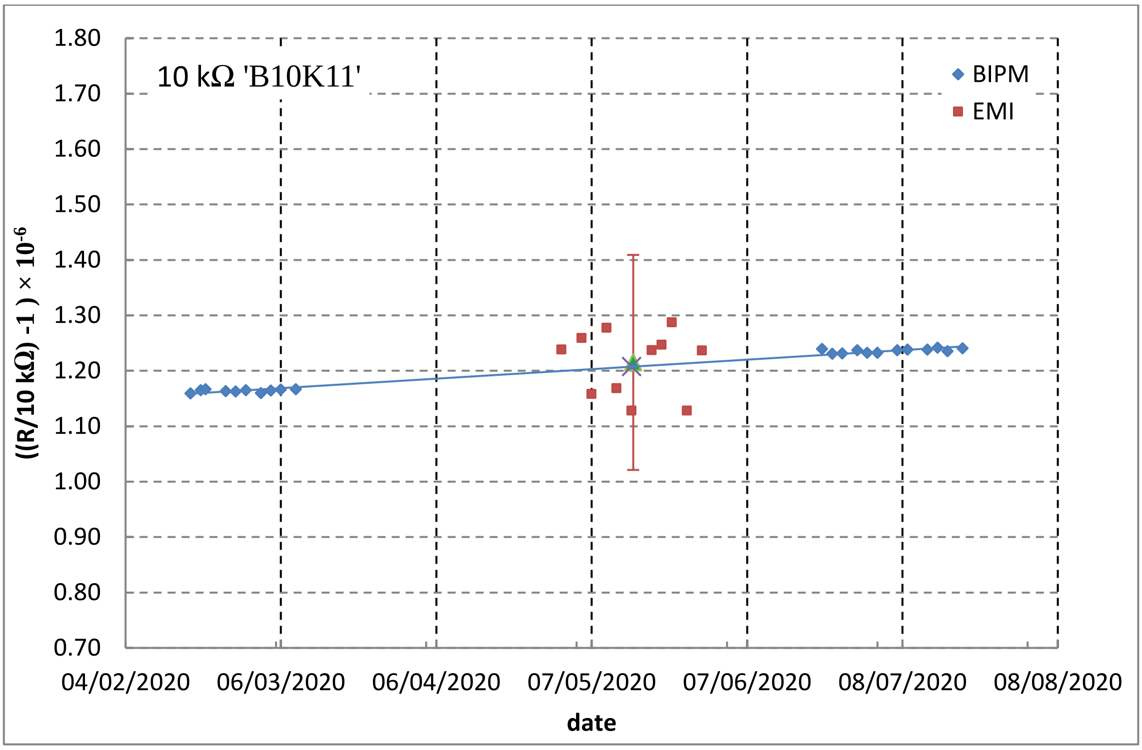

Figure 4: results for 10 kΩ standard B10K11
Uncertainty bar shows the combined expanded uncertainty of the comparison on the mean EMI results

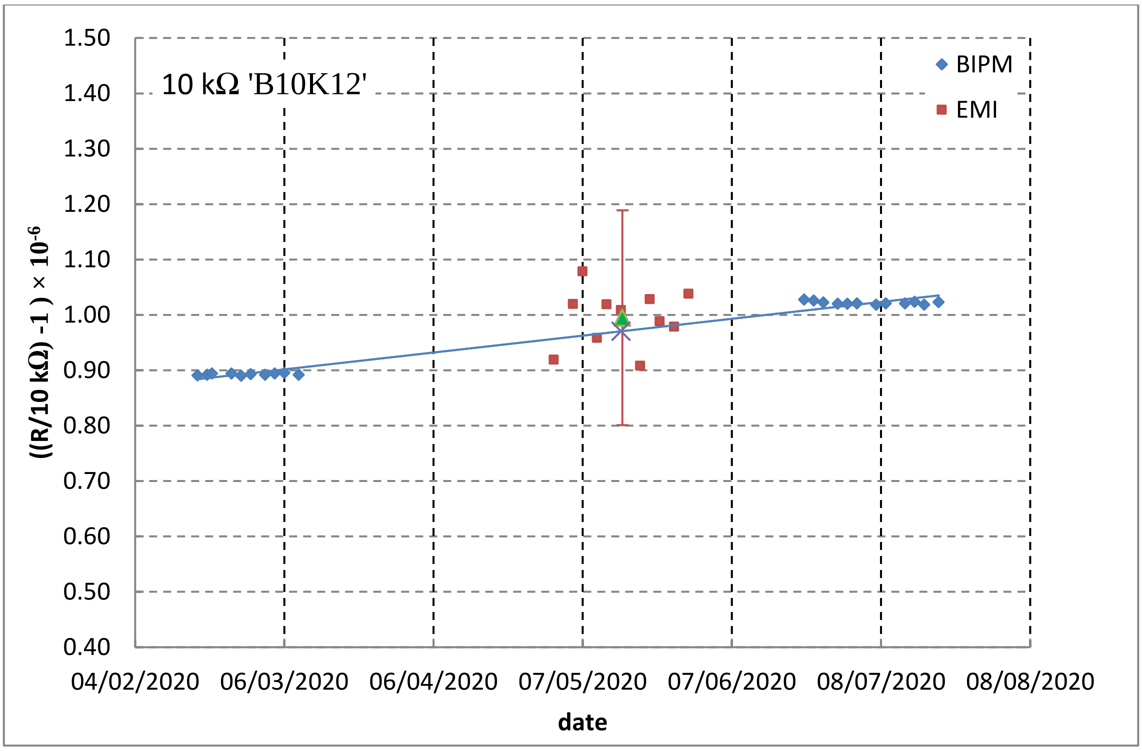

Figure 5: results for 10 kΩ standard B10K12
Uncertainty bar shows the combined expanded uncertainty of the comparison on the mean EMI results