Final Report

EUROMET.EM-K10 Key Comparison of Resistance Standards at 100 Ω

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1. Introduction

In the Mutual Recognition Arrangement (MRA) it is stated, that the metrological equivalence of national measurement standards will be determined by a set of key comparisons chosen and organised by the Consultative Committees of the CIPM working closely together with the Regional Metrology Organisations (RMO's). An international CIPM key comparison CCEM-K10 of "Resistance at 100 Ω" has been carried out with the Physikalisch-Technische Bundesanstalt (PTB) as the pilot laboratory.
In order to link the laboratories organised in EUROMET this EUROMET key comparison EUROMET.EM-K10 (also EUROMET project 636) has followed. All laboratories representing EUROMET in the CIPM comparison participated to establish a firm link between the CIPM and the RMO key comparisons.

Following the Guidelines for EUROMET key comparisons two institutes from the list of participants were nominated to help the pilot laboratory with the organisation. These are MIKES (A. Satrapinski) and METAS (B. Jeckelmann).

The travelling standards for this comparison were kindly supplied by the National Physical Laboratory (NPL), United Kingdom, by TEGAM, Geneva, Ohio, USA, and by MIKES, Finland.
The resistors used in set 1 (MIKES) had proven good stability in EUROMET project 487 [2]. A quick intercomparison showed that a relative uncertainty of less than 10^{-8} could be achieved. The resistors in set 3 had been tested in EUROMET project 435. It has been shown that these 100 Ω standard resistors also allow a comparison at a very low level of uncertainty ($< 10^{-8}$, 2σ)[1]. These are the same resistors, that have been used in the key comparison CCEM K-10.
The resistors used in set 2 had been checked in a bilateral test between NPL and PTB. Initially they had been measured at NPL at a temperature of 20,00°C, then been transported to PTB and re-measured at 23,00°C. The difference in the results including the correction for the temperature difference was not greater than $2 \cdot 10^{-8}$.

2. Participant list and time schedule

The pilot laboratory, 26 NMIs, and the BIPM agreed to participate in the comparison. The tables below list all participating laboratories in chronological order and the period of their measurements. The last column indicates the main events occurred during the comparison. In the column "Source of Traceability" QHR means that the laboratory has its own realisation of the unit Ω by means of the quantum Hall effect.. Otherwise the acronym of the metrological institute is given from which traceability is obtained.

Set 1

Acronym	*National Metrology Institute*	*Country*	*Period of Measurements*	*Mean Date of Measurement*	*Source of Traceability*	*Comment*
MIKES	Centre for Metrology and Accreditation		8. Apr. 2003			initial characterisation of the standards
		Finland	to	12. Apr. 2003	QHR	
			18. Apr. 2003			
SP	Swedish National Testing and Research Institute		12. May 2003			
		Sweden	to	13. May 2003	QHR	
			16. May 2003			
JV	Norwegian Metrology and Accreditation Service		27. May 2003			
		Norway	to	30. May 2003	QHR	
			3. Jun. 2003			
DFM	Danish Fundamental Metrology		27. Jun. 2003			
		Denmark	to	28. Jun. 2003	BIPM	
			30. Jun. 2003			
MIKES	Centre for Metrology and Accreditation		21. Jul. 2003			
		Finland	to	23. Jul. 2003	QHR	
			25. Jul. 2003			
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		4. Sep. 2003			
		Germany	to	10. Sep. 2003	QHR	
			16. Sep. 2003			
VNIIM	D.I. Mendeleyev Institute for Metrology		8. Oct. 2003			
		Russia	to	8. Oct. 2003	QHR	
			9. Oct. 2003			
MIKES	Centre for Metrology and Accreditation		7. Nov. 2004			final characterisation of the standards
		Finland	to	9. Nov. 2004	QHR	
			11. Nov. 2004			

Set 2

Acronym	*National Metrology Institute*	*Country*	*Period of Measurements*	*Mean Date of Measurement*	*Source of Traceability*	*Comment*
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		30. Jan. 2003			initial characterisation of the standards
		Germany	to	10. Apr. 2003	QHR	
			15. Jul. 2003			
OMH	National Office of Measures		8. Aug. 2003			
		Hungary	to	14. Aug. 2003	BIPM	
			19. Aug. 2003			
SASM	State Agency for Metrology and Technical Surveillance		25. Sep. 2003			
		Bulgaria	to	27. Sept. 2003	BEV	
			30. Sep.2003			
GUM	Glowny Urzad Miar		13. Oct. 2003			
		Poland	to	21. Oct. 2003	BIPM	
			30. Oct. 2003			
VMT	State Metrology Service/Institute for Semiconductor Physics		7. Nov. 2003			
		Lithuania	to	20. Nov. 2003	CMI	
			4. Dec. 2003			
LNMC	Latvian National Metrology Centre		8. Jan. 2004			
		Latvia	to	12. Jan. 2004	SP	
			12. Jan. 2004			
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		4. Feb. 2002			
		Germany	to	16. Feb. 2004	QHR	
			26. Feb. 2002			
EIM	Hellenic Institute of Metrology		8. Mar. 2004			
		Greece	to	28. Mar. 2004	QHR	
			3. Apr. 2004			
INRIM*	Istituto Nazionale di Ricerca Metrologica		20. Apr. 2004			
		Italy	to	21. Apr. 2004	QHR	
			22. Apr. 2004			
CEM	Centro Espanol de Metrologia		17. May 2004			
		Spain	to	24. May 2004	QHR	
			4. Jun. 2004			
INETI	Instituto Nacional de Engenharia, Tecnologia e Inovacao		15. Jun. 2004			
		Portugal	to	8. Jul. 2004	BIPM	
			28. Jul. 2004			
METAS	Federal Office of Metrology		4. Aug. 2004			
		Switzerland	to	20. Aug. 2004	QHR	
			23. Aug. 2004			
SIQ	Slovenian Institute for Quality		9. Sep. 2004			
		Slovenia	to	17. Sep. 2004	PTB	
			26. Sep. 2004			
DMDM	Directorate of Measures and Precious Metals		7. Oct. 2004			
		Serbia	to	17. Oct. 2004	BIPM	
			26. Oct. 2004			
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		16. Nov. 2004			final characterisation of the standards
		Germany	to	17. Dec. 2004	QHR	
			28. Jan. 2005			

* IEN, Istituto Elettrotecnico Nazionale Galileo Ferraris, before 1. January 2006

Set 3

Acronym	*National Metrology Institute*	*Country*	*Period of Measurements*	*Mean Date of Measurement*	*Source of Traceability*	*Comment*
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		9. Jul. 2003			initial characterisation of the standards
		Germany	to	2. Aug. 2003	QHR	
			18. Aug. 2003			
NPL	National Physical Laboratory		22. Sep. 2003			
		United Kingdom	to	25. Sep. 2003	QHR	
			30. Sep. 2003			
NML	National Metrology Laboratory		13. Oct. 2003			
		Ireland	to	22. Oct.2003	BIPM	
			28. Oct.2003			
LNE	Laboratoire National de métrologie et d'Essais		5. Nov. 2003			
		France	to	18. Nov. 2003	QHR	
			27. Nov. 2003			
BIPM	Bureau International de Poids et Mesures		1. Dec. 2003			
		International	to	7. Dec. 2003	QHR	
			15. Dec. 2003			
SMD	Belgian Calibration Service		29. Dec. 2003			
		Belgium	to	17. Jan. 2004	BIPM	
			6. Feb. 2004			
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		17. Feb. 2004			
		Germany	to	10. Mar. 2004	QHR	
			15. Apr. 2004			
CMI	Czech Metrology Institute		17. May 2004			
		Czech Republic	to	20. May 2004	QHR	
			24. May. 2004			
UME	Ulusal Metrologi Enstitüsü		28. Jun. 2004			
		Turkey	to	8. Jul. 2004	QHR	
			3. Jul. 2004			
NMISA	National Metrology Institute of South Africa		17. Aug 2004			
		South Africa	to	23. Aug. 2004	BIPM	
			26. Aug. 2004			
NMI	Nederlands Meetinstituut		26. Oct. 2004			
		The Netherlands	to	30. Oct. 2004	QHR	
			3. Nov. 2004			
BEV	Bundesamt für Eich- und Vermessungswesen		25. Nov. 2004			
		Austria	to	1. Dec. 2004	BIPM	
			8. Dec. 2004			
PTB	Physikalisch-Technische Bundesanstalt (Pilot)		21. Dec. 2004			final characterisation of the standards
		Germany	to	26. Jan. 2005	QHR	
			22. Feb. 2005			

3. Transfer standards and required measurements

3.1 The transfer standards

In order to restrict this comparison to a reasonable time scale three sets of resistors have been prepared to have three loops in parallel. The resistors are commercially available types with common four terminal connectors.

Set1, TinsleyTrN (MIKES):

- Standard Resistor 100 Ω Tinsley 5685A, S/N 267 908, Tinsley Tr1 in a pressure and temperature stabilised enclosure,
- Standard Resistor 100 Ω Tinsley 5685A, S/N 279 373, TinsleyTr2 in a pressure and temperature stabilised enclosure; this resistor includes a recorder for ambient conditions.

Set2, TinsleySet2:

- Standard Resistor 100 Ω Tinsley 5685A, S/N 267 918,
- Standard Resistor 100 Ω Tinsley 5685A, S/N 265 025,
- Standard Resistor 100 Ω Tinsley 5685A, S/N 263 417.

Set3, KC-Set:

- Standard Resistor 100 Ω TEGAM SR102, S/N A 2030397
- Standard Resistor 100 Ω Tinsley 5685A, S/N 267 919,
- Standard Resistor 100 Ω Tinsley 5685A, S/N 262 767,
- Standard Resistor 100 Ω Tinsley 5685A, S/N 268 168.

3.2 Required measurements

The measurand was the value of the resistance at DC, based on the conventional value of the von Klitzing constant $R_{\text{K-90}}$=25 812.807 Ω. In practice, DC meant that the waiting time between the end of a current reversal and the start of data acquisition should not be shorter than 5 s. Choice was left to the participants to either carry out a guarded measurement where the resistor case is used as a guard, or leave the resistor floating with respect to the case, or connect one point of the resistor to its case. The solution which was adopted should be mentioned in reporting the results. Together with the measurement results, a short description of the individual measuring methods used must be included for the final report.

After installation of the resistors in their respective thermostats a minimum settling time of one day was required. The measurements should have been carried out with these preferred conditions:

- direct comparison with the QHR using a CCC bridge,
- aimed uncertainty less than $2{\cdot}10^{-8}$ (95% confidence level),
- current through the resistor 5 mA,

- ambient temperature (23,00 ± 0,1) °C (for set 2 also (20,00 ± 0,1) °C); the deviation of the temperature from nominal should not exceed the given limit.

Participants not using the QHR as their primary standard of resistance must measure the resistors with their respective best measurement capability, preferably at 23°C, for Set2 a temperature of 20°C was also allowed. For these measurements the source of traceability had to be included in the measurement report.

The resistance temperature and ambient pressure should have been recorded and reported as well as the height of oil above the top plate of the Tinsley resistors in the oil bath. If known, the density of the oil in the oil bath should be reported. These resistors have a huge thermal time constant (several hours)! The measurements should be made at different dates during the period in the laboratory. The temperature and pressure coefficients of the standards have been determined to allow for corrections. They were intentionally not provided with the protocol. In case this information was needed for evaluation of the individual measurements it had been provided on request.

4. Measurements of the pilot laboratory, temperature and pressure coefficients

In loop 1 two resistors from MIKES have been used. The drift rate of these resistors is determined from the measurements, carried out by MIKES. All individual measurements are used. For the resistors #267 908 and #279 373 the drift behavior can be described by a linear equation,

- $R(\#267\ 908) = 100\cdot(1+(5632{,}214 - 0{,}0702\cdot t)\cdot 10^{-9})\ \Omega$
- $R(\#279\ 373) = 100\cdot(1-(944{,}148 - 0{,}0008\cdot t)\cdot 10^{-9})\ \Omega$

where t is the number of days since January 1st 2003. The standard deviations of the residuals for the fits are $6{,}65\cdot 10^{-9}$ and $9{,}54\cdot 10^{-9}$ respectively.

The resistors used in loop 2 and 3, and their temperature and pressure coefficients are listed in the table below. Some of the Tinsley resistors showed no significant pressure coefficient. With these values and the provided temperature and pressure data, all measured results of the participants have been corrected to nominal conditions which are 23,000°C and 1013.25 hPa.

Resistor serial number	α_{23} 10^{-9}K^{-1}	$u(\alpha_{23})$ 10^{-9}K^{-1}	β 10^{-9}K^{-2}	$u(\beta)$ 10^{-9}K^{-2}	p_k 10^{-9}hPa^{-1}	$u(p_\text{k})$ 10^{-9}hPa^{-1}
Tinsley 267 919	-483,4	2,1	-79,1	2	0,01	0,03
Tinsley 262 767	-35,7	2,1	-79,3	2	0,00	0,02
Tinsley 268 168	-635,6	2,1	-76,3	2	-0,04	0,02
Tegam A 2030397	79,5	2,1	-22,7	2	-0,29	0,13
Tinsley 267 918	-259,1	2,1	-74,0	2	-0,18	0,09
Tinsley 265 025	-360,1	2,1	-69,8	2	-0,05	0,05
Tinsley 263 417	-186,8	2,1	-72,3	2	-0,07	0,03

These resistors have repeatedly been measured by the pilot laboratory. Due to transportation effects, the overall drift of the standards is different from the drift during the period in the laboratory. Therefore all measurements of a laboratory are combined to a mean result given for a mean date. This result is taken from a linear regression analysis and the residual of the fit is included in the laboratory's uncertainty.

The drift rate of the resistors is determined by the measurements of the pilot laboratory. The calculation is based on all individual measurements. For all resistors the drift behavior has been described by a linear equation,

- $R(\#262\ 767) = 100\cdot(1-(3495{,}795 - 0{,}13472\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 7{,}44\cdot 10^{-9}$
- $R(\#268\ 168) = 100\cdot(1-(1248{,}353 - 0{,}08255\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 2{,}37\cdot 10^{-9}$
- $R(\#267\ 919) = 100\cdot(1-(5368{,}078 - 0{,}03411\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 9{,}49\cdot 10^{-9}$
- $R(\#2030\ 397) = 100\cdot(1+(167{,}521 + 0{,}36256\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 10{,}19\cdot 10^{-9}$
- $R(\#263\ 417) = 100\cdot(1-(4305{,}649 - 0{,}05053\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 13{,}56\cdot 10^{-9}$
- $R(\#267\ 918) = 100\cdot(1-(4438{,}199 - 0{,}01624\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 10{,}47\cdot 10^{-9}$
- $R(\#265\ 025) = 100\cdot(1-(3448{,}796 + 0{,}01433\cdot t)\cdot 10^{-9})\ \Omega$, $\sigma_\mathrm{r} = 10{,}57\cdot 10^{-9}$

where t is the number of days since January 1st 2003. The standard deviations of the residuals σ_r for the fits are also listed above. Since the residual for resistor #268 168 is so small that it would inevitably bias the results of loop3, a more statistical approach is chosen in that particular case. The residual of the fit is replaced by the standard deviation of the mean of the independent results.

5. Measurement method of the participants

The methods of measurement carried out by the participants are described briefly.

PTB – pilot laboratory, SP, JV, NPL, LNE, BIPM, CMI, UME, INRIM, EIM:

The measurements were made using the laboratory's cryogenic current comparator bridge. All resistors were measured against the QHR i=2 plateau.

MIKES:

The resistors were measured against the MIKES QHR standard using an AC cryogenic current comparator bridge. The measurements were performed in the frequency range from 0.1 Hz to 0.3 Hz with current values of 2.6 mA and 5 mA (rms value). No significant frequency dependence has been found so the values are considered to be equal to the DC values

DFM:

The measurements were made using a direct current comparator bridge. All resistors were measured against the maintained 10-kΩ standards (traceable to the BIPM) via a Hamon transfer device.

VNIIM:

The measurements were made using the VNIIM double bridge–comparator and Hamon-type transfer resistors. The resistors were measured against a maintained group of resistors, linked to the QHR i=2 plateau.

SMD:

The measurements were made using a direct current comparator bridge. All resistors were measured against the maintained 1-Ω standards (traceable to the BIPM) via a Hamon transfer resistor (two 1:10 steps).

NMi, METAS:

The measurements were made using a cryogenic current comparator bridge. All resistors were measured against the QHR plateau i=2 and i=4.

BEV:

The measurements were made using a direct current comparator bridge. All resistors were measured against 10-Ω, 100-Ω and 1-kΩ standard resistors, their values derived from the maintained 1-Ω standards (traceable to the BIPM).

SASM:

The measurements were made using a substitution method with a digital multimeter. All resistors were measured against the maintained 100-Ω standards (traceable to the BEV).

LNMC:

The measurements were made using a direct current comparator bridge. All resistors were measured against the maintained 100-Ω standards (traceable to the SP).

INETI, ZMDM, VMT, GUM, OMH, NML, NMISA:

The measurements were made using a direct current comparator bridge. All resistors were measured against the maintained 1-Ω standards (traceable to the BIPM) via a 10-Ω standard resistor (two 10:1 steps).

CEM:

The resistance reference in CEM is a 10 kΩ standards group, calibrated relative to QHE via a Josephson potentiometer. The 10 kΩ standards are in turn compared by transposition with a Hamon transfer device configured in its series mode, using an automatic bridge. The Hamon resistor in its parallel configuration is finally compared by substitution with the travelling standards using a manual current comparator bridge. The Hamon device and the travelling standards are immersed in an oil bath. Due to problems in the temperature control of the bath, the measurements were made at 23.2 ºC.

SIQ

The measurements were made using a direct current comparator bridge. All resistors were measured against the maintained 10-kΩ standards (traceable to the PTB) via a set of standard resistors (10 Ω to 1 kΩ in 10:1 steps).

6 Results

6.1 Participants result and differences from pilot

Due to similarities in the measurement objects and procedures, for the calculation of the reference value a similar procedure as for CCEM-K2 is chosen[3], which has also been accepted for the evaluation of the data for CCEM-K10[4]. In the long run all resistors show a linear drift behavior. The drift behavior of the travelling standards has been analyzed, taking all individual results of PTB respectively MIKES into account. For all resistors a single linear regression is chosen. Also the drift during the respective periods in the laboratories is linear. Due to unforeseeable transport effects the drift during the time in the laboratory is different from the overall drift. Hence, for the evaluation all short term drifts have been accounted for by a linear regression on the participant measurement results. A single value, for each resistor and for each participant, is calculated at the mean date of the participant measurements, based on the corresponding regression, and is used for further analysis. For the pilot laboratories (MIKES in loop 1 and PTB in loops 2 and 3) the single values for their periods of measurement are calculated from the overall drift behavior of the standards. The uncertainty of this value is calculated from the Type A and Type B uncertainties from the uncertainty budget, and the residual standard deviation of the regression (also considered as Type A). Values and uncertainties for each laboratory are listed in Appendix A, Tables 1-9. For some participants having supplied different Type A uncertainties for each resistance measurement an appropriate mean uncertainty has been calculated which is listed in the tables mentioned above.

The following analysis is carried out for the resistors in each loop.

In a next step, in each loop, for each participant and for each resistor, the difference between the single value and the corresponding value deduced from the fit to the pilot measurement results (V_r) is calculated.

$$D_i = x_i - V_r$$

This eliminates the drift from the results.

Then, for each participant (a total of 28 participations, subdivided in three loops), the weighted mean, $D_{i,\text{Loop\_k}}$, of the differences is calculated using weights proportional to $1/\sigma_r^2(j)$ where $\sigma_r(j)$ are the standard deviations of the residuals in the pilot fits for the resistors. The expanded relative uncertainty (k=2) for the $D_{i,\text{Loop\_k}}$ is defined as

$$U_{i,\text{Loop\_}k} = 2 \cdot \sqrt{\text{VAR}\left[D_{i,\text{Loop\_}k}\right]}.$$

where the variances of the $D_{i,\text{Loop\_k}}$ are defined as follows (n is the number of times the pilot has measured, m is the number of resistors):

- non-pilot laboratory

$$Var\left[D_{i,\text{Loop\_}k}\right] = \sigma_{B,i}^2 + \sigma_{A,i}^2 \cdot \frac{\sum_{j=1}^{m} \frac{1}{\sigma_r^4(j)}}{\left(\sum_{j=1}^{m} \frac{1}{\sigma_r^2(j)}\right)^2} + \frac{1 + \frac{1}{n} + \frac{(t_i - \bar{t}_{Pilot})^2}{\sum_{l=1}^{n} (t_{Pilot,l} - \bar{t}_{Pilot})^2}}{\sum_{j=1}^{m} \frac{1}{\sigma_r^2(j)}}$$

- pilot laboratory

$$Var\left[D_{Pilot,Loop\_k}\right]=\sigma_{B,Pilot}^{2}+\frac{\sigma_{A,Pilot}^{2}}{n}\cdot\frac{\sum_{j=1}^{m}\frac{1}{\sigma_{r}^{4}(j)}}{\left(\sum_{j=1}^{m}\frac{1}{\sigma_{r}^{2}(j)}\right)^{2}}$$

The standard uncertainties $\sigma_{B,i}$ and $\sigma_{A,i}$ are taken from the uncertainty budget of each participant, additionally, $\sigma_{A,i}$ also includes the weighted mean of the scatter of the resistance values during the measurement in the participants laboratory (root sum square).

The $D_{i,Loop\_k}$ for the pilot laboratory is the arithmetic mean of its individual measurements. Each $U_{i,Loop\_k}$ includes the variance of each resistor and thus a first estimate for the transport uncertainty is included. The measurement results and the differences from the fit are listed in tables 10 to 12.

The statistical significance of the results is checked by the χ^2-test, using the following equation:

$$\chi^{2}=\sum_{i=1}^{N}\frac{(D_{i}-D_{W})^{2}}{Var\left[D_{i}\right]}$$

where the value D_W is the weighted mean of the loop results.
For the loops 1 and 3 the test gives a reasonable value (< N, where N is the number of participants), for loops 2 the test gives evidence that the analysis includes insufficient information on the transport behavior of the standards. For the standards used in loop 2 the transportability of these resistors had been checked on their way from NPL to PTB with good agreement, during the course of the comparison they apparently show some jumps. This behavior may be partly attributed to thermal and mechanical shocks. Unfortunately due to technical problems the data recorded during the transport are incomplete. But there is evidence that the temperature during transport varied between 5°C and 35°C.The support group concluded to add an additional transport uncertainty for the standards used in loop 2. This additional uncertainty component σ_{Trans} is estimated such that the χ^2-test is passed, the value used is $\sigma_{Trans} = 35\cdot10^{-9}$. For this calculation the result from GUM has not been considered, since it deviates more than 4 standard deviations from the loop reference value.
In loop 3 the residual of the fit function for resistor #268 168 is significantly smaller than the standard deviation of the results obtained by laboratories that use the QHE. For the evaluation it is concluded that taking the standard deviation of the results is a better estimate for the fit residual. Furthermore an additional transport uncertainty $\sigma_{Trans\_3} = 7\cdot10^{-9}$ is introduced. Although this appeared not to be necessary for the individual loop, it improved the overall uncertainty of the CRV after combination of the loops (see next paragraph). With this additional transport uncertainty the combination of the loops also fulfills the χ^2-test.

6.2 Combining the loops, comparison reference value and its uncertainty

The link between the loops is given by the PTB as pilot laboratory since this is the only common laboratory to all loops. For the combination of the loops the difference $D_{i,COMB}$ between each participants $D_{i,Loop\,k}$ and the respective PTB $D_{PTB,Loop\,k}$ is calculated. By this definition the $D_{PTB,COMB}$ is 0. The uncertainties remain unchanged, so $U_{\mathrm{i,COMB}} = U_{\mathrm{i,Loop\_k}}$. This was achieved by adding a transport uncertainty also for loop 3 (see above) so that the combined loops also fulfill the χ^2-test. For $U_{\mathrm{PTB,COMB}}$ the weighted mean of the $U_{\mathrm{PTB,Loop\_k}}$ is chosen.

The comparison reference value, X_{CRV}, and its associated uncertainty, U_{CRV}, is determined from the weighted mean of the $D_{\mathrm{i,COMB}}$ with the $U_{\mathrm{i,COMB}}$ used as weight. In this calculation only one value for the pilot laboratory is considered. To exclude a possible correlation, only those laboratories having their own representation of the Ohm, based on the QHE, are taken into consideration (see also 7). The values are calculated as follows:

$$X_{\mathrm{CRV}} = U_{\mathrm{CRV}}^2 \cdot \sum_{i=1}^{p} \frac{D_{i,\mathrm{COMB}}}{U_{i,\mathrm{COMB}}^2}, \quad U_{\mathrm{CRV}} = \frac{1}{\sqrt{\sum_{i=1}^{p} \frac{1}{U_{i,\mathrm{COMB}}^2}}}$$

$$X_{\mathrm{CRV}} = 4{,}0\cdot 10^{-9},\ U_{\mathrm{CRV}} = 6{,}0\cdot 10^{-9}$$

6.3 Degrees of equivalence with respect to the CRV

The equivalence with the key comparison reference value and its uncertainty is calculated as follows

$$D_{i,\mathrm{CRV}} = D_{i,\mathrm{COMB}} - X_{\mathrm{CRV}}$$

$$U_{i,\mathrm{CRV}} = \sqrt{U_{i,\mathrm{COMB}}^2 - U_{\mathrm{CRV}}^2}\ .$$

and, where a laboratory does not contribute to the X_{CRV}

$$U_{i,\mathrm{CRV}} = \sqrt{U_{i,\mathrm{COMB}}^2 + U_{\mathrm{CRV}}^2}\ .$$

These values are listed in table 16 and shown in graphs 10 and 11.

A significant part of the uncertainty is related to the poor transport behavior of the resistors, compared to the measurement capabilities of some laboratories, particular those deriving their resistance value from the quantum Hall effect by means of a CCC. Therefore it is concluded that the determination of a bilateral degree of equivalence is not very meaningful. So as a result only the differences of each laboratory to the comparison reference value is listed. Laboratories that claim an uncertainty, smaller than the transport uncertainty are marked in this list. The difference to the reference value is a result of the transport behavior of the resistors and cannot be attributed to the measurement capabilities of the laboratory.

6.4 Link to CCEM–K10

For linking the EUROMET.EM-K10 to the respective CCEM.EM-K10 a procedure similar to that proposed for K8 will be followed [5]. Four laboratories have participated in both, the CCEM and the EURAMET comparison K10.

Laboratory	$D_{\text{i-KCRV}}$ / 10^{-9}	U_{i} / 10^{-9}	$D_{\text{i-CRV}}$ / 10^{-9}	U_{i} / 10^{-9}
MIKES	12,15	17,1	5,31	17,1
METAS	-4,93	11,1	-45,22	71,8
BIPM	-1,48	18,7	7,85	17,5
PTB	0,14	7	-4,04	9,7

With the definitions of the differences $D_{\text{i-KCRV}} = D_{\text{i,COMB}} - X_{\text{KCRV}}$, $D_{\text{i-CRV}} = D_{\text{i}}'_{\text{,COMB}} - X_{\text{CRV}}$, and $(X_{\text{CRV}} - X_{\text{KCRV}})_{\text{i}} = D_{\text{i-KCRV}} - D_{\text{i-CRV}}$, from these four values of $(X_{\text{CRV}} - X_{\text{KCRV}})_{\text{i}}$, a weighted mean of the references „link value“ and its uncertainty, U_{LINK}, can be calculated. Then,

$$X_{\text{CRV}} - X_{\text{KCRV}} = 3{,}30 \cdot 10^{-9} \text{ and } U_{\text{LINK}} = 9{,}80 \cdot 10^{-9}$$

Results and graphs are shown in the appendix B.

7 Effect of correlation among the laboratory differences

Since not all participating laboratories have their own independent realization of the unit of resistance, some results are correlated. All results with no independent realization of the unit of resistance have been excluded from the determination of the comparison reference value.

8 Conclusion

The results of all laboratories except one show good equivalence with the comparison reference value. It can also be concluded that the results of laboratories, which directly derive their unit of resistance from the quantum Hall effect, agree within $\pm 3 \cdot 10^{-8}$. This is less than one order of magnitude worse than a direct comparison of QHR-systems and limited by the transportability of the transfer standards.

One key point the pilot laboratory wants to raise is the non-uniformity of the uncertainty budgets. Although clear guidelines were given and a sample table was provided not all participants submitted uncertainty budgets in the desired form. Even after a second request, there is still no uniformity and it took a great effort to harmonize the budgets as much as possible. This should have consequences in future comparisons.

9 References

[1] B. Schumacher et al. "Transport Behavior of Commercially Available 100-Ω Resistors", IEEE-IM **50**, 242-246.

[2] A. Satrapinski et al. "Comparison of Four QHR Systems Within One Month Using a Temperature and Pressure Stabilized 100-Ω Resistor", IEEE-IM **50**, 238-241.

[3] D.G. Jarrett and R.F. Dziuba, "CCEM-K2 Key Comparison of 10-MΩ and 1-GΩ Resistance Standards", IEEE-IM **52**, 474-477.

[4] B. Schumacher, "Final report on CCEM-K10: Key comparison of resistance standards at 100 Ω", 2007 Metrologia **44** 01004 doi: 10.1088/0026-1394/44/1A/01004

[5] G. Marullo Reedtz, R. Cerri, “Linking the Results of Key Comparisons CCEM-K8 and EUROMET.EM-K8”, Metrologia.

Appendix A: Measurement Results

Table 1

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 267 908			
institute	mean date	result ($\cdot 10^{-9}$)	u_i ($\cdot 10^{-9}$)	$u_{i\text{-Resid}}$ ($\cdot 10^{-9}$)	U_i ($\cdot 10^{-9}$)
MIKES	12.04.2003	5625,12	6,8	6,69	19,1
SP	13.05.2003	5599,62	12,5	6,56	28,2
JV	30.05.2003	5604,32	3,6	2,2	8,4
DFM	28.06.2003	5620,00	190		380
MIKES	23.07.2003	5617,96	8,6	6,69	21,8
PTB	10.09.2003	5603,53	2,2	6,82	14,3
VNIIM	08.10.2003	5660,00	40		80
MIKES	09.11.2004	5584,57	9,3	6,69	22,9

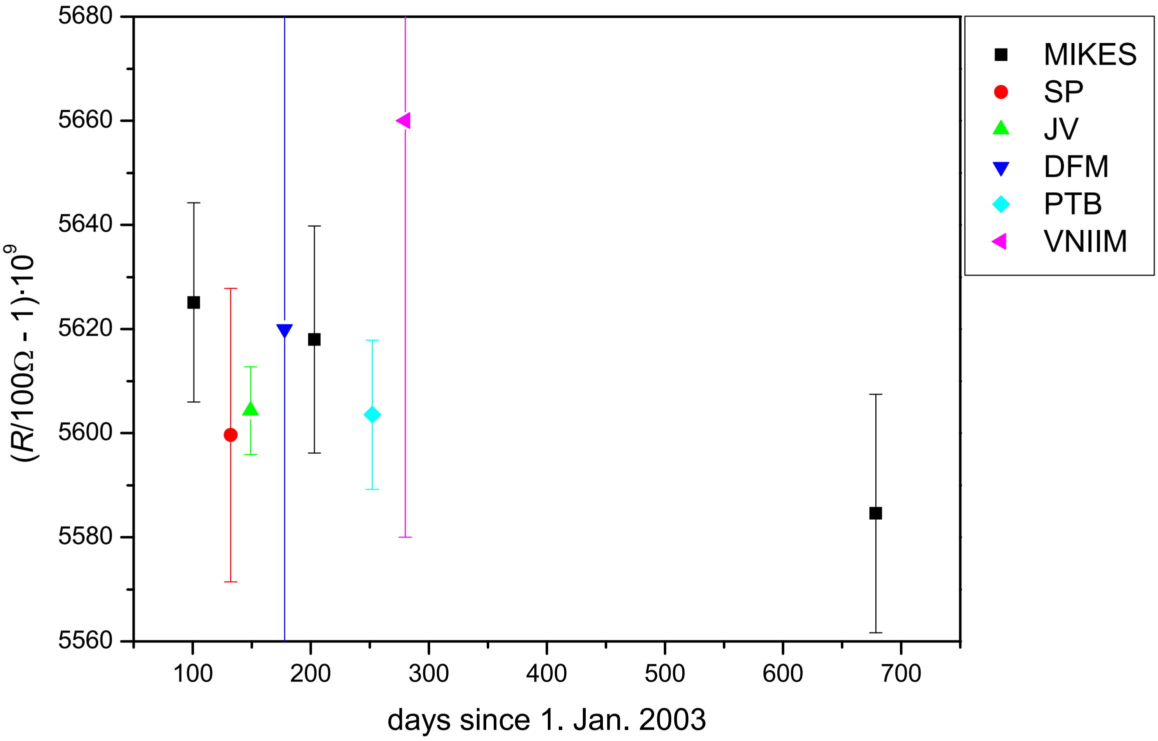


Figure 1

Results as given in Table 1:

Table 2

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_\mathrm{i\text{-}Resid}$ of the linear fit of the laboratory's result.

		Tinsley no. 279 373			
institute	mean date	result ($\cdot 10^{-9}$)	u_i ($\cdot 10^{-9}$)	$u_\mathrm{i\text{-}Resid}$ ($\cdot 10^{-9}$)	U_i ($\cdot 10^{-9}$)
MIKES	12.04.2003	-944,23	6,8	9,54	23,4
SP	13.05.2003	-960,68	12,5	6,56	28,2
JV	29.05.2003	-956,60	3,6	8,89	19,2
DFM	28.06.2003	-940,00	190		380
MIKES	23.07.2003	-944,31	8,6	9,54	25,7
PTB	10.09.2003	-950,36	2,2	6,2	13,2
VNIIM	08.10.2003	-990,00	40		80
MIKES	09.11.2004	-944,68	9,3	9,54	26,7

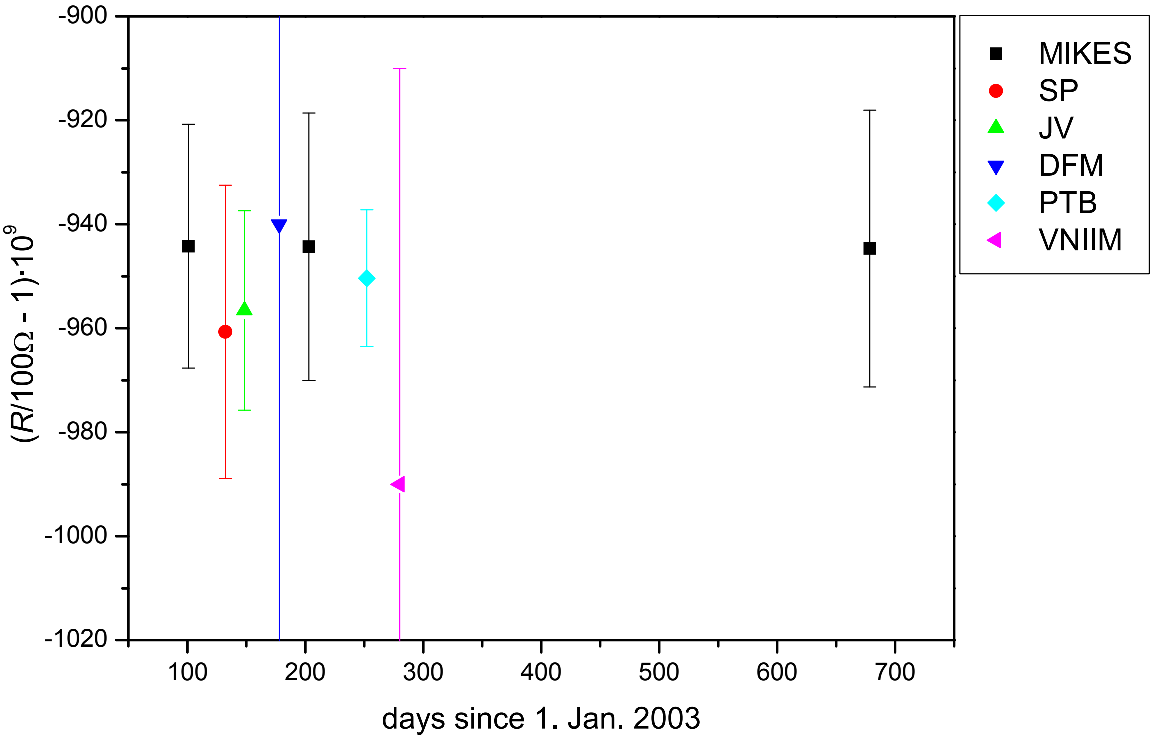


Figure 2

Results as given in Table 2:

Table 3

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 265 025			
institute	mean date	result ($\cdot 10^{-9}$)	u_i ($\cdot 10^{-9}$)	$u_{i\text{-Resid}}$ ($\cdot 10^{-9}$)	U_i ($\cdot 10^{-9}$)
PTB	11.04.2003	-3450,24	2,2	10,6	21,65
OMH	14.08.2003	-3337,41	900	39,8	1801,76
SASM	27.09.2003	-3963,33	600	43,8	1203,19
GUM	21.10.2003	-4308,24	184	52,7	382,80
VMT	20.11.2003	-3535,75	100	18,76	203,49
LNMC	12.01.2004	-2452,25	1000	0	2000,00
PTB	16.02.2004	-3454,69	2,2	10,6	21,65
EIM	28.03.2004	-3593,53	20	6,2	41,88
INRIM	21.04.2004	-3496,93	15	20,4	50,64
CEM	24.05.2004	-3505,15	21	24	63,78
INETI	09.07.2004	-3741,37	120	47,1	257,82
METAS	13.08.2004	-3567,59	3	0,78	6,20
SIQ	17.09.2004	-3396,39	230	0	460,00
DMDM	17.10.2004	-2603,66	920	25,43	1840,70
PTB	17.12.2004	-3459,07	2,2	10,6	21,65

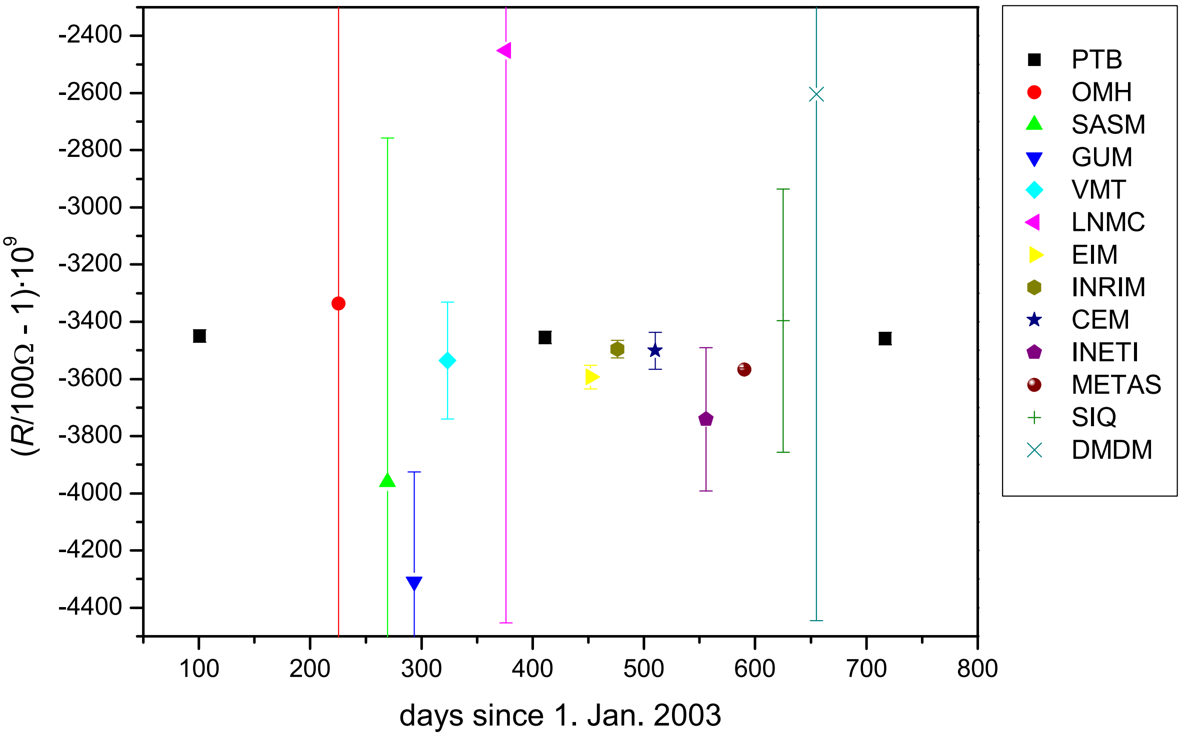


Figure 3

Results as given in Table 3:

Table 4

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 267 918			
institute	mean date	result ($\cdot 10^{-9}$)	u_i ($\cdot 10^{-9}$)	$u_{i\text{-Resid}}$ ($\cdot 10^{-9}$)	U_i ($\cdot 10^{-9}$)
PTB	11.04.2003	-4436,65	2,2	10,5	21,40
OMH	14.08.2003	-4016,10	900	36,8	1801,50
SASM	27.09.2003	-4787,56	600	22	1200,81
GUM	21.10.2003	-5167,05	184	48,4	380,52
VMT	20.11.2003	-4468,88	100	20,93	204,33
LNMC	12.01.2004	-4111,18	1000	0	2000,00
PTB	16.02.2004	-4431,52	2,2	10,5	21,40
EIM	28.03.2004	-4436,16	20	5,75	41,62
INRIM	21.04.2004	-4428,70	15	2,34	30,36
CEM	24.05.2004	-4409,29	21	24,11	63,95
INETI	08.07.2004	-4628,35	120	36,42	250,81
METAS	13.08.2004	-4387,15	3	0,69	6,16
SIQ	17.09.2004	-4296,10	230	0	460,00
DMDM	17.10.2004	-3455,37	920	35,22	1841,35
PTB	17.12.2004	-4426,56	2,2	10,5	21,40

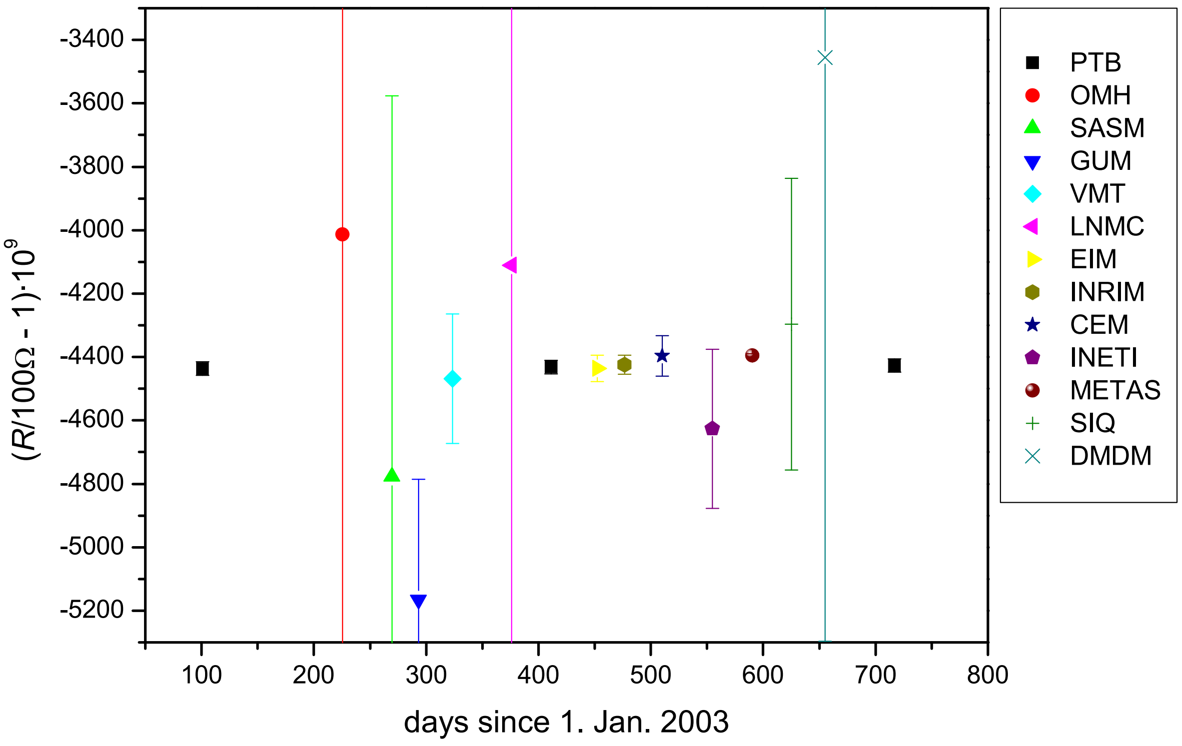


Figure 4

Results as given in Table 4:

Table 5

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 263 417			
institute	mean date	result ($\cdot 10^{-9}$)	u_i ($\cdot 10^{-9}$)	$u_{i\text{-Resid}}$ ($\cdot 10^{-9}$)	U_i ($\cdot 10^{-9}$)
PTB	11.04.2003	-4300,56	2,2	13,56	27,47
OMH	14.08.2003	-3921,28	900	37,2	1801,54
SASM	27.09.2003	-4671,62	600	55,5	1205,12
GUM	21.10.2003	-5045,82	184	47	379,82
VMT	21.11.2003	-4380,53	100	18,04	203,23
LNMC	08.01.2004	-2908,68	1000		2000,00
PTB	16.02.2004	-4284,86	2,2	13,56	27,47
EIM	23.03.2004	-4377,53	20	5,7	41,59
INRIM	21.04.2004	-4321,50	15	5,7	32,09
CEM	25.05.2004	-4312,70	21	25,05	65,38
INETI	07.07.2004	-4491,54	120	31,1	247,93
METAS	14.08.2004	-4331,50	3	0,7	6,16
SIQ	17.09.2004	-4199,92	230		460,00
DMDM	17.10.2004	-3380,98	920	33,2	1841,20
PTB	17.12.2004	-4269,47	2,2	13,56	27,47

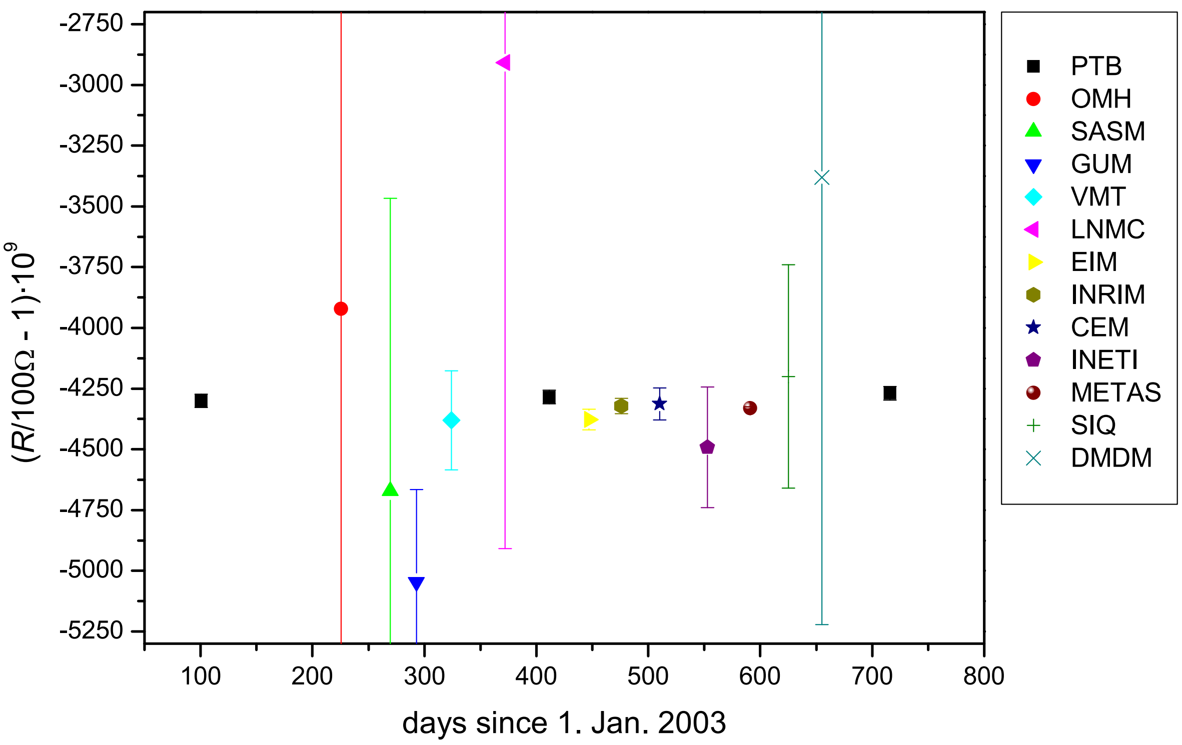


Figure 5

Results as given in Table 5:

Table 6

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tegam no. 2030397			
institute	mean date	result ($\cdot 10^{-9}$)	u_i ($\cdot 10^{-9}$)	$u_{i\text{-Resid}}$ ($\cdot 10^{-9}$)	U_i ($\cdot 10^{-9}$)
PTB	02.08.2003	244,75	2,2	10,19	20,84
NPL	25.09.2003	280,43	10	3,65	21,3
NML	22.10.2003	470,22	210	2,67	420
LNE	18.11.2003	302,72	1,1	1,13	3,16
BIPM	07.12.2003	306,84	2,6	2,98	7,91
SMD	17.01.2004	348,74	50		100
PTB	10.03.2004	325,20	2,2	10,19	20,84
CMI	20.05.2004	355,81	50	4,48	100
UME	08.07.2004	375,17	12,2	5,77	27,0
CSIR/NML	23.08.2004	546,49	300	12,47	600
NMi	30.10.2004	433,87	5,9	4,53	14,9
BEV	01.12.2004	352,01	165	5,84	330
PTB	26.01.2005	441,67	2,2	10,19	20,84

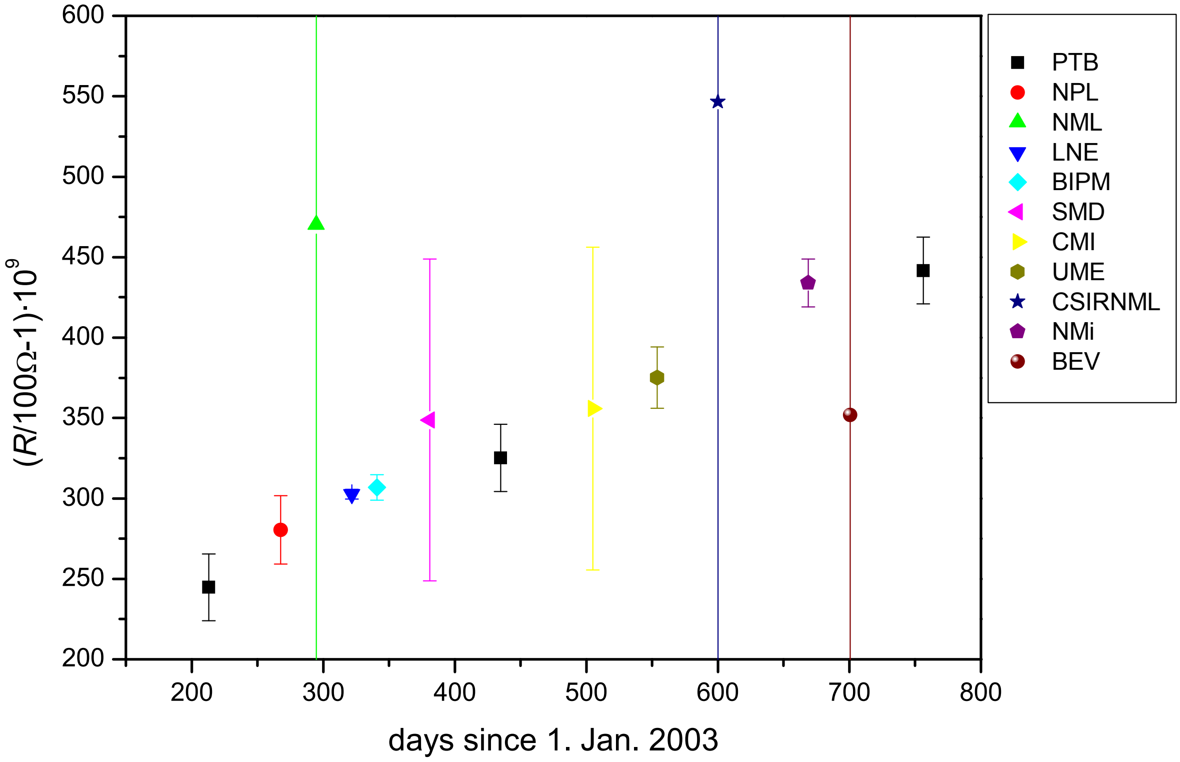


Figure 6

Results as given in Table 6:

Table 7

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 268 168			
institute	mean date	result $(\cdot 10^{-9})$	u_i $(\cdot 10^{-9})$	$u_{i\text{-Resid}}$ $(\cdot 10^{-9})$	U_i $(\cdot 10^{-9})$
PTB	02.08.2003	-1230,77	2,2	2,37	6,47
NPL	26.09.2003	-1195,60	10	4,73	22,1
NML	22.10.2003	-1014,75	210	2,22	420
LNE	15.11.2003	-1218,04	1,1	2,37	5,23
BIPM	08.12.2003	-1214,07	2,6	2,28	6,92
SMD	17.01.2004	-1180,45	50		100
PTB	10.03.2004	-1212,45	2,2	2,37	6,47
CMI	18.05.2004	-1181,62	50	2,48	100
UME	05.07.2004	-1191,15	7,3	2,35	15,3
CSIR/NML	22.08.2004	-1005,58	300	16,48	601
NMi	30.10.2004	-1190,46	5,9	3,86	14,1
BEV	01.12.2004	-1252,35	165	4,53	330,
PTB	25.01.2005	-1186,01	2,2	2,37	6,47

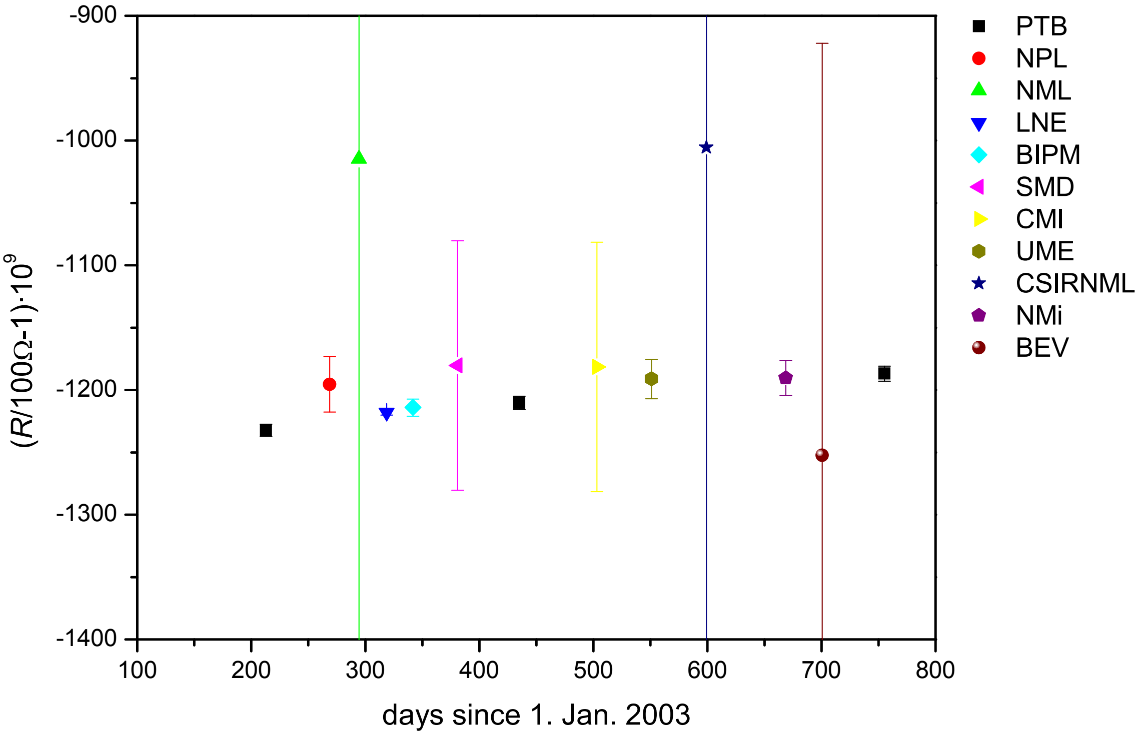


Figure 7

Results as given in Table 7:

Table 8

Summary of results, calculated for a mean date. The corresponding uncertainty U_i is calculated from the standard uncertainty u_i, given in the participants uncertainty budget, and the residual uncertainty $u_{i\text{-Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 267 919			
institute	mean date	result $(\cdot 10^{-9})$	u_i $(\cdot 10^{-9})$	$u_{i\text{-Resid}}$ $(\cdot 10^{-9})$	U_i $(\cdot 10^{-9})$
PTB	02.08.2003	-5360,81	2,2	9,48	19,46
NPL	25.09.2003	-5350,55	10	2,99	20,9
NML	22.10.2003	-5153,40	210	9,58	420
LNE	17.11.2003	-5346,67	1,1	2,39	5,26
BIPM	07.12.2003	-5337,91	2,6	1,41	5,91
SMD	17.01.2004	-5311,45	50		100
PTB	10.03.2004	-5353,24	2,2	9,48	19,46
CMI	18.05.2004	-5344	50	2,14	100
UME	04.07.2004	-5321,52	7,8	0,96	15,7
CSIR/NML	21.08.2004	-5180,42	300	18,3	601
NMi	31.10.2004	-5337,76	5,9	7,28	18,7
BEV	01.12.2004	-5388,58	165	5,73	330
PTB	25.01.2005	-5342,32	2,2	9,48	19,46

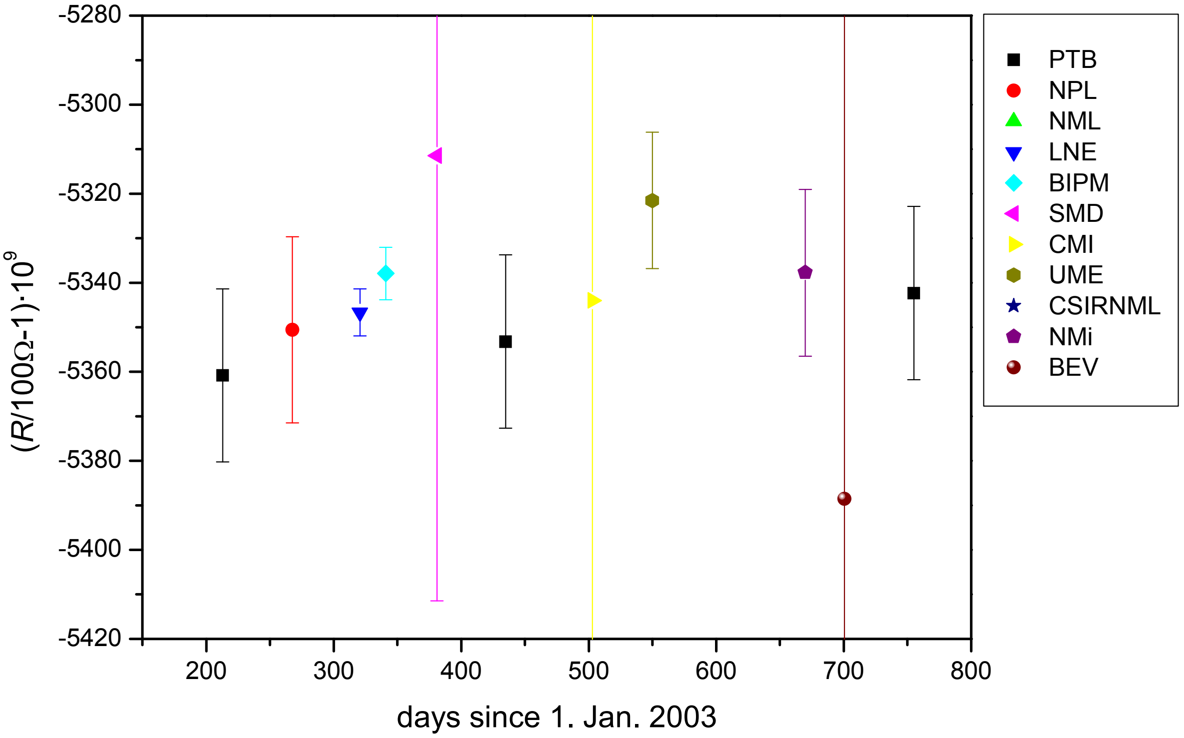


Figure 8

Results as given in Table 8:

Table 9

Summary of results, calculated for a mean date. The corresponding uncertainty U_{i} is calculated from the standard uncertainty u_{i}, given in the participants uncertainty budget, and the residual uncertainty $u_{\mathrm{i\text{-}Resid}}$ of the linear fit of the laboratory's result.

		Tinsley no. 262 767			
institute	mean date	result $(\cdot 10^{-9})$	u_{i} $(\cdot 10^{-9})$	$u_{\mathrm{i\text{-}Resid}}$ $(\cdot 10^{-9})$	U_{i} $(\cdot 10^{-9})$
PTB	02.08.2003	-3467,08	2,2	7,44	15,52
NPL	25.09.2003	-3456,24	10	4,94	22,3
NML	22.10.2003	-3254,65	210	3,1	420
LNE	16.11.2003	-3433,73	1,1	0,25	2,26
BIPM	07.12.2003	-3440,89	2,6	2,19	6,80
SMD	17.01.2004	-3410,14	50		100
PTB	10.03.2004	-3437,22	2,2	7,44	15,52
CMI	18.05.2004	-3418,50	50	2,41	100
UME	06.07.2004	-3414,71	7,4	3,89	16,6
CSIR/NML	22.08.2004	-3252,59	300	7,83	600
NMi	31.10.2004	-3397,33	5,9	8,16	20,1
BEV	01.12.2004	-3464,59	165	5,38	330
PTB	25.01.2005	-3393,96	2,2	7,44	15,52

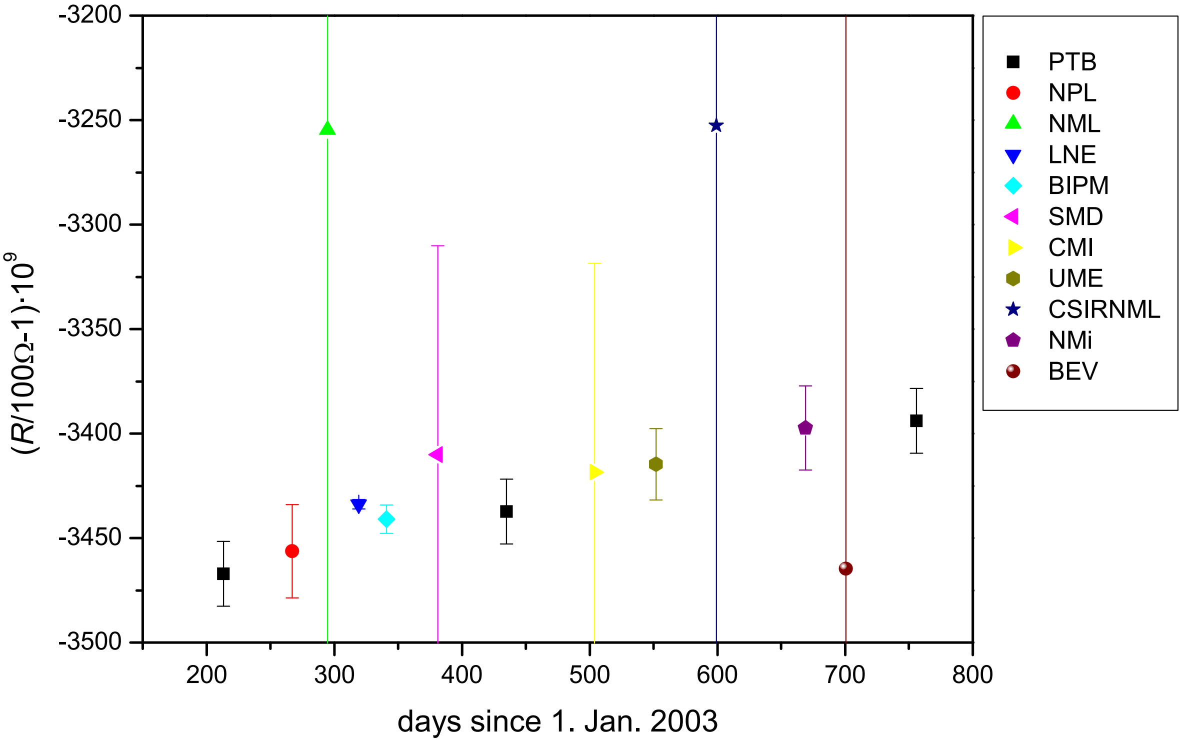


Figure 9

Results as given in Table 9:

Repeat measurement by GUM

Since the first results of GUM were not satisfying, it was decided to repeat the comparison. On the occasion of a visit at GUM one of the Tinsley resistors (SN 262 767) was measured by the pilot and by GUM. The results were:

Table 9a

institute	mean date	result ($\cdot 10^{-9}$)
PTB	28.8.2006	-3307,0
	31.8.2006	-3305,0
	1.9.2006	-3306,0
GUM	11.9.2006	-3585,8
	12.9.2006	-3588,4
	14.9.2006	-3559,0
PTB	14.12.2006	-3284,5
	15.12.2006	-3285,0
	15.2.2007	-3278,2
	16.2.2007	-3277,5
	23.3.2007	-3278,1

A linear regression to the pilot laboratory data was applied, and the mean difference for GUM calculated.
The results are linked to the comparison via the difference of the pilot laboratory with the reference value.

$$D_{\mathrm{GUM,CRV}} = -281{,}9\cdot 10^{-9},\ U_{\mathrm{GUM,CRV}} = 500\cdot 10^{-9}$$

Appendix B:

Lists of the reference values for each loop and for the combination of all loops

Summary of the values V_r, deduced from the fits to the pilots results, and the corresponding differences D_i of the laboratories

Table 10, loop 1:

		Tinsley no. 267 908		Tinsley no. 279 373		weighted mean	
institute	mean date	V_r ($\cdot 10^{-9}$)	D_i ($\cdot 10^{-9}$)	V_r ($\cdot 10^{-9}$)	D_i ($\cdot 10^{-9}$)	$D_{\mathrm{i,Loop\_1}}$ ($\cdot 10^{-9}$)	$U_{\mathrm{i,Loop\_1}}$ ($\cdot 10^{-9}$)
SP	13.05.2003	5622,92	-23,30	-944,25	-16,43	-21,04	29,89
JV	30.05.2003	5621,75	-17,43	-944,26	-12,34	-15,76	16,58
DFM	28.06.2003	5619,71	0,29	-944,29	4,29	1,60	378,30
PTB	10.09.2003	5614,51	-10,98	-944,34	-6,01	-9,35	16,37
VNIIM	08.10.2003	5612,55	47,45	-944,37	-45,63	16,85	63,99
MIKES$_{\mathrm{mean}}$	24.11.2003					0,00	18,13

The significance is χ^2=2,5 (N=6).

Table 11, loop 2:

		Tinsley no. 265 025		Tinsley no. 267 918		Tinsley no. 263 417		weighted mean		
institute	mean date	V_r ($\cdot 10^{-9}$)	D_i ($\cdot 10^{-9}$)	V_r ($\cdot 10^{-9}$)	D_i ($\cdot 10^{-9}$)	V_r ($\cdot 10^{-9}$)	D_i ($\cdot 10^{-9}$)	$D_{\mathrm{i,Loop\_2}}$ ($\cdot 10^{-9}$)	$U_{\mathrm{i,Loop\_2*}}$ ($\cdot 10^{-9}$)[*]	$U_{\mathrm{i,Loop\_2}}$ ($\cdot 10^{-9}$)[**]
OMH	14.08.2003	-3452,03	114,61	-4434,54	418,44	-4294,25	372,97	292,26	1737,2	1738,6
SASM	27.09.2003	-3452,66	-510,67	-4433,82	-353,74	-4292,03	-379,59	-419,46	1196,9	1199,0
GUM	21.10.2003	-3453,01	-855,24	-4433,44	-733,62	-4290,84	-754,97	-784,86	257,6	266,9
VMT	21.11.2003	-3453,43	-82,32	-4432,95	-35,93	-4289,27	-91,26	-66,39	125,0	143,3
LNMC	08.01.2004	-3454,19	1001,94	-4432,09	320,92	-4286,85	1378,17	824,73	1820,1	1821,4
EIM	23.03.2004	-3455,28	-138,25	-4430,85	-5,31	-4283,06	-94,47	-76,54	43,4	82,4
INRIM	21.04.2004	-3455,62	-41,30	-4430,46	1,76	-4281,60	-39,90	-24,27	35,5	78,5
CEM	25.05.2004	-3456,11	-49,04	-4429,92	20,63	-4279,88	-32,82	-18,26	52,9	87,8
INETI	07.07.2004	-3456,76	-284,61	-4429,19	-199,16	-4277,71	-213,83	-235,08	243,6	253,4
METAS	14.08.2004	-3457,35	-110,24	-4422,58	35,43	-4275,48	-56,01	-41,18	17,2	72,1
SIQ	17.09.2004	-3457,76	61,36	-4428,05	131,95	-4274,07	74,15	91,71	450,5	455,9
DMDM	17.10.2004	-3458,19	854,53	-4427,56	972,19	-4272,55	891,57	908,75	1220,7	1222,7
PTB$_{\mathrm{mean}}$	14.02.2004							0	8,0	70,5

[*] Uncertainty without additional transport uncertainty. The significance is χ^2=32,5 (N=13).

[**] Transport uncertainty $\sigma_{\mathrm{Trans\_2}} = 35\cdot 10^{-9}$ included, the significance is χ^2=11,1 (N=13); $U_{\mathrm{i,Loop\_2}} = 2\cdot\mathrm{SQRT}((U_{\mathrm{i,Loop\_2*}}/2)^2+\sigma_{\mathrm{Trans\_2}}^2)$.

Table 12, loop 3:

		Tegam no. 2030397		Tinsley no. 268 168		Tinsley no. 267 919		Tinsley no. 262 767		weighted mean		
institute	mean date	V_r $(\cdot 10^{-9})$	D_i $(\cdot 10^{-9})$	V_r $(\cdot 10^{-9})$	D_i $(\cdot 10^{-9})$	V_r $(\cdot 10^{-9})$	D_i $(\cdot 10^{-9})$	V_r $(\cdot 10^{-9})$	D_i $(\cdot 10^{-9})$	$D_{i,Loop\_3}$ $(\cdot 10^{-9})$	$U_{i,Loop\_3*}$ $(\cdot 10^{-9})^*$	$U_{i,Loop\_3}$ $(\cdot 10^{-9})^{**}$
NPL	25.09.2003	264,58	15,84	-1226,17	30,57	-5358,95	8,40	-3459,81	3,57	12,6	23,6	27,4
NML	22.10.2003	274,32	195,90	-1224,04	209,29	-5358,03	204,63	-3456,11	201,46	202,7	411,5	411,7
LNE	16.11.2003	284,18	18,54	-1222,04	3,99	-5357,14	10,47	-3452,79	19,05	13,9	11,3	18,0
BIPM	07.12.2003	291,08	15,76	-1220,14	6,06	-5356,45	18,53	-3449,88	8,99	11,9	12,1	18,5
SMD	17.01.2004	305,66	43,08	-1216,90	36,45	-5355,08	43,63	-3444,47	34,33	38,6	98,7	99,7
CMI	18.05.2004	350,62	5,19	-1206,83	25,20	-5350,92	6,92	-3427,94	9,44	11,3	100,6	101,6
UME	06.07.2004	368,38	6,79	-1202,87	11,71	-5349,32	27,80	-3421,43	6,72	12,6	18,6	23,3
CSIR/NML	22.08.2004	385,10	161,39	-1198,89	193,32	-5347,68	167,25	-3415,09	162,50	169,7	600,6	600,8
NMi	31.10.2004	409,98	23,89	-1193,15	2,69	-5345,23	7,48	-3405,66	8,33	10,1	17,5	22,4
BEV	01.12.2004	421,55	-69,54	-1190,51	-61,84	-5344,18	-44,40	-3401,41	-63,18	-59,9	328,9	329,2
PTB_{mean}	13.04.2004									0,0	5,1	14,9

*Uncertainty without additional transport uncertainty. The significance is $\chi^2=9$ ($N=11$).

**Transport uncertainty $\sigma_{Trans\_3} = 7\cdot 10^{-9}$ included, the significance is $\chi^2=3,3$ ($N=11$) ; $U_{i,Loop\_3} = 2\cdot SQRT((U_{i,Loop\_3*}/2)^2+\sigma_{Trans\_2}{}^2)$.

Table 13

List of the uncertainty components σ_A and σ_B, used in the evaluation of loop 1:

	weighted mean		Tinsley no. 267 908		Tinsley no. 279 373	
institute	$\sigma_{A,i}$ $(\cdot 10^{-9})$	$\sigma_{B,i}$ $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$
mikes	7,90	8,40	6,98	8,40	9,77	8,40
SP	7,13	12,20	7,13	12,20	7,13	12,20
JV	4,51	3,50	2,34	3,50	8,93	3,50
DFM	5,00	189,00	5,00	189,00	5,00	189,00
PTB	6,64	1,20	6,84	1,20	6,22	1,20
VNIIM	13,46	29,70	13,43	29,70	13,51	29,70

For each resistor, the σ_B is taken from the uncertainty budget, the σ_A is the root sum square of the participants type A uncertainty as stated in the budget and the scatter of the resistor during the measurement in the participants laboratory.

Table 14

List of the uncertainty components σ_A and σ_B, used in the evaluation of loop 2:

	weighted mean		Tinsley no. 265 025		Tinsley no. 263 417		Tinsley no. 267 918	
institute	$\sigma_{A,i}$ $(\cdot 10^{-9})$	$\sigma_{B,i}$ $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$
PTB	11,23	1,20	10,58	1,20	13,57	1,20	10,48	1,20
OMH	302,40	850,00	302,63	850,00	302,30	850,00	302,25	850,00
SASM	38,05	598,00	43,80	598,00	55,50	598,00	22,01	598,00
GUM	174,26	77,00	175,12	77,00	173,49	77,00	173,87	77,00
VMT	101,88	15,00	101,74	15,00	101,61	15,00	102,17	15,00
LNMC	0,00	910,00	0,00	910,00	0,00	910,00	0,00	910,00
EIM	6,31	20,00	6,58	20,00	6,11	20,00	6,16	20,00
INRIM	10,43	14,80	20,51	14,80	6,07	14,80	3,14	14,80
CEM	24,47	20,80	24,19	20,80	25,23	20,80	24,30	20,80
INETI	41,73	119,00	49,14	119,00	34,11	119,00	39,02	119,00
METAS	0,83	3,00	0,88	3,00	0,81	3,00	0,80	3,00
SIQ	10,00	225,00	10,00	225,00	10,00	225,00	10,00	225,00
DMDM	31,06	610,00	25,47	610,00	33,23	610,00	35,25	610,00

For each resistor, the σ_B is taken from the uncertainty budget, the σ_A is the root sum square of the participants type A uncertainty as stated in the budget and the scatter of the resistor during the measurement in the participants laboratory.

Table 15

List of the uncertainty components σ_A and σ_B, used in the evaluation for loop 3:

	weighted mean		Tinsley no. 268 168		Tinsley no. 267 919		Tegam no. 2030397		Tinsley no. 262 767	
institute	$\sigma_{A,i}$ $(\cdot 10^{-9})$	$\sigma_{B,i}$ $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$	σ_A $(\cdot 10^{-9})$	σ_B $(\cdot 10^{-9})$
PTB	7,43	1,20	2,42	1,20	9,50	1,20	10,20	1,20	7,46	1,20
NPL	5,56	9,90	5,94	9,90	4,68	9,90	5,13	9,90	6,11	9,90
NML	50,27	204,00	50,05	204,00	50,91	204,00	50,07	204,00	50,10	204,00
LNE	1,86	0,72	2,61	0,72	2,63	0,72	1,58	0,72	1,13	0,72
BIPM	3,27	2,00	3,31	2,00	2,78	2,00	3,83	2,00	3,25	2,00
SMD	7,70	48,90	7,70	48,90	7,70	48,90	7,70	48,90	7,70	48,90
CMI	3,45	50,00	3,19	50,00	2,93	50,00	4,91	50,00	3,13	50,00
UME	5,36	7,10	2,96	7,10	3,53	7,10	11,55	7,10	4,52	7,10
CSIR/NML	24,08	300,00	25,92	300,00	27,11	300,00	23,57	300,00	21,48	300,00
NMI	6,53	5,70	4,11	5,70	7,41	5,70	4,74	5,70	8,28	5,70
BEV	19,94	164,00	19,73	164,00	20,04	164,00	20,07	164,00	19,94	164,00

For each resistor, the σ_B is taken from the uncertainty budget, the σ_A is the root sum square of the participants type A uncertainty as stated in the budget and the scatter of the resistor during the measurement in the participants laboratory.

Difference of the participants with respect to the combined reference value

Table 16:

NMI	linked loops		equivalence with comparison reference value	
	$D_{\mathrm{i,COMB}}$ (10^{-9})	$U_{\mathrm{i,COMB}}$ (10^{-9})	$D_{\mathrm{i,CRV}}$ (10^{-9})	$U_{\mathrm{i,CRV}}$ (10^{-9})
MIKES	0,00	18,1	5,31	17,1
SP	-21,04	29,9	-15,74	29,3
JV	-15,76	16,6	-10,45	15,4
DFM	1,60	378,3	6,91	378,4
VNIIM	16,85	64,0	22,16	63,7
OMH	292,26	1738,6	288,22	1738,6
SASM	-419,46	1199,0	-423,51	1199,0
GUM	*-784,86*	*266,9*	*-788,90*	*267,0*
GUM**	-277,9	500,0	-281,9	500,0
VMT	-66,39	143,3	-70,43	143,4
LNMC	824,73	1821,4	820,69	1821,4
EIM*	-76,54201	82,4	-80,58	82,1
INRIM*	-24,27	78,5	-28,31	78,2
CEM*	-18,26	87,8	-22,30	87,6
INETI	-235,08	253,4	-239,13	253,5
METAS*	-41,18	72,1	-45,22	71,8
SIQ	91,71	455,9	87,67	455,9
DMDM	908,75	1222,7	904,71	1222,7
NPL	12,62	27,4	8,58	26,8
NML	202,69	411,7	198,65	411,8
LNE*	13,91	18,0	9,87	16,9
BIPM*	11,90	18,5	7,85	17,5
SMD	38,61	99,7	34,56	99,9
CMI	11,26	101,6	7,22	101,4
UME	12,55	23,3	8,51	22,5
CSIR/NML	169,68	600,8	165,64	600,8
NMi*	10,05	22,4	6,01	21,6
BEV	-59,89	329,2	-63,93	329,2
$\mathrm{PTB}_{\mathrm{mean}}^{*}$	0	11,4	-4,04	9,7

The acronyms of the laboratories whose results are not used for the calculation of the comparison reference value, are shown in italics. *Denotes laboratories, that claim an uncertainty, smaller than the transport uncertainty. For these laboratories the result reflects the limited knowledge on the behavior of the travelling standards and not the capability of the laboratory. ** denotes that the result was obtained by a repeated measurement in October 2006

The significance is χ^2=24,4 (N=27).

Figure 10:

Equivalence with the comparison reference value, all differences are in 10^{-9}. The uncertainty bars indicate the expanded uncertainty (k=2).

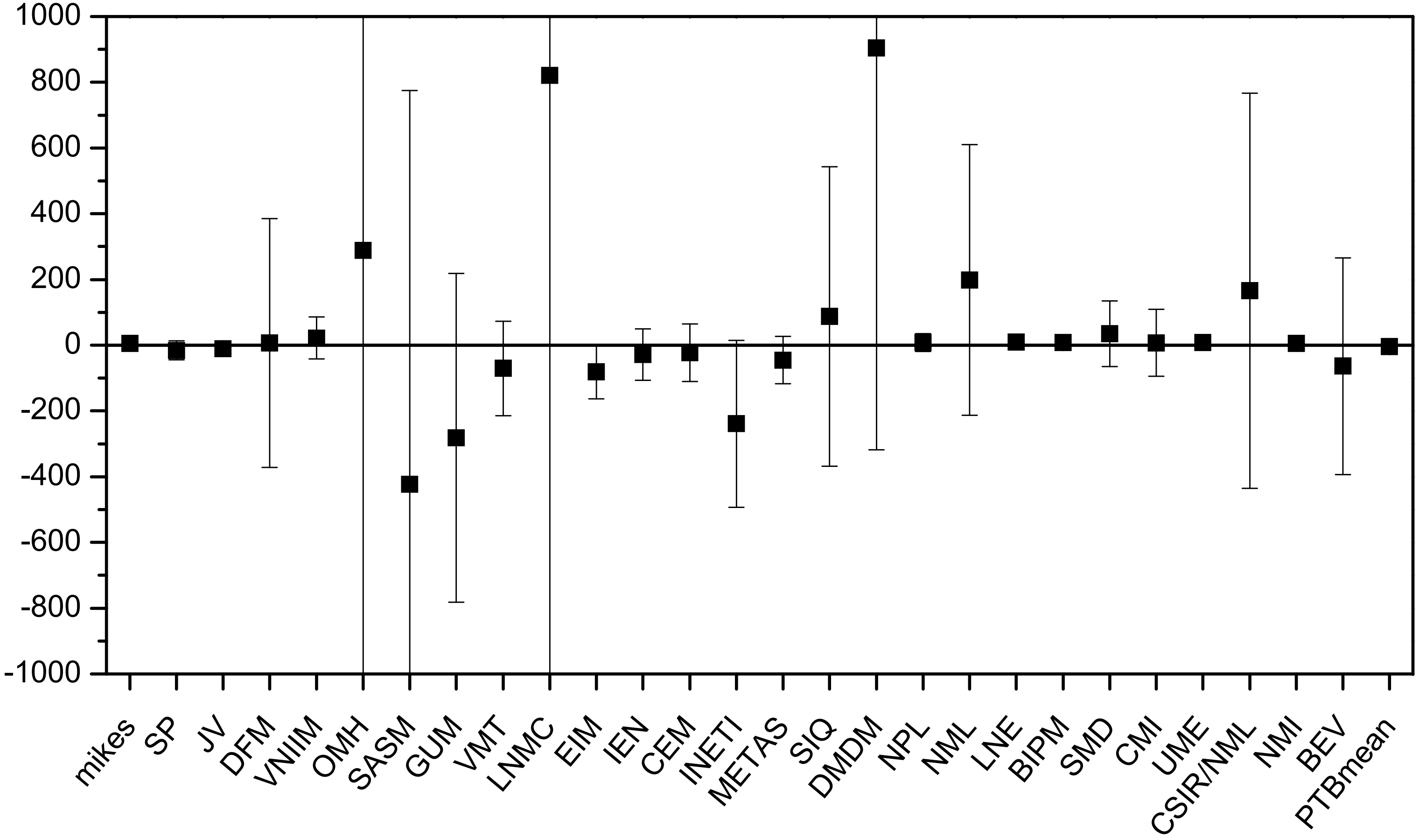
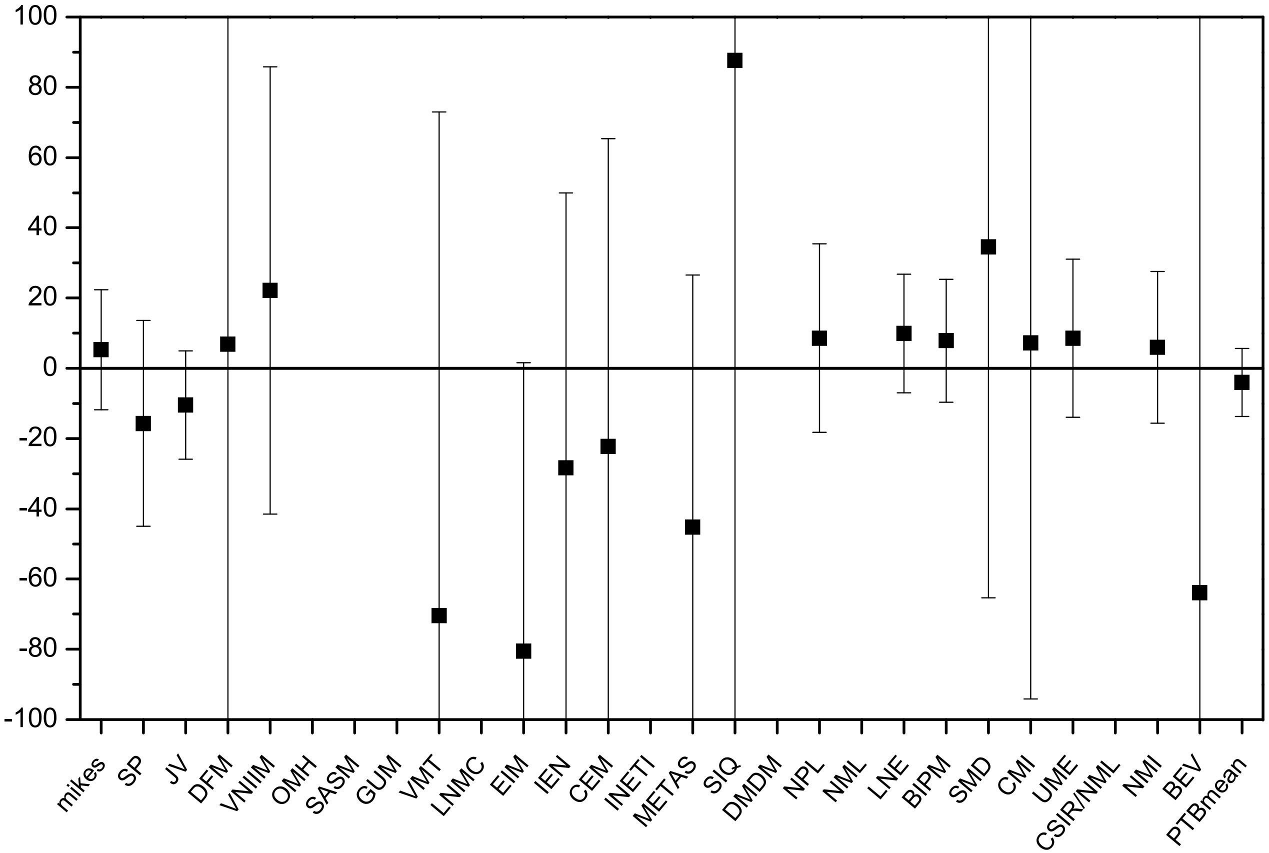


Figure 11:

Equivalence with the comparison reference value, all differences are in 10^{-9} (expanded scale). The uncertainty bars indicate the expanded uncertainty (k=2).

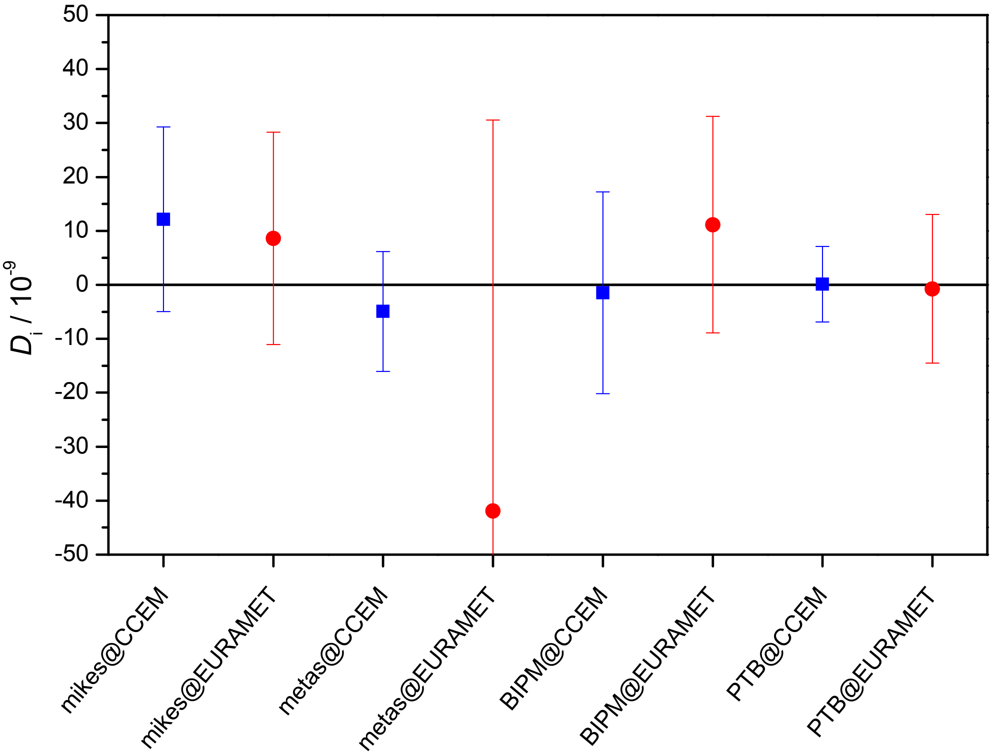


For the laboratories having participated in both comparisons, Table 17 and figure 12 contain the differences and associated uncertainties of their value relative to X_R as obtained in the CCEM comparison and calculated as if they had participated only in the EUROMET comparison.

Table 17:

	CCEM comparison		EURAMET comparison	
Laboratory	$D_{\text{i-KCRV}}$ / 10^{-9}	U_i / 10^{-9}	$D_{\text{i-KCRV}}$ / 10^{-9}	U_i / 10^{-9}
mikes	12,15	17,1	8,61	19,7
metas	-4,93	11,1	-41,92	72,5
BIPM	-1,48	18,7	11,15	20,1
PTB	0,14	7	-0,74	13,8

Figure 12:



For the laboratories having only participated in the EUROMET comparison, table 18 contains the difference and uncertainty of their results in terms of the CCEM-K10 KCRV

$$D_{\text{i-KCRV}} = D_{\text{i,comb}}\text{-X}_{\text{KCRV}} = (D_{\text{i,comb}} - X_{\text{CRV}}) + (X_{\text{CRV}} - X_{\text{KCRV}}) \text{ and}$$

$$U_{\text{i}} = (U_{\text{i,EUR}}^{2} + U_{\text{LINK}}^{2})^{1/2}$$

Table 18:

Laboratory	$D_{\text{i-KCRV}}$ / 10^{-9}	U_{i} / 10^{-9}
SP	-12,44	30,90
JV	-7,15	18,25
DFM	10,21	378,53
VNIIM	25,46	64,45
OMH	291,52	1738,63
SASM	-420,21	1199,04
GUM	-785,60	267,18
GUM**	-278,6	500,10
VMT	-67,13	143,73
LNMC	823,99	1821,43
EIM*	-77,28	82,68
INRIM*	-25,01	78,81
CEM*	-19,00	88,15
INETI	-235,83	253,69
SIQ	90,97	456,01
DMDM	908,01	1222,74
NPL	11,88	28,54
NML	201,95	411,92
LNE*	13,17	19,54
SMD	37,86	100,38
CMI	10,52	101,87
UME	11,81	24,54
CSIR/NML	168,94	600,88
NMi*	9,31	23,72
BEV	-60,63	329,35

Appendix C

Uncertainty Budgets of the Participants

1. PTB

Mathematical model:

$$R_{100} = R_{\mathrm{H}} \cdot V_{\mathrm{CCC}} \cdot k_{\mathrm{leak}} \cdot k_{\mathrm{bridge}}$$

Uncertainty budget

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_{H}	$R_{\mathrm{K\text{-}90}}/2$	0	rect/B	1	0	
V_{CCC}	$N_{\mathrm{S}}/N_{\mathrm{P}}$	$0{,}83\cdot10^{-9}$	rect/B	1	$0{,}83\cdot10^{-9}$	inf
k_{leak}	1	$0{,}1\cdot10^{-9}$	rect/B	1	$0{,}1\cdot10^{-9}$	inf
k_{bridge}	1	$0{,}8\cdot10^{-9}$	rect/B	1	$0{,}8\cdot10^{-9}$	inf
k_{Temp}	1	$0{,}14\cdot10^{-9}$	rect/B	1	$0{,}14\cdot10^{-9}$	inf
k_{press}	1	$0{,}02\cdot10^{-9}$	rect/B	1	$0{,}02\cdot10^{-9}$	inf
k_{read}	1	$0{,}5\cdot10^{-9}$	normal/A	1	$0{,}5\cdot10^{-9}$	9
R_{100}	100 Ω				$1{,}3\cdot10^{-9}$	411
		RSS of Type A uncertainties			$0{,}5\cdot10^{-9}$	
		RSS of Type B uncertainties			$1{,}2\cdot10^{-9}$	

2. SP Uncertainty budgets

The standard uncertainty has been determined in accordance with the Guide to the expression of Uncertainty in Measurement (GUM), ISO, 1995.

The model for the measurements is:

$$Rx = QHR \cdot (1 + \delta QHR + \delta qpl) \cdot r \cdot (1 + \delta cwr + \delta clr + \delta csr + \delta rxl)$$

Where

Rx is the unknown 100 Ω resistor.
QHR is the realised quantum Hall resistance at plateau i=2 with the exact numerical value 12906,4035 Ω.
δQHR is the relative error of the realised Hall resistance due to imperfect quantization and effects of imperfect contacts on the Hall sample.
δqpl is the relative error due to the QHR probe leakage resistance.
r is the ratio Rx/QHR measured by the CCC bridge.
δcwr is the relative error due to the CCC winding ratio deviation from nominal.
δclr is the relative error due the CCC leakage resistance.
δcsr is the relative error due to the error of the shunt resistor value.
δrxl is the relative error due to the error of the internal lead resistance of the shunted resistor.

The relative standard uncertainty is given by :

$$\frac{u(Rx)}{Rx} = \sqrt{\left(\frac{u(r)}{r}\right)^2 + \sum u(\delta_i)^2}$$

Where $\dfrac{u(r)}{r}$ is the standard deviation of the mean for the measurement results.

Uncertainty budget for R_X = Tr1

Quan-tity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$,(10^{-9})	Probability distribution / method of evaluation (A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$,(nΩ/Ω)	Degree of freedom ν_i
QHR	12906,4035 Ω	0.0	-	1	0.0	$\propto$
r	0.007748135260	2.8	Normal/A	1	2.8	6
δQHR	0	12.0	Normal/B	1	12.0	$\propto$
δqpl	0	0.9	Rectangular/B	1	0.9	$\propto$
δcwr	0	0.8	Normal/A	1	0.8	4
δclr	0	0.6	Rectangular/B	1	0.6	$\propto$
δcsr	0	1.3	Normal/B	1	1.3	$\propto$
δrxl	0	0.2	Normal/B	1	0.2	$\propto$
R_X	100.0005600 Ω				12.5	2359

3. **JV** Budget of uncertainty

The model for the measurement is:

$$R_x = R_s \cdot r \cdot (1 + \delta_{wind} + \delta_{leak} + \delta_{bal} + \delta_{shunt} + \delta_{rect})$$

The components are:

R_x: the unknown resistor
R_s: the QHR, i = 2 = 12906.4035 Ω
r : the ratio measured by the CCC-bridge
δ_{wind}: the relative winding ratio error
δ_{leak}: the relative error due to leakage resistance
δ_{bal}: the relative error due to bridge balancing
δ_{shunt}: the relative error due to the stability and calibration of the shunt resistor
δ_{rect}: the relative error due to noise rectification

The relative standard uncertainty is then given by :

$$\frac{u(R_x)}{R_x} = \sqrt{\left(\frac{u(R_s)}{R_s}\right)^2 + \left(\frac{u(r)}{r}\right)^2 + \sum(\delta_i)^2}$$

Which give the following uncertainty budget for R_x:

Uncertainty budget for R_X = TR1:

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (ppb)	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$,(ppb)	Degree of freedom ν_i
R_s	12906.4035 Ω	1	rectangular / B	1	1	∝
r	$7.748135295 \cdot 10^{-3}$	0.8	normal / A	1	0.8	17
δ_{wind}	0	1	normal / B	1	1	∝
δ_{leak}	0	1	rectangular / B	1	1	∝
δ_{bal}	0	2.5	normal / B	1	2.5	11
δ_{shunt}	0	0.6	normal / B	1	0.6	∝
δ_{rect}	0	1	rectangular / B	1	1	∝
R_X	100.000 560 49 Ω				3.6	46

Budget of uncertainty for the QHR measurements.
The type A uncertainty used for the measured ratio, r, is the standard deviation of the mean (3,2 ppb/√18 = 0.8 ppb) . For the resistor TR2 the standard deviation of the mean is 1.7 ppb.

The effective degrees of freedom is larger than 46 for TR1 and larger than 40 for TR2.

The standard uncertainty for the two resistors is:

U(TR1) = 0.36 μΩ (k=1) and U(TR2) = 0.36 μΩ (k=1)

The uncertainty in the temperature and pressure measurements are not included in this estimate of the measurement uncertainty.

4. MIKES Budget of uncertainty

Mathematical model:
When CCC bridge is balanced, the value R_{100} of the unknown resistor is obtained from the relationship:
$R_{100} = R_H(2) * (1 + \delta_{RH}) * k_{ccc} (1 + k_{br})* (1 + k_{Rx})$, where:

$R_H(2)$ is the realised quantum Hall resistance at plateau i=2 with the value 12906,4035 Ω.
δ_{RH} is the relative error of the realised Hall resistance due to imperfect quantization, imperfect contacts on the Hall sample and the QHR probe leakage resistance
k_{ccc} is the nominal ratio of CCC windings
$k_{br} = \sqrt{\Sigma (\delta_i)^{\wedge 2}}$, is the relative combined error of CCC bridge,
components of k_{br} are: V_{ccc}, k_{leak}, k_{comp}, k_{gain}, k_{noise}, k_{acdct}, k_{switch}, k_{offs}
V_{ccc} is the relative winding ratio error
k_{leak} is the relative error due to leakage currents in voltage link
k_{comp} is the relative error due to calibration of compensation current
k_{gain} is the relative error due to the gain in FB circuit
k_{noise} is the relative error due to noise rectification
k_{acdct} is the relative error due to extrapolation from 0.2 Hz to dc
k_{switch} is the relative error due to contact resistance in rotary switch of I_{comp}
k_{offs} is the relative error due to zero offset
k_{Rx} is the relative errors due to unexcluded temperature-pressure and 1/f-related resistance variation of the unknown resistor
k_{read} is the deviation of R_{100} from the nominal value measured by the CCC bridge

The relative combined standard uncertainty is given by :

$$u(R_x) / R_x = \sqrt{ [u (R_{H(2)}) /R_{H(2)}]^2 + [u (k_{read})/ k_{read}]^2 + \Sigma u (\delta_i)^2 }$$

Where $u(k_{read}) / k_{read}$ is the relative standard deviation of the measurement results.

Error budget in determination of 100 Ohm , Tr2, (SN 279373) from QHR by CCC bridge. **(an example of estimation of the measurements in July 2004)**

Quantity	Estimate	Standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_H	$R_{K-90}/2$	0	rect/B	1	-	inf
δ_{RH}	0	$2 \cdot 10^{-9}$	rect/B	1	$2\cdot10^{-9}$	inf
k_{ccc}	N_S/N_P	0	rect/B	1	-	inf
V_{CCC}	0	$1.3\cdot10^{-9}$	rect/B	1	$1.3\cdot10^{-9}$	inf
k_{leak}	0	$3.0\cdot10^{-9}$	rect/B	1	$4.0\cdot10^{-9}$	inf
k_{comp}	0	$1.0\cdot10^{-9}$	rect/B	1	$1.0\cdot10^{-9}$	inf
k_{gain}	0	$2.5\cdot10^{-9}$	rect/B	1	$2.5\cdot10^{-9}$	inf
k_{noise}	0	$4.0\cdot10^{-9}$	rect/B	1	$4.0\cdot10^{-9}$	inf
k_{acdc}	0	$2.0\cdot10^{-9}$	rect/B	1	$2.0\cdot10^{-9}$	inf
k_{switch}	0	$3.0\cdot10^{-9}$	rect/B	1	$3.0\cdot10^{-9}$	inf
k_{offs}	0	$3.5\cdot10^{-9}$	rect/B	1	$3.5\cdot10^{-9}$	inf
k_{Rx}	0	$1.0\cdot10^{-9}$	rect/B	1	$1.0\cdot10^{-9}$	inf
k_{read}	$-0.9446\cdot10^{-9}$	$2.5\cdot10^{-9}$	normal/A	1	$2.5\cdot10^{-9}$	5
R_{100}	99.99990554				$8.6\cdot10^{-9}$	9
		RSS of Type A uncertainties			$2.5\cdot10^{-9}$	
		RSS of Type B uncertainties			$8.2\cdot10^{-9}$	

5. DFM

Uncertainty budget

The DFM uncertainty budget is based on the folowing model equation:

$$\varepsilon_X=\varepsilon_S+\delta\varepsilon_{SP}+\delta_{T,S}+(r+\delta r_S+\delta r_C)-\delta_{T,X}$$

Where ε_X is the relative deviation from nominal (RDN) of the unknown resistor, ε_S is the RDN of the reference/transfer resistor and $\delta\varepsilon_{SP}$ is the series-parallel transfer error. The term $\delta_{T,S}$ is the temperature correction of the reference, r is the ratio as measured, δr_S is the specification of the current comparator bridge, δr_C is the correction of the bridge error, and $\delta_{T,S}$ is the temperature correction of the unknown.

Table 21: uncertainty

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
ε_S	$-13{,}268\cdot10^{-6}$	$0{,}15\cdot10^{-6}$	normal/B	1	$0{,}15\cdot10^{-6}$	30
$\delta\varepsilon_{SP}$	0	$0{,}058\cdot10^{-6}$	rect/B	1	$0{,}058\cdot10^{-6}$	inf
$\delta_{T,S}$	0	$0{,}08\cdot10^{-6}$	rect/B	1	$0{,}08\cdot10^{-6}$	inf
r	$18{,}877\cdot10^{-6}$	$0{,}005\cdot10^{-6}$	normal/A	1	$0{,}005\cdot10^{-6}$	30
δr_S	0	$0{,}058\cdot10^{-6}$	rect/B	1	$0{,}058\cdot10^{-6}$	inf
δr_C	$0{,}008\cdot10^{-6}$	$0{,}001\cdot10^{-6}$	normal/B	1	$0{,}001\cdot10^{-6}$	30
ε_X	$5{,}617\cdot10^{-6}$				$0{,}189\cdot10^{-6}$	75
		RSS of Type A uncertainties			$0{,}005\cdot10^{-6}$	
		RSS of Type B uncertainties			$0{,}189\cdot10^{-6}$	

6. METAS

Mathematical model:

$$R_{100} = R_{\mathrm{H}} \cdot V_{\mathrm{CCC}} \cdot k_{\mathrm{leak}} \cdot k_{\mathrm{bridge}} \cdot k_{\mathrm{noise}}$$

uncertainty for Tegam SR102 #A2030397

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
V_{CCC}	$N_{\mathrm{S}}/N_{\mathrm{P}}$	$0{,}1\cdot10^{-9}$	normal/A	1	$0{,}1\cdot10^{-9}$	10
k_{leakS}	1	$1\cdot10^{-9}$	normal/A	1	$1\cdot10^{-9}$	10
k_{leakG}	1	$0{,}40\cdot10^{-9}$	normal/A	1	$0{,}40\cdot10^{-9}$	5
k_{divider}	1	$0{,}40\cdot10^{-9}$	normal/A	1	$0{,}40\cdot10^{-9}$	10
k_{GainV}	1	$0{,}22\cdot10^{-9}$	rect/B	1	$0{,}22\cdot10^{-9}$	50
k_{GainS}	1	$0{,}29\cdot10^{-9}$	rect/B	1	$0{,}29\cdot10^{-9}$	50
k_{uncomp0}	1	$0{,}58\cdot10^{-9}$	rect/B	1	$0{,}58\cdot10^{-9}$	50
k_{Temp}	1	$2{,}6\cdot10^{-9}$	rect/B	1	$2{,}6\cdot10^{-9}$	50
k_{press}	1	$0{,}6\cdot10^{-9}$	rect/B	1	$0{,}6\cdot10^{-9}$	50
k_{read}	1	$0{,}4\cdot10^{-9}$	normal/A	1	$0{,}4\cdot10^{-9}$	8
R_{100}	100 Ω				$3{,}0\cdot10^{-9}$	80
		RSS of Type A uncertainties			$0{,}4\cdot10^{-9}$	
		RSS of Type B uncertainties			$3{,}0\cdot10^{-9}$	

7. VNIIM

Uncertainty budget for R_X

Components of the uncertainty	Relative standard uncertainty, $\times 10^8$	Method of evaluation	Degree of freedom	Comment
Repeated measurements	1.2	Type A	9	n=10 (number of measurements)
Representation by the VNIIM QHR	1.8	Type B	3	k = 3 (VNIIM Interim Report)
Stability of the group standard	2.0	same	6	30 % (accuracy of the uncertainty estimate)
Resolution of the bridge for 1:1 comparison	1.0	-″-	13	20 % (accuracy of the uncertainty estimate)
Transfer by Hamon type set	0.5	-”-	60	k = 2 (VNIIM Interim Report)
Temperature of the group standard	0.5	-”-	4	30 % (accuracy of the uncertainty estimate)
Pressure	0.2	-”-	2	50 % (accuracy of the uncertainty estimate)
Temperature of the travelling standard	0.2	-”-	3	40 % (accuracy of the uncertainty estimate)
Resistance of the travelling standard Rx	**Combined standard relative uncertainty $3.2 \cdot 10^{-8}$** Expanded relative uncertainty $8 \cdot 10^{-8}$		$\nu_{eff} = 17$	Coverage factor **k =2.11**

8. BIPM (Measurements in terms of a reference 100 Ω resistance, R_{100B})

Uncertainty budget for measurement of the 100 Ω resistances in terms of R_{K-90}

Source of uncertainty	Standard uncertainty in parts in 10^9	*Type*	Probability distribution
Imperfect quantization of the Hall resistance	1	B	Triangular
CCC imperfect winding ratio	1	B	Triangular
Calibration of the resistive current divider and null detector interpolation	1	B	Triangular
Leakage resistances	0.2	B	Triangular
Possible noise rectification	1	B	Triangular
RMS total	**2**	B	

9. CEM

In the analysis of uncertainty for this comparison, the following mathematical model is considered:

$$R_x = \frac{r_{H10k} R_{10k} (1 + mt)}{N_H} \frac{r_x}{r_{H100}} \qquad (1)$$

, where:

- R_x: Value of the 100 Ω travelling standard, relative to Quantum Hall Resistance.
- R_{10k}: Value of the 10 kΩ standard resistor, referred directly to Quantum Hall Resistance.
- N_H: Ratio series-parallel of the Hamon device, nominally equal to 100
- m: Drift rate of the10 kΩ standard resistor, estimated from its history.
- t: Time elapsed since last comparison of the 10 kΩ standard resistor with QHE.
- r_{H10k}: Ratio Hamon device in series to the 10 kΩ standard. Nominal value equal to 1.
- r_{H100}: Ratio Hamon device in parallel to the tare resistor. Nominal value equal to 10.
- r_x: Ratio R_x to the tare resistor. Nominal value equal to 10.

It results the following formula for the relative uncertainty:

$$\frac{u^2(R_x)}{(R_x)^2} = \frac{u^2(R_{10k})}{(R_{10k})^2} + u^2(m)t^2 + \frac{u^2(r_{H10k})}{(r_{H10k})^2} + \frac{u^2(N_H)}{(N_H)^2} + \frac{u^2(r_x)}{(r_x)^2} + \frac{u^2(r_{H100})}{(r_{H100})^2} \qquad (2)$$

1. Uncertainty budget for the three standards. The shown estimates are only nominal values.

2. Measurements with ambient conditions.

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
R_{10k}	10000 Ω	10×10^{-9}	Normal/A+B	1	10×10^{-9}	∞
m	3×10^{-8} /yr	1×10^{-8} /yr	Student/A	0.7 yr	7×10^{-9}	7
N_H	100	7×10^{-9}	Rectangular/B	1	7×10^{-9}	∞
r_{H10k}:	1	15×10^{-9}	Normal/A+B	1	15×10^{-9}	∞
r_{H100}	10	3×10^{-9}	Rectangular/B	1	3×10^{-9}	∞
r_x:	10	3×10^{-9}	Rectangular/B	1	3×10^{-9}	∞
R_X					$2,1 \times 10^{-8}$	ν_{eff} =81

10. GUM

The example of calibration results an uncertainty budget for R_X Tinsley 5685A No 265025

Quantity X_i	Estimate x_i	Standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
r	9,99974775	1,667E-08	normal	10	1,667E-07 Ω	9
r'	0	1,155E-06	rectangular	10	1,155E-05 Ω	∞
r''	0	5,773E-07	rectangular	10	5,773E-06 Ω	∞
R_s	10,00021231	1,300E-06 Ω	normal	10	1,300E-05 Ω	19
δR_s	0	0 Ω	normal	10	0 Ω	
$\delta' R_s$	-3,06807E-06	1,501E-07 Ω	rectangular	10	1,501E-6 Ω	8
$\delta'' R_s$	0	0 Ω	normal	10	0 Ω	
$\delta' R_x$	0	0 Ω	rectangular	1	0 Ω	
$\delta'' R_x$	0	0 Ω	normal	1	0 Ω	
R_x	99,9994827 Ω				1,84E-05 Ω	$\nu_{eff} = 75$
Relative value of standard uncertainty u(R_x) = 1,84 E-07						

The value of the test current was equal during the calibration to (3 ± 0,045) mA
A resistance R_x of the calibrated standard resistor was measured using the current comparator resistance bridge type Guildline 9975 and reference standard resistor type ZIP 321.
The source of traceability is from BIPM.
The resistor 1 Ω, tape P321 was calibrated in BIPM. The resistor 10 Ω tape P321 was calibrated to 1 Ω. The resistor 10 Ω was used to comparison the resistor 100 Ω The value resistor of 10 Ω was corrected by influence of temperature and pressure (air and oil).The resistance standards were maintained in an oil bath in (23 ± 0,01) ^{O}C. The temperature was measured by a platinum resistance thermometer and resistance bridge type 5840 – produced by Tinsley. The pressure in the oil bath was referred to the height of oil above tope plate and density of oil.
The resistance R_x was calculated from the reading r of the current comparator resistance bridge by:

$$R_x = (r + r' + r'')(R_s + \delta R_s + \delta' R_s + \delta'' R_s) - \delta' R_x - \delta'' R_x,$$

in which $r = R_x / R_s$ is the resistance ratio of calibrated and reference standard resistors, R_s is the conventional true resistance value of the reference standard resistor, δR_s is the correction of the reference resistance due to drift, $\delta' R_s$ and $\delta' R_x$ are the temperature related resistance variations of the reference and unknown resistor,
$\delta'' R_s$ and $\delta'' R_x$ are the pressure related resistance variations of the reference and unknown resistor.
δR_s = 0 because reference value was calibrated (to resistor 1 Ω) each day in during the period of measurement.
$\delta'' R_s$ = 0 because pressure coefficient for the reference resistor is unknown.
There wasn't make correction value resistance R_x on regard influences the temperature and pressure.

11. OMH

Euromet key comparison No 636 – measurement results **(Annex 3)**

List of the principal components of the uncertainty budget to be evaluated.

Uncertainty budget for R_1; R_2; R_3

Quantity X_i		Estimate x_i	Standard uncertainty $u(x_i)$	Probability distribution / method of evaluation (A, B)	Sensitivity coefficient c_i	Uncertainty contribution $c_{i*}\,u_i(R_x)$	Degree of freedom ν_i
OMH 100 Ω working standard		100,00153 Ω	60 μΩ	normal, B	1	60 μΩ	infinite
Adjustment in R_s side		0,00153 Ω	5 μΩ	normal, B	1	5 μΩ	infinite
Deviation from reference temp.		0,02 °C	0,05 °C	rectangular, B	1040 μΩ/°C	52 μΩ	infinite
Temp. coefficient		1040 μΩ/°C	0,5 μΩ/°C	rectangular, B	0 °C	0 μΩ	infinite
Measurement system of Comparator Bridge	R_x side	0 μΩ	1 turn	rectangular, B	0,1 μΩ/turn	0,1 μΩ	infinite
	R_s side	0 μΩ	1 turn	rectangular, B	0,1 μΩ/turn	0,1 μΩ	infinite
	Indication	0 μΩ	3 μA	rectangular, B	10 μΩ/μA	30 μΩ	infinite
Nullindicator		0 nV	17 nV	rectangular, B	1/10 mA	1,7 μΩ	infinite
Indication of measurement	R_1 (No:267918)	-0,00040 Ω	30 μΩ	normal, A	1	30 μΩ	8
	R_2 (No:263417)	-0,00039 Ω					
	R_3 (No:265025)	-0,00033 Ω					
R_1 (No:267918)		99,99960 Ω				90,2 μΩ (0,9 ppm)	ν_{eff}>>100 (648)
R_2 (No:263417)		99,99961 Ω					
R_3 (No:265025)		99,99967 Ω					

Expanded uncertainty is (assuming normal distribution and an expansion coefficient $k = 2$): 0,18 mΩ, that is 1,8 ppm.

12. DMDM

Scheme for uncertainty budget for 10 Ω : 1 Ω measurements

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
r	9.998449405	1.3E-09	normal/type A	1.0	1.3E-09	190
R_S	0.99999295 Ω	1.7E-09	rectangular/type B	10.0	1.7E-08	∞
$\delta R_{S,drift}$	0.00 μΩ	3.0E-09	rectangular/type B	10.0	3.0E-08	∞
$\delta R_{S,pressure}$	0.00 μΩ	2.0E-09	rectangular/type B	10.0	2.0E-08	∞
$\delta R_{X\text{-}S,temperature}$	0.00 μΩ	6.0E-07	rectangular/type B	1	6.0E-07	∞
$\delta R_{ratio\ accuracy}$	0.00 μΩ	1.4E-08	rectangular/type B	1	1.4E-08	∞
$\delta R_{resolution}$	0.00 μΩ	3E-10	rectangular/type B	1	3E-10	∞
$\delta R_{linearity}$	0.00 μΩ	3E-09	rectangular/type B	1	3E-09	∞
$\delta R_{connection}$	0.00 μΩ	1.15E-07	rectangular/type B	1	1.15E-07	∞
R_X	9.9983789 Ω				6.1E-07	ν_{eff} 1E+13

where:

r - ratio (mean value) R_X and R_S, read on the DCC bridge;

R_S - resistance of the reference 1 Ω standard (mean value of the group of four standard resistors) at the mean temperature during the measurements;

$\delta R_{S,drift}$ - drift of the resistance of the reference standard since its last calibration (No correction applied. Uncertainty is estimated from its calibration history.);

$\delta R_{S,pressure}$ - resistance change of the reference standard due to pressure;

$\delta R_{X\text{-}S,temperature}$ - temperature-related resistance variation of the reference standard and standard under test;

$\delta R_{ratio\ accuracy}$ - resistance variation due to ratio accuracy of the DCC bridge;

$\delta R_{resolution}$ - resistance variation due to resolution of the readout of the DCC bridge;

$\delta R_{linearity}$ - resistance variation due to nonlinearity of the DCC bridge;

$\delta R_{connection}$ - resistance variation due to connection.

13. UME

The model function of uncertainty can be written as below

$$\mathbf{R_{(100\ ohm)} = (R_{QH} + \delta_Q) \times (r + \delta_r + \delta_L + \delta_B + \delta_{Shunt}) + \delta_T}$$

$R_{(100\ ohm)}$: Value of the 100 ohm resistor that compared with the Quantum Hall Resistance

R_{QH} : Quantum Hall resistance

δ_Q : Deviation due to imperfect quantization in the sample

r : Ratio that is determined by the CCC measurements

δ_r : Deviation due to winding ratio

δ_L : Deviation due to leakage resistance

δ_B : Deviation due to bridge balancing

δ_{Shunt} : Deviation due to internal shunt resistance calibration

δ_T : Measurement uncertainty of temperature of the oil / air bath

Proposed scheme for an uncertainty budget for R_X (S/N: 262767 against to Quantum Hall standard)

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Divisor	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
S (δ_S)		$2{,}3\text{x}10^{-9}$	Normal / A	1	1	$2{,}3\text{x}10^{-9}$	16
Measurement uncertainty of temperature of the oil bath (δ_T)		$0{,}1\text{x}10^{-9}$	Rectangular / B	1,7321	1	$0{,}06\text{x}10^{-9}$	100
Winding ratio (δ_r)		$5\text{x}10^{-9}$	Rectangular / B	1,7321	1	$2{,}9\text{x}10^{-9}$	1000
Leakage resistance (δ_L)		$3{,}5\text{x}10^{-9}$	Rectangular / B	1,7321	1	$2\text{x}10^{-9}$	50
Bridge balancing (δ_B)		$9\text{x}10^{-9}$	Rectangular / B	1,7321	1	$5{,}2\text{x}10^{-9}$	1000
Internal shunt resistance calibration (δ_{Shunt})		$1{,}5\text{x}10^{-9}$	Rectangular / B	1,7321	1	$0{,}9\text{x}10^{-9}$	50
Imperfect quantization in the sample (δ_Q)		$5\text{x}10^{-9}$	Rectangular / B	1,7321	1	$2{,}9\text{x}10^{-9}$	1000
R_X	**99,9996577**	u_c				$7{,}35\text{x}10^{-9}$	ν_{eff} =986

Coverage factor k=2

total uncertainty =u_c x k = $14{,}7\text{x}10^{-9}$

Proposed scheme for an uncertainty budget for R_X (S/N: 267919 against to Quantum Hall standard)

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Divisor	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
S (δ_S)		$3{,}4 x 10^{-9}$	Normal / A	1	1	$3{,}4 x 10^{-9}$	12
Measurement uncertainty of temperature of the oil bath (δ_T)		$1{,}5 x 10^{-9}$	Rectangular / B	1,7321	1	$0{,}9 x 10^{-9}$	100
Winding ratio (δ_r)		$5 x 10^{-9}$	Rectangular / B	1,7321	1	$2{,}9 x 10^{-9}$	1000
Leakage resistance (δ_L)		$3{,}5 x 10^{-9}$	Rectangular / B	1,7321	1	$2 x 10^{-9}$	50
Bridge balancing (δ_B)		$9 x 10^{-9}$	Rectangular / B	1,7321	1	$5{,}2 x 10^{-9}$	1000
Internal shunt resistance calibration (δ_{Shunt})		$1{,}5 x 10^{-9}$	Rectangular / B	1,7321	1	$0{,}9 x 10^{-9}$	50
Imperfect quantization in the sample (δ_Q)		$5 x 10^{-9}$	Rectangular / B	1,7321	1	$2{,}9 x 10^{-9}$	1000
R_X	**99,9994677**	u_c				$7{,}8 x 10^{-9}$	ν_{eff} =302

Coverage factor k=2

total uncertainty = u_c x k = $15{,}6 x 10^{-9}$

Proposed scheme for an uncertainty budget for R_X (S/N: 268168 against to Quantum Hall standard)

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Divisor	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
S (δ_S)		$1{,}8x10^{-9}$	Normal / A	1	1	$1{,}8x10^{-9}$	15
Measurement uncertainty of temperature of the oil bath (δ_T)		$1{,}9x10^{-9}$	Rectangular / B	1,7321	1	$1{,}1x10^{-9}$	100
Winding ratio (δ_r)		$5x10^{-9}$	Rectangular / B	1,7321	1	$2{,}9x10^{-9}$	1000
Leakage resistance (δ_L)		$3{,}5x10^{-9}$	Rectangular / B	1,7321	1	$2x10^{-9}$	50
Bridge balancing (δ_B)		$9x10^{-9}$	Rectangular / B	1,7321	1	$5{,}2x10^{-9}$	1000
Internal shunt resistance calibration (δ_{Shunt})		$1{,}5x10^{-9}$	Rectangular / B	1,7321	1	$0{,}9x10^{-9}$	50
Imperfect quantization in the sample (δ_Q)		$5x10^{-9}$	Rectangular / B	1,7321	1	$2{,}9x10^{-9}$	1000
R_X	**99,9998804**	u_c				$7{,}3x10^{-9}$	ν_{eff} =1470

Coverage factor k=2

total uncertainty = u_c x k = $14{,}6x10^{-9}$

Proposed scheme for an uncertainty budget for R_X (S/N: A2030397 against to Quantum Hall standard)

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Divisor	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$	Degree of freedom ν_i
S (δ_S)		1 x10^{-8}	Normal / A	1	1	1x10^{-8}	6
Measurement uncertainty of temperature of the air bath (δ_T)		0,5x10^{-9}	Rectangular / B	1,7321	1	0,3x10^{-9}	100
Winding ratio (δ_r)		5x10^{-9}	Rectangular / B	1,7321	1	2,9x10^{-9}	1000
Leakage resistance (δ_L)		3,5x10^{-9}	Rectangular / B	1,7321	1	2x10^{-9}	50
Bridge balancing (δ_B)		9x10^{-9}	Rectangular / B	1,7321	1	5,2x10^{-9}	1000
Internal shunt resistance calibration (δ_{Shunt})		1,5x10^{-9}	Rectangular / B	1,7321	1	0,9x10^{-9}	50
Imperfect quantization in the sample (δ_Q)		5x10^{-9}	Rectangular / B	1,7321	1	2,9x10^{-9}	1000
R_X	**100,0000370**		u_c			1,22x10^{-8}	ν_{eff} =33

Coverage factor k=2

total uncertainty = u_c x k = 24,4x10^{-9}

14. NPL

Equation and Method of Evaluation

The equation describing the CCC at balance is:

$$R_X = R_Q \frac{1 + I_B / I_X}{N} \qquad (1)$$

R_X = unknown resistance (on master current source side of bridge)

R_Q = QHR resistance (on slave current source side of bridge)

I_X = current in R_X

I_B = current applied to a balance winding on the comparator to maintain zero detector voltage

N = turns ratio of comparator

The ratio I_B/I_X is determined by a calibration step during the measurement, when a resistor of nominal value 10^4 R_X is added in parallel to R_X.

For the purposes of estimating the type B uncertainty, we do not attempt to individually calculate the uncertainties due to I_B/I_X and N. We consider the CCC to be represented by a single term, K_{CCC} in the equation, so $R_X = R_Q K_{CCC}$. We then conduct a series of loop-closure tests to evaluate $u(K_{CCC})$.

R_X is subject to an additional uncertainty due to the uncertainty in measuring the temperature and pressure at the time of measurement. We denote the effect of the temperature and pressure dependence on the measurement by R_T and R_P. Equation (1) becomes

$$R_X = R_Q K_{CCC} - R_T - R_P \qquad (2)$$

To estimate $u(R_Q)$, we performed measurements of the same resistor using plateaux i=2 and i=4 of the QHR device. A total of seven 100 Ω resistors (the four comparison resistors plus three NPL standards) were measured against both plateaux, and a rectangular distribution was assigned to cover the full range of $R_X(i=2) – R_X(i=4)$. The rectangular distribution had a *full* width of 9.9 ppb, giving a standard uncertainty contribution $u(R_Q) = 9.9/2\sqrt{3} = 2.9$ ppb.

To estimate K_{CCC}, we performed loop-closure measurements whereby a resistor was measured directly against the QHR device, and also in two stages via a 100 Ω buffer resistor. A total of four 100 Ω resistors were each measured directly against the QHR and using four buffer routes (two buffer resistors, and two CCC bridges), and a rectangular distribution assigned to cover the full range of R_X(direct) – R_X(via buffer). The rectangular distribution had a *full* width of 13.2 ppb, giving a standard uncertainty contribution $u(K_{CCC}) = 13.2/2\sqrt{3} = 3.8$ ppb. Measurements involving the Teagam resistor were not included in evaluation of the distribution width of R_Q or K_{CCC}, as no air pressure measurements were made to correct for the pressure dependence of this resistor.

Uncertainty Budget

The particular budget shown is for the Teagam air-bath resistor. The air pressure was not recorded at the time of the measurements, hence the large uncertainty assigned to pressure.

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$ (ppb)	Degree of freedom ν_i
R_Q	R_{K-90}/i	2.9 ppb	B(rectangular)	1	2.9	Inf.
K_{CCC}	-	3.8 ppb	B(rectangular)	1	3.8	Inf
T	23 ^{0}C	0.1 ^{0}C	B(normal)	$7.95*10^{-8}$/K	8.0	Inf
P	1005 hPa	20 hPa	B(rectangular)	$-2.9*10^{-10}$/hPa	3.3	Inf
Random		3.6 ppb	A(normal)	1	3.6	>30
R_X					10.5	*large*
R_X *(k=2)*					**21.0**	*large*

Since u(T) is a significant contribution to the total uncertainty, and is different for each resistor, the following table summarises the temperature contribution and the total uncertainty for each resistor.

Resistor	u(T)	c_T (10^{-9} K^{-1})	$U_T(R_X)$	**R_X(k=2)**
267919	0.01	-483.4	-4.8	**15.3**
262767	0.01	-35.7	-0.4	**12.0**
268168	0.01	-635.6	-6.4	**17.5**
A2030397	0.1	79.5	8.0	**21.0**

Note: The total uncertainty is likely to be a slight over-estimate due to a correlation between $u(R_Q)$ and $u(K_{CCC})$. This is because certain types of CCC winding leakage error can affect the agreement between measurements on two QHR plateaux as well as loop-closure measurements.

15. NML Measurement Uncertainty Analysis

Since a multivariate approach, employing statistical estimation, is used to arrive at the values of the unknown resistors, it is not possible to present the entire uncertainty budget in the scheme proposed in Annex 3 of the measurement protocol. Instead, the uncertainties of the main input quantities are presented in the table below. Note that all values refer to fractional deviations from the nominal values of the resistors or the ratios form their nominal values. For standard uncertainties arrived at by a type A evaluation, a value of 10^4 is ascribed to the degrees of freedom. The combined standard uncertainty and correlation coefficient for the measurement results are also reported below.

Quantity X_i	Estimate x_i	Standard uncertainty $u(x_i)$	Prob. Distr. / method of evaluation(A,B)	Degree of freedom ν_i
Bridge Reading	$+5.76 \times 10^{-6}$	5.0×10^{-8}	Type A	24
Calibration Correction to Bridge	$+0.03 \times 10^{-6}$	1.0×10^{-8}	Normal/type B	10000
Bridge correction for non-linearity, drift and temperature	0.00×10^{-6}	12×10^{-8}	Uniform/type B	10000
Correction for leakage resistance	0.00×10^{-6}	2.0×10^{-8}	Normal/ type B	10000
Certified value of reference 10 □ resistor	-5.30×10^{-6}	19×10^{-8}	Normal/ type B	10000
Drift correction to reference resistor	0.00×10^{-6}	1×10^{-8}	Normal/ type B	10000
Temperature correction to reference resistor	0.00×10^{-6}	1×10^{-8}	Uniform/ type B	10000
R_X	$+ 0.49 \times 10^{-6}$			4700

Table: Estimates and Standard Uncertainties of the Input Quantities

Combined Standard Uncertainty : $2.1 \times 10^{-7} * R_X$

Correlation coefficient between the measured values of any two of the four unknown resistors is 98%.

16. LNE

Because the resistance measurement is not perfect, the master equations are written including the main and significant correction terms:

$$R_X = (R_{K\text{-}90}/2)\mathrm{k}_W(1+\alpha Q+\alpha \mathrm{k}_W+\alpha_S+\alpha gl+\alpha dl)[1 + \varepsilon\, \mathrm{k}_W']$$

$$\mathrm{k}_W = N_S/N_P = 15/1936 \qquad \mathrm{k}_W' = N_A/N_S = 15/15$$

$$\varepsilon = \varepsilon^- + (\varepsilon^+ - \varepsilon^-)\mathrm{k}_V(1+\alpha \mathrm{k}_{VC}+\alpha \mathrm{k}_{VNL})$$

$$\mathrm{k}_V = |V^-|/(|V^+| + |V^-|)$$

With:

Quantities	**Origin of the corrections**
αQ	Quantization error
$\alpha \mathrm{k}_W$	Winding ratio error
α_S	SQUID Open loop finite gain error
αgl	Leakage to ground
αdl	Direct leakage
$\alpha \mathrm{k}_{VC}$	Voltage ratio error due to primary current drift
$\alpha \mathrm{k}_{VNL}$	Voltage ratio error due to the non linearity of the voltage measurement

Type B standard uncertainty budget

Quantity X_i	**Estimate** x_i **x 10^9**	**Relative standard uncertainty** $U(x_i)$ **x 10^9**	**Probability distribution/ Method of evaluation (A,B)**	**Sensitivity coefficient** C_i **(Ω)**	**Standard uncertainty contribution** $U_i(R_X)$ **(μΩ)**	**Degree of freedom** ν_i
αQ	0	0.5	Gaussian/B	100	0.05	8
$\alpha \mathrm{k}_W$	0	0.2	Gaussian/B	100	0.02	5
α_S	0	0.03	Gaussian/B	100	0.003	5
αgl	+0.4	0.2	Gaussian/B	100	0.02	6
αdl	0	0.2	Rectangular/B	100	0.02	8
$\alpha \mathrm{k}_{VC}$	0	4×10^5	Gaussian/B	2.510^{-5}	0.01	8
$\alpha \mathrm{k}_{VNL}$	0	2×10^5	Gaussian/B	2.510^{-5}	0.005	8
Max[U(ε^+), U(ε^-)]	0	0.35	Gaussian/B	100	0.035	17
$\boldsymbol{R_X}$					**0.072**	ν_{eff}**=26**

17. SMD

Mathematical model:

$$R_{100} = R_S \cdot A$$

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_S	100	$18\cdot10^{-9}$	normal/B	1	$18\cdot10^{-9}$	50
δR_{S1}	0	$11{,}5\cdot10^{-9}$	rect/B	1	$11{,}5\cdot10^{-9}$	inf
δR_{S2}	0	$0{,}06\cdot10^{-9}$	rect/B	1	$0{,}06\cdot10^{-9}$	inf
δR_{S3}	0	$20\cdot10^{-9}$	normal/B	1	$20\cdot10^{-9}$	50
δR_{S4}	0	$23{,}1\cdot10^{-9}$	rect/B	1	$23{,}1\cdot10^{-9}$	inf
$\delta \mathit{Lin}$	0	$28{,}9\cdot10^{-9}$	rect/B	1	$28{,}9\cdot10^{-9}$	inf
$\delta \mathit{Leak}$	0	$5{,}77\cdot10^{-9}$	rect/B	1	$5{,}77\cdot10^{-9}$	inf
δR_{x1}	0	$11{,}5\cdot10^{-9}$	rect/B	1	$11{,}5\cdot10^{-9}$	inf
δR_{x2}	0	$0{,}06\cdot10^{-9}$	rect/B	1	$0{,}06\cdot10^{-9}$	inf
A	1	$7{,}71\cdot10^{-9}$	normal/A	1	$7{,}71\cdot10^{-9}$	5
R_{100}	100 Ω				$49{,}5\cdot10^{-9}$	1000
		RSS of Type A uncertainties			$7{,}71\cdot10^{-9}$	
		RSS of Type B uncertainties			$48{,}9\cdot10^{-9}$	

18. CMI

Mathematical model:

Evaluation of error of resistance and of effective degrees of freedom

Equation used for evaluation of the uncertainty of an unknown resistor:

$$\boxed{\begin{aligned} & R_X + \delta R_{XT} + \delta R_{XW} + \delta R_{XAT} + \delta R_C + \delta R_{IT} = \\ & = \left(R_S + \delta R_{SD} + \delta R_{ST} + \delta R_{SW} + \delta R_{SAT}\right) \times \left(P + \delta QHR_{ND} + \delta QHR_P + \delta QHR_L\right) \end{aligned}}$$

or:

$$\boxed{\begin{aligned} R_X = & \left(R_S + \delta R_{SD} + \delta R_{ST} + \delta R_{SW} + \delta R_{SAT}\right) \times \left(P + \delta QHR_{ND} + \delta QHR_P + \delta QHR_L\right) - \\ & - \delta R_{XT} - \delta R_{XW} - \delta R_{XAT} - \delta R_C - \delta R_{IT} \end{aligned}} \qquad (2)$$

where (see also calibration certificates from CMI):

R_X - unknown resistor
R_S - reference standard
P - ratio R_X / R_S

I. Unknown resistor

δR_{XT} - error of the R_X due to a temperature deviation of the oil bath

δR_{XW} - error of the R_X due to a power dissipation (effect of $P = R_X I^2$ heating)

δR_{XAT} - error of the R_X due to an atmospheric pression

δR_C - error due to a connection

δR_{IT} - error due to an influence of the transport of the calibrated resistor between laboratories

II. Reference standard

δR_{SD} - drift in value of the reference standard since its last calibration

δR_{ST} - error of the R_S due to a temperature deviation of the oil bath

δR_{SW} - error of the R_S due to a power dissipation (effect of $P = R_S I^2$ heating)

δR_{SAT} - error of the R_S due to an atmospheric pression

III. Measurement system (CRYOGENIC QHR 2010)

δQHR_{ND} - the error in the detector circuit is associated with the stability of the zero setting of the detector, the stability of the thermal emfs in the circuit, and the resolution of the detector system of a CCC (measuring of the ratio 100 Ω (QHR) / 100 Ω)

δQHR_p - these include the errors of the ratio caused by the instability of the turns ratio of a CCC (turns ratio accuracy)

δQHR_L - these include the errors of the linearity caused by the non - linearity of the turns ratio of a CCC

Sensitivity coefficients and effective degrees of freedom are calculated from eq. (2) accordingly to [1].

References

[1]: *International Organization for Standardization*: Guide to the Expression of Uncertainty in Measurement, 1993

Literatura:

[1]: Guide to the Expression of Uncertainty in Measurement; International Organization for Standartization, 1993
[2]: Vyjadřování nejistot měření při kalibracích; Dokumenty EAL, č. publikace: EAL – R2

Příloha: Tabulka

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncer-tainty contri-bution $u_i(R_x)$	Degree of freedom ν_i
R_S	100. 000 414 Ω	35 x 10^{-9}	normal	1	35 x 10^{-9}	∞
δ R_{SD}	+ 50 x 10^{-9}	50 x 10^{-9} / √ 3	rectangular	1	29 x 10^{-9}	∞
δ QHR_{ND}	0	10 x 10^{-9} / √ 3	rectangular	100. 000 414 Ω	6 x 10^{-9}	∞
δ QHR_p	0	1 x 10^{-9} / √ 3	rectangular	100. 000 414 Ω	0.6 x 10^{-9}	∞
δ QHR_L	0	2 x 10^{-9} / √ 3	rectangular	100. 000 414 Ω	1 x 10^{-9}	∞
δ R_{XT}	0	5 x 10^{-9} / √ 3	rectangular	-1	3 x 10^{-9}	∞
δ R_{ST}	0	5 x 10^{-9} / √ 3	rectangular	1	3 x 10^{-9}	∞
δ R_{XW}	0	5 x 10^{-9} / √ 3	rectangular	-1	3 x 10^{-9}	∞
δ R_{SW}	0	5 x 10^{-9} / √ 3	rectangular	1	3 x 10^{-9}	∞
δ R_{XAT}	0	2 x 10^{-9} / √ 3	rectangular	-1	1 x 10^{-9}	∞
δ R_{SAT}	0	2 x 10^{-9} / √ 3	rectangular	1	1 x 10^{-9}	∞
δ R_C	0	2 x 10^{-9} / √ 3	rectangular	-1	1 x 10^{-9}	∞
δ R_{IT}	0	30 x 10^{-9} / √ 3	rectangular	-1	17 x 10^{-9}	∞
repeatability u_A	99. 999 658 Ω	1 x 10^{-9}	normal	1	1 x 10^{-9}	29
R_X		99. 999 658 Ω			50 x 10^{-9}	ν_{eff} 181 250 000

$$R_{100} = R_S \cdot k_{read}$$

Table 1: uncertainty for Tegam SR102 #A2030397

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_S	100 Ω	$35\cdot10^{-9}$	normal/B	1	$35\cdot10^{-9}$	inf
δR_{SD}	0	$29\cdot10^{-9}$	rect/B	1	$29\cdot10^{-9}$	inf
δQHR_{ND}	0	$6\cdot10^{-9}$	rect/B	1	$6\cdot10^{-9}$	inf

δQHR_{P}	0	$0{,}6\cdot10^{-9}$	rect/B	1	$0{,}6\cdot10^{-9}$	inf
δQHR_{L}	0	$1\cdot10^{-9}$	rect/B	1	$1\cdot10^{-9}$	inf
δR_{XT}	0	$3\cdot10^{-9}$		1	$3\cdot10^{-9}$	inf
δR_{ST}	0	$3\cdot10^{-9}$		1	$3\cdot10^{-9}$	inf
δR_{XW}	0	$3\cdot10^{-9}$		1	$3\cdot10^{-9}$	inf
δR_{SW}	0	$3\cdot10^{-9}$		1	$3\cdot10^{-9}$	inf
δR_{XAT}	0	$1\cdot10^{-9}$		1	$1\cdot10^{-9}$	inf
δR_{SAT}	0	$1\cdot10^{-9}$	rect/B	1	$1\cdot10^{-9}$	inf
δR_{C}	0	$1\cdot10^{-9}$	rect/B	1	$1\cdot10^{-9}$	inf
δR_{IT}	0	$17\cdot10^{-9}$		1	$17\cdot10^{-9}$	inf
k_{read}	1	$2\cdot10^{-9}$	normal/A	1	$2\cdot10^{-9}$	29
R_{100}	100 Ω				$50\cdot10^{-9}$	inf
		RSS of Type A uncertainties			$2\cdot10^{-9}$	
		RSS of Type B uncertainties			$50\cdot10^{-9}$	

19. NMISA

Mathematical model:

$$R_{100} = (R_{\mathrm{ref}} + \delta R_{\mathrm{drift}} + \delta R_{\mathrm{bridge}} + \delta R_{\mathrm{temp}} + \delta R_{\mathrm{power}})\cdot R_{\mathrm{bridge}} + \delta R_{\mathrm{meas}}$$

Uncertainty budget

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_{ref}	10	$300\cdot10^{-9}$	normal/B	1	$300\cdot10^{-9}$	inf
$\delta R_{\mathrm{drift}}$	0	$14\cdot10^{-9}$	normal/B	1	$14\cdot10^{-9}$	inf
$\delta R_{\mathrm{bridge}}$	0	$9{,}2\cdot10^{-9}$	rect/B	1	$9{,}2\cdot10^{-9}$	inf
R_{bridge}	10	$5{,}8\cdot10^{-9}$	rect/B	1	$5{,}8\cdot10^{-9}$	inf
δR_{Temp}	0	$12{,}1\cdot10^{-9}$	rect/B	1	$12{,}1\cdot10^{-9}$	inf
$\delta R_{\mathrm{power}}$	0	$1{,}2\cdot10^{-9}$	rect/B	1	$1{,}2\cdot10^{-9}$	inf
δR_{meas}	0	$20\cdot10^{-9}$	normal/A	1	$20\cdot10^{-9}$	5
R_{100}	100 Ω				$301\cdot10^{-9}$	inf
		RSS of Type A uncertainties			$20\cdot10^{-9}$	
		RSS of Type B uncertainties			$300\cdot10^{-9}$	

20. NMi

The model equation used in the uncertainty analysis for the 1 mW measurements is as follows:

$R_{100} = R_{QHE} * (1 + \delta R_{Qquant}) * (1 - \delta R_{100env}) * \mathrm{CCCratio}$

with:

$\mathrm{CCCratio} = (N_p / N_s) * r * (1 + \delta r_{system}) * (1+ \delta r_{i24}) * (1 + \delta r_{leak}) * (1+ \delta r_{wind}) * (1 + \delta r_{cal})$

where the occurring quantities are explained as follows:

Quantity	Definition
R_{100}	Value of the unknown 100 Ohm resistor
R_{QHE}	Quantum Hall Effect resistance value; a constant, equal to 12906.4035 Ω for the *i* = 2 plateau
δR_{Qquant}	Imperfect quantisation of the QHE. Estimated to be less than 4 parts in 10^9 (see paragraph 3.1)
δR_{100env}	Residual effect of environment on the 100 Ohm resistor estimated to be a most 3 parts in 10^9; the main effect of the varying environment on the resistor is already contained in the type A uncertainty of the *r* values.
CCCratio	Resistance ratio measured by the CCC
N_p	Number of primary windings (exact)
N_s	Number of secondary windings (exact)
r	Resistance ratio as found in the CCC measurement (by the analysis program) based on analysing the CCC feedback signals for plus and minus current
δr_{system}	System error of complete CCC system; estimated to be at most 7 parts in 10^9 based on the tests of the system (see paragraph 3.2). The estimate is based on the consistency of 'triangle' measurements, and includes residual systematic effects not mentioned below.
δr_{i24}	Inconsistency of *i* = 2 and *i* = 4 QHE measurements, see the graphs in Appendix A for the results of each of the four resistors. In the calculation, 70 % of the apparent difference in the *i* = 2 and *i* = 4 value is taken as the maximum estimated contribution to the uncertainty.
δr_{leak}	Leakage effect in CCC. Estimated to be less than 3 parts in 10^9, since the measured leakage resistance of the connecting cables and of some parts inside the CCC bridge is larger than 10^{13} Ω.
δr_{wind}	Winding error of CCC. Based on binary calibration of the windings of the CCC this is estimated to be less than 1 part in 10^9.
δr_{cal}	Calibration of the CCC feedback signal. Uncertainty is better than 2 parts in 10^4 of the feedback signal. Given the measured deviation from nominal value of the resistors, this uncertainty varies from 2.5 to 4 parts in 10^9.

This results in the following uncertainty budget for the TEGAM resistor:

Quantity	Value	Standard Uncertainty	Degrees of Freedom	Distribution	Sensitivity Coefficient	Uncertainty Contribution
R_{QHE}	12906.4035 Ω					
δR_{Qquant}	0.0	$2.31 \cdot 10^{-9}$	$\tilde{1}$	rect; B	100	$231 \cdot 10^{-9}$ Ω
δR_{100env}	0.0	$1.73 \cdot 10^{-9}$	$\tilde{1}$	rect; B	-100	$-173 \cdot 10^{-9}$ Ω
N_p	16.0					
N_s	2065.0					
r	0.999988538	$1.40 \cdot 10^{-9}$	14	normal; A	100	$140 \cdot 10^{-9}$ Ω

Quantity	**Value**	**Standard Uncertainty**	**Degrees of Freedom**	**Distribution**	**Sensitivity Coefficient**	**Uncertainty Contribution**
δr_{system}	0.0	$4.04 \cdot 10^{-9}$	50	rect; B	100	$404 \cdot 10^{-9}\ \Omega$
δr_{i24}	0.0	$1.62 \cdot 10^{-9}$	50	rect; B	100	$162 \cdot 10^{-9}\ \Omega$
δr_{leak}	0.0	$1.73 \cdot 10^{-9}$	$\tilde{1}$	rect; B	100	$173 \cdot 10^{-9}\ \Omega$
δr_{wind}	0.0	$577 \cdot 10^{-12}$	$\tilde{1}$	rect; B	100	$58 \cdot 10^{-9}\ \Omega$
δr_{cal}	0.0	$1.44 \cdot 10^{-9}$	$\tilde{1}$	rect; B	100	$144 \cdot 10^{-9}\ \Omega$
R_{100}	100.0000432 Ω	$5.9 \cdot 10^{-7}\ \Omega$	192			

For the resistance values at the current level of 4.8 mA an extra contribution $\delta R_{100power}$ is added to the uncertainty budget. This has the value given in Table 1, with an uncertainty ($k = 1$, normal) of 7 parts in 10^9. For the resistor given above, this increases the standard uncertainty from $5.9 \cdot 10^{-7}\ \Omega$ to $7.1 \cdot 10^{-7}\ \Omega$ (corresponding to a total uncertainty of 7 parts in 10^9). At the same time, due to the low number of degrees of freedom in the power effect measurement, the degrees of freedom in the final result reduce from 192 to 17.

21. BEV

Mathematical model:

$$R_{100} = (R_{\mathrm{ref}} + \delta R_{\mathrm{drift}} + \delta R_{\mathrm{bridge}} + \delta R_{\mathrm{temp}} + \delta R_{\mathrm{pressure}})\cdot\ \delta R_{\mathrm{meas}}$$

uncertainty budget

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_{ref}	100	$150\cdot10^{-9}$	normal/B	1	$150\cdot10^{-9}$	50
$\delta R_{\mathrm{drift}}$	0	$28{,}9\cdot10^{-9}$	rect/B	1	$28{,}9\cdot10^{-9}$	inf
$\delta R_{\mathrm{bridge}}$	0	$57{,}7\cdot10^{-9}$	rect/B	1	$57{,}7\cdot10^{-9}$	inf
δR_{Temp}	0	$3{,}2\cdot10^{-3}$ K	normal/B	$5\cdot10^{-6}$ /K	$16\cdot10^{-9}$	50
$\delta R_{\mathrm{press}}$	0	0,58 hPa	rect/B	$1\cdot10^{-9}$ /hPa	$0{,}6\cdot10^{-9}$	inf
δR_{meas}	1	$19{,}2\cdot10^{-9}$	normal/A	1	$19{,}2\cdot10^{-9}$	14
R_{100}	100 Ω				$165\cdot10^{-9}$	74
		RSS of Type A uncertainties			$19{,}2\cdot10^{-9}$	
		RSS of Type B uncertainties			$164\cdot10^{-9}$	

22. SASM

Mathematical model:

$$R_{100} = [R_S + \delta R_{dr} + R_S\cdot\ \alpha(t_m\text{-}t_{cal}) + R_S\cdot\alpha\cdot\delta t]\cdot r_{RES}\cdot r_m\cdot r_{stab}$$

uncertainty budget

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_S	100	$250\cdot10^{-9}$	normal/B	1	$250\cdot10^{-9}$	50
δR_{dr}	0	2,64	rect/B	$140\cdot10^{-9}$	$370\cdot10^{-9}$	inf
t_m	23,047	$0{,}125\cdot10^{-3}$	rect/B	$0{,}289\cdot10^{-3}$	$36{,}1\cdot10^{-9}$	inf
t_{cal}	23,00	$0{,}751\cdot10^{-3}$	rect/B	$0{,}289\cdot10^{-3}$	$217\cdot10^{-9}$	inf
δt	-0,0072	0,542	normal/B	$90\cdot10^{-9}$	$48{,}8\cdot10^{-9}$	50
r_{res}	1	$4{,}08\cdot10^{-9}$	triangular/B	1	$4{,}08\cdot10^{-9}$	inf
r_m	1	$0{,}1\cdot10^{-9}$	normal/A	1	$0{,}1\cdot10^{-9}$	59
r_{stab}	1	$327\cdot10^{-9}$	triangular/B	1	$327\cdot10^{-9}$	inf
R_{100}	100 Ω				$598\cdot10^{-9}$	1623
		RSS of Type A uncertainties			$0{,}5\cdot10^{-9}$	
		RSS of Type B uncertainties			$598\cdot10^{-9}$	

23. VMT

EUROMET key comparison EUROMET.EM-K10 "100 Ω Standard Resistor"
Institute: State Metrology Service/Semiconductor Physics Institute (VMT/PFI), Lithuania

Uncertainty budget for Rx s/n number 263417
Date: 2003.12.04

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution/ method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(Rx)$	Degree of freedom n_i
X1	0,99999995	1,000E-07	Normal / B	99,9995657	1,000E-07	infinity
X2	-3,530E-09	2,502E-01	Rectangular / B	99,9995616	8,833E-10	12
X3	0	2,887E-09	Rectangular / B	99,9995707	2,887E-09	12
X4	9,999967766	6,642E-09	Normal / A	9,99998841	6,642E-09	140
X5	0	7,500E-09	Normal / A	-99,9995592	-7,500E-09	infinity
X6	9,999988817	3,634E-09	Normal / A	9,9999671	3,634E-09	140
X7	0	8,000E-09	Normal / A	-99,9995606	-8,000E-09	infinity
X8	1	2,887E-09	Rectangular / B	199,999122	5,774E-09	infinity
X9	0	1,085E-07	Rectangular / B	-1,00000004	-1,085E-09	12
Rx	99,99956048				1,011E-07	4951196

Model function:
Rx = (X1 + X2 +X3) * (X4 / (X5 + 1)) * (X6 / (X7 + 1)) * X8 * X8 - X9
where :
X_1 resistance of reference standard (Ohm)
X_2 drift of reference standard since last calibration (Ohm)
X_3 temperature correction of reference standard (Ohm)
X_4 10:1 ratio: intermediate standard / reference standard (relative)
X_5 bridge 10:1 ratio error (relative)
X_6 100:10 ratio: unknown standard / intermediate standard (relative)
X_7 bridge 100: ratio error (relative)
X_8 bridge linearity error (relative)
X_9 temperature correction of unknown standard (Ohm)
Rx resistance of unknown standard (Ohm)

24. LNMC

Uncertainty budget

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_S	100 Ω	$500\cdot10^{-9}$	normal/B	1	$500\cdot10^{-9}$	inf
k_drift	1	$460\cdot10^{-9}$	rect/B	1	$460\cdot10^{-9}$	inf
k_Temp	1	$580\cdot10^{-9}$	rect/B	1	$580\cdot10^{-9}$	inf
k_bridge	1	$170\cdot10^{-9}$	rect/B	1	$170\cdot10^{-9}$	inf
R_{100}	100 Ω				$910\cdot10^{-9}$	inf
		RSS of Type A uncertainties				
		RSS of Type B uncertainties			$910\cdot10^{-9}$	

25. EIM

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
Temperature sensor accuracy	0	0,03°C	rect/B	-1,9·10-5 Ω/°C	$-3\cdot10^{-7}$ Ω	inf
Temperature measurement repeatability	1	0,004°C	normal/A	-1,9·10-5 Ω/°C	$-1\cdot10^{-7}$ Ω	9
System Accuracy	0	$2\cdot10^{-6}$ Ω	normal/B	1	$20\cdot10^{-7}$ Ω	inf
Measurement mean	100 Ω	$2\cdot10^{-7}$ Ω	normal/A	1	$2\cdot10^{-7}$ Ω	98
R_{100}	100 Ω				$20\cdot10^{-7}$ Ω	$1,4\cdot10^{6}$
		RSS of Type A uncertainties			$2,2\cdot10^{-7}$ Ω	
		RSS of Type B uncertainties			$20\cdot10^{-7}$ Ω	

26. INRIM

Uncertainty budget

The model equation is:

$$R_X + \delta_{R_X} = R_H(1+\delta_{H,Q}+\delta_{H,L})\frac{N_S}{N_P}(1+\delta_r+\delta_I)\left[1+\beta(1+\delta_\beta+\frac{\delta_{\beta,0}}{\beta})\frac{N_C}{N_S}+\frac{V_D+\delta_V}{R_H I_P}\right] \qquad (1)$$

with the following meaning of the symbols:

R_X	unknown resistance;
δ_{Rx}	repeatability of the measurement;
R_H	quantum Hall resistance (i = 2);
$\delta_{H,Q}$	deviations of measured R_H from ideal value due to insufficient quantisation;
$\delta_{H,L}$	deviations of measured R_H from ideal value due to leakage and circuit bias current;
N_P, N_S, N_C	number of turns of the primary, secondary and compensation windings, respectively;
δ_r, δ_I	deviations of the current ratio from nominal, due to the CCC ratio error and to an imperfect current balance, respectively;
δ_β	ratio error of the Kelvin-Varley divider;
$\delta_{\beta,0}$	bias of the Kelvin-Varley divider;
V_D	voltage unbalance;
I_P	primary current;
δ_V	error of the voltage reading (uncompensated thermal voltages, detector resolution and instability).

All corrections in eq. (1) will be neglected. The following equation of the relative variance can be derived:

$$u^2(R_X) = u_A^2(R_X) + u_{H,Q}^2 + u_{H,L}^2 + u_r^2 + u_I^2 + \beta^2\left(\frac{N_C}{N_S}\right)^2 u_\beta^2 + \left(\frac{N_C}{N_S}\right)^2 u_{\beta,0}^2 + \frac{1}{(R_H I_P)^2}u_V^2 \qquad (2)$$

where $u_A(R_X)$ is a type A standard uncertainty.

Uncertainty budget for resistor 263417

Resistor number	Quantity X_i	Standard uncertainty $u(x_i)$	Prob. distr./ Type (A,B)	Sensitivity coefficient c_i	Rel. uncert. contribution $u_i(R_x)$	Deg. of freedom ν_i
263417	δ_{Rx}	2.1E-09	gauss / A	1	2.1E-09	5
	$\delta_{H,Q}$	4.0E-09	rett / B	1	4.0E-09	inf.
	$\delta_{H,L}$	3.9E-09	rett / B	1	3.9E-09	inf.
	δ_r	2.0E-09	rett / B	1	2.0E-09	inf.
	δ_I	12E-09	rett. /B	1	12E-09	inf.
	δ_β	1.2E-03	rett. / B	0.6E-06	0.72E-09	inf.
	$\delta_{\beta,0}$	0.34E-09	rett. / B	0.5	0.17E-09	inf.
	δ_V	2.0E-09 V	rett / B	2.22 V^{-1}	4.4E-09	12
					$u(R_X)$= 15E-09$^{(*)}$	ν_{eff} = 1142

27. INETI

Mathematical model:

$$R_x = (R_{S0} + \delta R_D + \delta R_{TS}) \cdot (Y_1 \cdot Y_2) - \delta R_{TX}$$

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
$R_{S0}+\delta R_D$	1	$202\cdot 10^{-9}$	rect/B	1	$117\cdot 10^{-9}$	inf
δR_{TS}	0	$0{,}27\cdot 10^{-9}$	rect/B	1	$0{,}27\cdot 10^{-9}$	inf
δR_{TX}	0	0	rect/B	1	0	inf
Y_1	10	$10{,}0\cdot 10^{-9}$	rect/B	1	$10{,}0\cdot 10^{-9}$	inf
Y_1	0	$21{,}6\cdot 10^{-9}$	normal/A	1	$21{,}6\cdot 10^{-9}$	11
Y_2	10	$10{,}0\cdot 10^{-9}$	rect/B	1	$10{,}0\cdot 10^{-9}$	inf
Y_2	0	$13{,}7\cdot 10^{-9}$	normal/A	1	$13{,}7\cdot 10^{-9}$	11
R_X	100 Ω				$120\cdot 10^{-9}$	inf
		RSS of Type A uncertainties			$14\cdot 10^{-9}$	
		RSS of Type B uncertainties			$119\cdot 10^{-9}$	

28. SIQ

Mathematical model:

$$R_X = R_S \cdot (1+k\_\text{tc})/(Ratio \cdot k\_\text{rep} + k\_\text{lin}))$$

R_S: reference standard resistor
k_tc: temperature coefficient of the standard resistor
Ratio: measured ratio
k_rep: repeatability
k_lin: linearity of the DCC bridge

Quantity	estimate	standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution	Degree of Freedom
R_S	1 kΩ	$210\cdot10^{-9}$	normal/B	1	$210\cdot10^{-9}$	inf
k_tc	0	$58\cdot10^{-9}$	rect/B	1	$58\cdot10^{-9}$	inf
Ratio	10	$58\cdot10^{-9}$	normal/A	-1	$58\cdot10^{-9}$	inf
k_rep	0	$10\cdot10^{-9}$	rect/B	-1	$10\cdot10^{-9}$	19
k_lin	0	$5{,}8\cdot10^{-9}$	rect/B	-1	$5{,}8\cdot10^{-9}$	inf
R_X	100 Ω				$230\cdot10^{-9}$	411
		RSS of Type A uncertainties			$10\cdot10^{-9}$	
		RSS of Type B uncertainties			$225\cdot10^{-9}$	