REPORT

EUROMET project no. 710

EUROMET.EM-S18 supplementary comparison of 1Ω and 10 kΩ resistance standards

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Abstract:

This report presents the result of the supplementary comparison of measurements on 1 Ω and 10 kΩ resistance standards performed by the national institutes of Sweden, Norway, Denmark and Finland.

Content

1. Introduction

Many national laboratories use the quantum Hall effect as a means to establish a reference standard of dc resistance. When the lowest possible uncertainty is desirable for the scaling from the Quantum Hall Resistance (QHR) to conventional resistance standards a cryogenic current comparator (CCC) is often used. The scaling from QHR to a 100 Ω resistance standard is often the first step in the scaling process. A EUROMET key comparison of "Resistance at 100 Ω", EUROMET.EM-K10 began in the spring of 2003 and will soon be completed. Comparison of the scaling from QHR or 100 Ω to 1 Ω and 10 kΩ is also very important. Such a BIPM comparison was carried out in 1991.

In May to November 2003 the national institutes of Denmark, Finland, Norway and Sweden performed a supplementary comparison on 1 Ω and 10 kΩ resistance standards. This comparison allows for a clear and unequivocal comparison of the measurement results and will show the equivalence of measuring results obtained with various quantized Hall systems or other measurement systems for resistance in the participating national institutes.

2. Participants and schedule

There were four laboratories participating in this comparison. The laboratories are listed in Table 1.

Laboratory		**Country**
JV	Norwegian Metrology Service	Norway
DFM	Danish Institute for Fundamental Metrology	Denmark
MIKES	Centre for Metrology and Accreditation	Finland
SP	Swedish National Testing and Research Institute	Sweden

Table 1: List of participants

2.1 Comparison schedule

Circulation Time Schedule
Supplementary Comparison EUROMET project 710
(2003-05-20)
Set A1 & A2

Institution	Country	Start date	Time for measurement and transportation	Comment
SP (start 10 kΩ)	Sweden	May 2003	4 weeks	10 kΩ
JV (start 1 Ω)	Norway	June 2003	4 weeks	10 kΩ +1 Ω
DFM	Denmark	July 2003	4 weeks	10 kΩ +1 Ω
MIKES	Finland	August 2003	4 weeks	10 kΩ +1 Ω
SP (pilot 10 kΩ)	Sweden	September 2003	4 weeks	10 kΩ +1 Ω
JV (pilot 1 Ω)	Norway	November 2003	4 weeks	1 Ω

2.2 Organisation of the comparison

For this supplementary comparison JV was pilot laboratory for the 1Ω travelling standards and SP for the 10 kΩ travelling standard. The comparison was performed in parallel with the ongoing 100 Ω EUROMET key comparison, EUROMET.EM-K10. The 1 Ω and 10 kΩ travelling standards were circulated together with two 100 Ω resistors. All travelling standards were transported by car and kept in temperature regulated enclosures. After circulation the travelling standards were returned to the pilot laboratories.
Each laboratory had 4 weeks to perform the measurements.

3. The travelling standards and measurement instructions

3.1 Description of the travelling standards

The travelling resistance standards were one 10 kΩ ESI standard and two CSIRO 1Ω standards. The 10 kΩ standard had a temperature stabilised enclosure with internal temperature measurement using a Pt 100 thermometer. The 1Ω standards where kept in oilbaths at each laboratory but transported in a temperature regulated box. DFM used the temperature regulated box instead of an oilbath.

Set A1, 10 kΩ (SP):

- Standard Resistor 10 kΩ ESI , “QHR transfer standard” with temperature stabilised enclosure, S/N SP910102
- Pt 100 measurement cable
- Power supply 12V
- Instruction Manual.

Set A2, CSIRO 1 Ω (JV):

- Standard Resistor 1 Ω CSIRO, S/N 64179,
- Standard Resistor 1 Ω CSIRO, S/N 64187
- Temperature regulated transport box

3.2 Measurement instructions

The measurand is the value of the resistance at DC, based on the conventional value of the von Klitzing constant R_{K-90} = 25 812,807 Ω. In practice, DC means that the waiting time between the end of a current reversal and the start of data acquisition should not be shorter than 8 s for the 10 kΩ travelling standard and 4 s for the 1 Ω travelling standards. It is up to the participants to either make a guarded measurement where the resistor case is guarded, leave the resistor floating with respect to the case, or connect one point of the resistor to its case. The solution adopted should be reported. A short description of the methods used should be included in the final report together with the measurement results.

After installation of the travelling standards in their respective bath and thermostat in the laboratory, a minimum settling time of two days is required for the 1 Ω travelling

standards and three days for the 10 kΩ travelling standard.
The measurements should be carried out with the following preferred conditions:

- direct or indirect (using a 100 Ω resistor) comparison with the QHR using a CCC bridge,
- aimed uncertainty less than $5 \cdot 10^{-8}$ (95% confidence level),
- 50 mA through the 1 Ω resistor,
- applied measurement voltage 10 kΩ standard 0.5V – 1V (max applied 3 V)
- oil bath temperature for the 1Ω standards (23,00 ± 0,01) °C
- ambient temperature for the10 kΩ enclosure (20,0 – 24,0) °C

Laboratories not using the QHR as their primary standard of resistance shall measure the resistors with their respective best measurement capability. For these measurements the source of traceability has to be included in the measurement report.

The travelling standard temperature and ambient pressure should be recorded and reported as well as the height of oil above the top plate of the resistors in the oil bath. If known, the density of the oil in the oil bath should be reported. Resistors may have a long thermal time constant (several hours)! The measurements should be made at different dates during the period in the laboratory. The temperature and pressure coefficients of the travelling standard have been determined to allow for corrections. They are not given in the protocol.

4. Method of measurements

Different measurement set-ups were used in the comparison. One of the laboratories used a conventional DCC bridge and has traceability to BIPM through 10 kΩ resistor standards. The scaling from 10 kΩ to 1 Ω is achieved primarily via two Hamon transfer devices.
The three remaining laboratories have traceability to their own QHR standards. The travelling standards were compared directly against 100 Ω standards using CCC resistance bridges. The 100 Ω standards were calibrated against QHR. The 10 kΩ travelling standard pilot laboratory (SP) measurements was directly against the QHR using the dc-CCC bridge. Two of these three laboratories use the same type of commercial dc-CCC bridge. These bridges are based upon a National Physical Laboratory (NPL) design [1]. The third laboratory uses a self designed ac-CCC bridge (0.1 Hz to 0.3 Hz) [2].
Further details are given in appendix A.

5. Behaviour of the travelling standards

The pilot laboratories measured the respective travelling standards before and after the circulation. From these measurements the drift and stability of the travelling standards have been evaluated. The estimated drift rate has also been compared with the historical drift rate of the travelling standards. All three travelling standards had a drift rate as expected from the historical data during the comparison. The temperature regulation of the 10 kΩ standard also showed a very good behaviour with small temperature variations between the measurements at the participating laboratories.

6. Measurement results

6.1 Results of the participating institutes

Each laboratory has reported a mean value, R_{lab}, the temperature, the mean date, ambient conditions and the measurement uncertainty for each travelling standard. The single values from which the mean values are estimated and complete uncertainty budgets are also reported.
The travelling standards have small temperature dependencies. The two 1 Ω travelling standards also have a small pressure dependence. These two effects are corrected for in tables 2-4. The corrected values for all three resistors are shown in figures 1-3. Due to the small variations of the temperature of the 10 kΩ travelling standard enclosure the temperature corrections were for all laboratories below 3 nΩ/Ω for the 10 kΩ travelling standard.

Laboratory	**Date**	**1 Ω sn. 64179 (Ω)**	**Exp. uncertainty (nΩ)**
JV	04.06.2003	0,999 999 004	34
DFM	07.07.2003	0,999 999 022	540
MIKES	09.08.2003	0,999 998 983	36
SP	08.10.2003	0,999 998 973	39
JV	29.10.2003	0,999 999 013	34

Table 2: Corrected measurement results for 1Ω travelling standard sn. 64179

Laboratory	**Date**	**1 Ω sn. 64189 (Ω)**	**Exp. uncertainty (nΩ)**
JV	04.06.2003	0,999 993 273	34
DFM	07.07.2003	0,999 993 302	540
MIKES	09.08.2003	0,999 993 251	36
SP	08.10.2003	0,999 993 250	72
JV	30.10.2003	0,999 993 293	34

Table 3: Corrected measurement results for 1Ω travelling standard sn. 64189

Laboratory	**Date**	**10 kΩ sn. SP910102 (Ω)**	**Exp. uncertainty (mΩ)**
SP	15.05.2003	9 999,930 65	0,26
JV	03.06.2003	9 999,930 62	0,09
DFM	25.06.2003	9 999,930 28	1,78
MIKES	04.08.2003	9 999,930 70	0,50
SP	28.09.2003	9 999,930 81	0,26

Table 4: Corrected measurement results for 10 kΩ travelling standard sn.SP910102

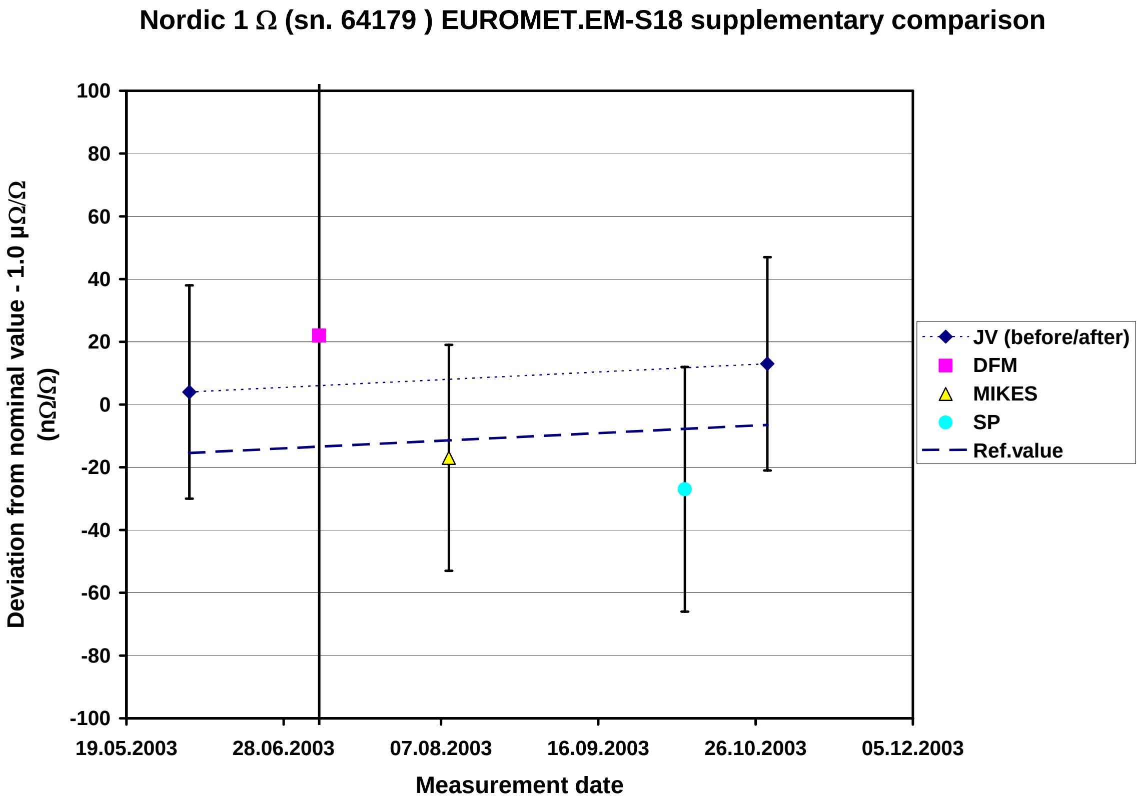


Figure 1: Results for 1Ω travelling standard , sn. 64179, with expanded uncertainty

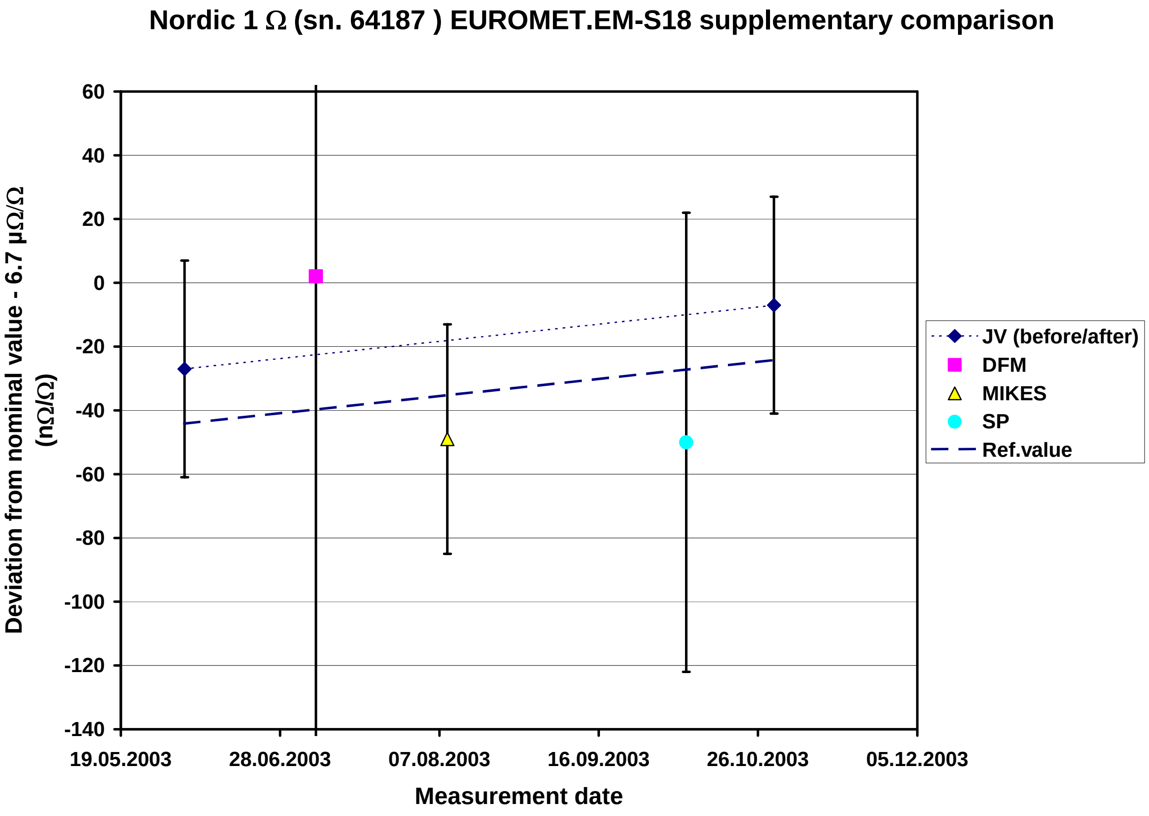


Figure 2: Results for 1Ω travelling standard , sn. 64187, with expanded uncertainty

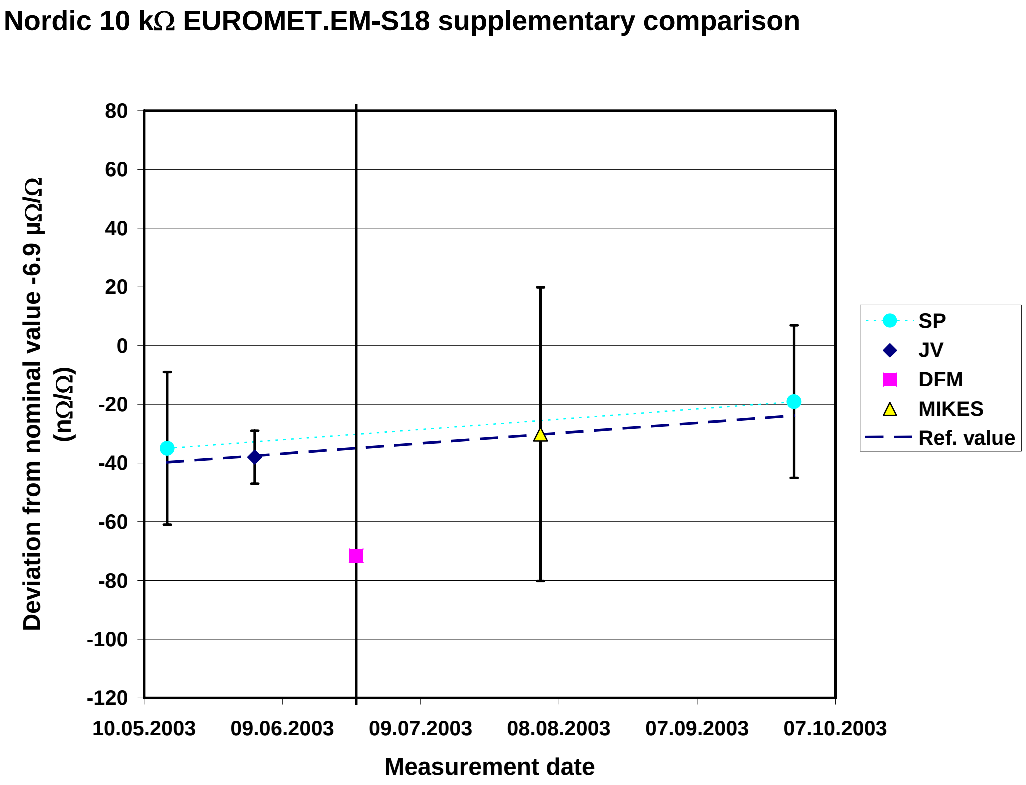


Figure 3: Results for 10 kΩ travelling standard, sn. SP910102, with expanded uncertainty

6.2 Calculation of the reference value and its uncertainty

For a stable reference standard it is possible to estimate the reference value using the method of least squares. Stability of the travelling standards was provided by organisation of careful transportation and we assume that change of the resistors was small. The reference value can be determined by the weighted mean of the laboratories measurements in that case.
The measurements results from the pilot laboratories and old drift data indicate that some drift in the standards has to be included in the evaluation of the reference values. Our proposal for the determination of the reference value is to determine the weighted fit to a straight line, *Y=a+bX*, where only parameter *a* is fitted. The parameter *b* is calculated from the pilot laboratory results.

$$a = \frac{\sum_i w_i y_i - b\sum_i w_i x_i}{\sum_i w_i}$$

where the weight w_i is

$$w_i = \frac{c}{u(x_i)^2}$$

with *c=0,5* for the pilot laboratory and *c=1* for the other laboratories.

In addition to the weight based on the measurement uncertainty, the pilot laboratories are given a weight of *0,5* because they contribute with two measurement results. The calculated reference values are shown as a dashed line in figure 1-3. The standard uncertainty of the reference value is determined from

$$\frac{1}{u^2(x_{ref})} = \frac{1}{u^2(x_1)} + \cdots\cdots + \frac{1}{u^2(x_N)}$$

6.3 Degree of equivalence (DoE) of the participating institutes with respect to the reference value

The laboratories degree of equivalence with respect to the reference value are calculated according to the CCEM guidelines [3].

For institute i=1, …., N the degree of equivalence is given by

$$d_i = x_i - x_{ref}$$

with the expanded uncertainty

$$U(d_i) = 2\,u(d_i)$$

when $u(d_i)$ is given by

$$u^2(d_i) = u^2(x_i) - u^2(x_{ref})$$

Laboratory	**DoE ± U(DoE)**		
	1Ω sn 64179 (nΩ/Ω)	**1Ω sn 64187 (nΩ/Ω)**	**10 kΩ sn SP910102 (nΩ/Ω)**
JV	19 ±27	17 ±25	0 ± 3
DFM	35 ±540	42 ±539	-37 ± 178
MIKES	-6 ±29	-14 ±27	0 ± 49
SP	-19 ±33	-23 ±68	5 ± 25

Table 5: Degrees of equivalence (DoE) with expanded uncertainty for the individual travelling standards.

Since two travelling standards have been used in the 1 Ω measurements, it is normal to report only one combined DoE. The two 1 Ω results for each laboratory can be defined to be $R_{1,i}$ and $R_{2,i}$. The measurants we want to compare are the average results from the two 1 Ω measurements.

$$R_{avg,i} = \frac{R_{1,i} + R_{2,i}}{2}$$

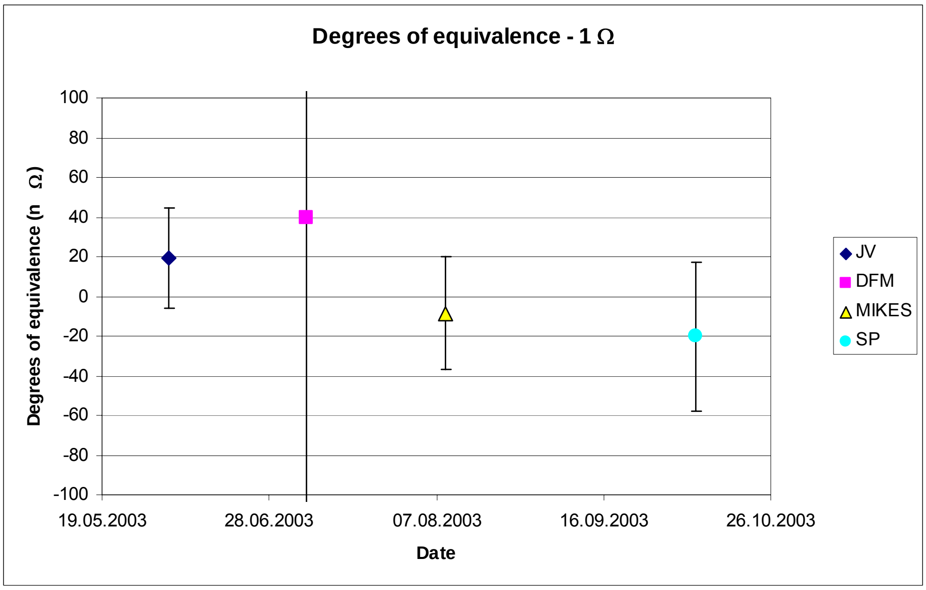
with the uncertainty

$$u(R_{avg,i}) = \sqrt{u^2(R_{0,i}) + \frac{1}{4}\left(u^2(\partial R_{1,i}) + u^2(\partial R_{2,i})\right)}$$

$R_{0,i}$ are the type B components with an uncertainty u($R_{0,i}$) which represent the system uncertainty common for both 1 Ω measurements. $\delta R_{1,i}$ and $\delta R_{2,i}$ are the type A components for each 1 Ω measurements with the uncertainty u($\delta R_{1,i}$) and u($\delta R_{2,i}$). Then with $R_{avg,i}$ and $u(R_{avg,i})$ for each laboratory a reference value and DoE among the laboratories are calculated in the same way as for the individual 1 Ω standards shown above.

Laboratory	**DoE ± U(DoE)**	
	1Ω (nΩ/Ω)	**10 kΩ sn SP910102 (nΩ/Ω)**
JV	19 ± 25	0 ± 3
DFM	40 ± 540	-37 ± 178
MIKES	-9 ± 28	0 ± 49
SP	-20 ± 38	5 ± 25

Table 6: Degrees of equivalence (DoE) with expanded uncertainty for the 1Ω and 10 kΩ measurements.



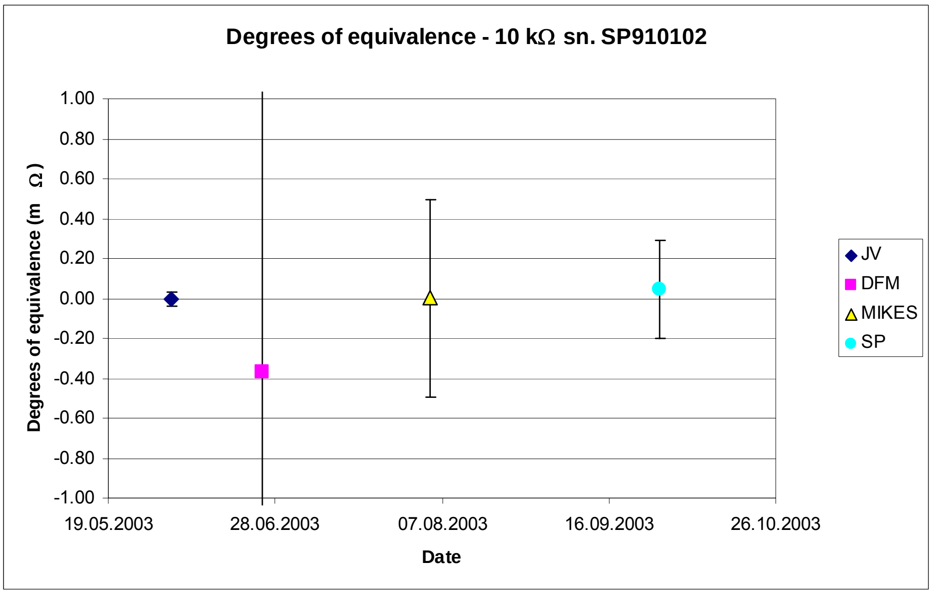


Figure 4: DoE with respect to the reference value for the 1Ω and 10 kΩ measurements.

7. Summary and conclusion

The comparison indicates that it is possible to compare resistance standards and estimate systematic effects in measurement equipment with uncertainties of parts in 10^{-8} for 1Ω and 10 kΩ using conventional resistance standards and careful transport. Personally transported standards, maintained at stable temperature, showed very stable behaviour. This is important for the possibility to check for systematic uncertainties in resistance scaling in the range 1Ω - 10 kΩ with CCC-bridges of different design. The results of the 1 Ω comparison show good agreement among the participants. The agreement of the 10 kΩ comparison is excellent.

References

[1] J.M. Williams and A. Hartland, "An Automated Cryogenic Current Comparator Resistance Ratio Bridge", *IEEE Trans. Instrum. Meas.*, Vol. 40, pp 267-270, 1991.

[2] H. Seppä and A. Satrapinski, "AC Resistance Bridge Based on the Cryogenic Current Comparator", *IEEE Trans. Instrum. Meas.*, Vol. 46, No.2, pp.463-466, 1997

[3] M.G. Cox, "The evaluation of key comparison data", *Metrologia*, 2002, **39**, 589-595

Appendix A: Method of measurements

The measurement method of SP

The measurements are based on the quantum Hall effect. The QHE cryostat has a variable temperature insert and QHE samples can be cooled to 1,6 K. The magnet can produce operating fields to 12 Tesla at 4,2 K. The Hall sample is mounted on a header of the TO-8 type and the room temperature connections are gold plated bind post terminals.

The used QHE sample is characterised with a constant current source connected to the current terminals. The measurements have been performed following the technical guidelines in "*Delahaye F 1989 Metrologia 26 63-8*".

The realised QHE resistance, the QHR, is compared with room temperature resistance standards with nominal values 100 Ω and 10 kΩ with a cryogenic current comparator (CCC) or a Josephson potentiometer (10 kΩ measurements). All results reported here are from CCC measurements.

The CCC system is manufactured by Oxford Instruments under a technology transfer agreement with National Physical Laboratory UK [1].

The CCC measurements are performed in a positive-negative-positive sequence with two current reversals. The total number of measurements is the same in positive and negative polarity.

For measurements below the 100 Ω level the CCC measurements have to be made in two steps. In the case of the measurements on the 1 Ω standards reported here, a temperature controlled 100 Ω resistance standard is used as a transfer standard.

The measurement method of MIKES

The 1 Ω travelling standards were measured with the MIKES ac Cryogenic Current Comparator (CCC) Resistance Bridge. The 1 Ω resistors were compared directly with the two 100 Ω transfer standards, Tr1 and Tr2. These 100 Ω resistors were measured in July 2003 against the MIKES QHR standard.

The 1 Ω standards were measured at the frequencies 0,1 Hz – 0,3 Hz, and with the currents (rms value) in the 100 Ω: 0,35 mA, 0,5 mA and 0,7 mA, (or equivalent dc currents in 1 Ω – 35 mA, 50 mA, and 70 mA).

The temperature of the 1 Ohm resistors was measured by temperature meter Hart Scientific (model 1529R), with the calibrated Pt100 sensors placed inside the wells of the resistors.

The 100 Ω references are in their own enclosures with the maintained temperatures: T_{Tr1} = 30,73 °C, and T_{Tr2} = 30,25 °C. These temperatures are maintained permanently with the instabilities of 0,003 °C and, at these temperatures the temperature coefficients are close to zero.

The 10 kΩ was compared also directly with the two 100 Ω transfer standards, Tr1 and Tr2 with the MIKES ac Cryogenic Current Comparator (CCC) Resistance Bridge.

The application of low frequency currents allows comparing the imaginary components of the measured standards. During measurements with the 10 kΩ a bigger imaginary part and a higher frequency dependence in the real part was noticed, compared to other types of 10 kΩ. This indicates probably additional dissipation in capacitive components of that standard. The increased frequency dependence and the uncertainty of interpolation were the reasons of an additional contribution to the uncertainty budget of that resistor.

The measurement method of JV

JV's system is an automatic QHR-system from Oxford Instruments. It consists of a 16T magnet with a top loading ^{3}He insert and a Cryogenic Current Comparator (CCC). The CCC-bridge is produced by Oxford Instruments under a technology transfer agreement with NPL [1]. The CCC bridge is used in the calibration of the 1 Ω and 10 kΩ resistors using a 100 Ω resistor as reference standard. This reference standard is calibrated against the QHR standard.

The measurement method of DFM

DFM achieves at present its traceability for resistance from BIPM. A set of three ESI SR104 10 kΩ resistors serve as DFM reference standard, and each of these are in turn calibrated by BIPM over a three year period.
Scaling over the range 1 Ω to 10 kΩ is achieved primarily via two Hamon transfer devices, Guildline 9350/1 kΩ and Guildline 9350/10 kΩ. The scaling is independently checked via a set of thermalised resistors in a 1:2:5 sequence. Working standards at 10 kΩ, 100 Ω, etc. are used as check standards.

Resistance ratio measurements are performed using a Guildline 6675A Direct Current Comparator Bridge. The resistors are connected to a Measurement International 4220A Four-Terminal Matrix scanner. The case of the ESI and Hamon transfer resistors is connected to the reference potential via the screen of the potential carrying interconnecting cable. The screen of the current carrying cable is connected to the reference point at the scanner side.
A program connect the resistors, set up the measurement parameters of the bridge, start the measurement, and read the resistance ratio measured by the bridge. Temperature measurements, Pt 100 sensor readings, as well as recording of the environmental conditions are performed simultaneously.

Appendix B: Uncertainty budgets

The uncertainty budget of MIKES for 1Ω

Table II Error budget in determination of 1 Ω from the 100 Ω by CCC bridge.

Source	Input Quantity X_i	Estimate x_i	Standard Uncertainty of input $u(x_i)$	Probability Distribution / method of evaluation (A,B)	Sensitivity Coefficient c_i (ΔR_x / ΔX_i)	Uncertainty Contribution $u_i(R_x)$ = $u(x_i)$* (ΔR_x / ΔX_i)	Degree of Freedom ν_i
100 Ohm	Reference resistance, Calibration from QHR, low frequency fluctuation.	100 Ω	0.8 10^{-8}	Square / B	1	0.8 10^{-8}	Infinite
	Reference resistance, Current dependence,	Power coefficient	3 10^{-9} μΩ/Ω/ mW	Square / B	1	0.3 10^{-8}	Infinite
CCC bridge	Ratio, compensation current error	I_{comp} / I_1	5 10^{-5}	Square / B	2 10^{-5}	0.1 10^{-8}	Infinite
	Ratio, CCC winding error, dc -1 Hz	N_1 / N_2, 16:1600	1.0 10^{-9}	Square / B	1	0.1 10^{-8}	Infinite
	Ratio, dynamic error, FB gain	I_1 / I_2	5 10^{-6}	Square / B	5 10^{-4}	0.25 10^{-8}	Infinite
	Noise rectification, squid voltage bias changes, flux jumps, nonlinearly of null-detector	Voltage	2.5 10^{-2} V	Square / B	2 10^{-7} / V	0.5 10^{-8}	Infinite
	Error due to uncompensated ac currents and leakage currents in voltage link	Voltage	1.0 10^{-8}	Square / B	1	1.0 10^{-8}	Infinite
	Zero offset in bridge	Voltage	3.5 10^{-9}	Square / B	1	0.35 10^{-8}	Infinite
	Extrapolation to dc	Ratio	0.6 10^{-8}	Square / B	1	0.6 10^{-8}	Infinite
	Contact resistance in rotary switch of I_{comp}	Resistance μΩ/Ω	3 10^{-9}	Normal / A	1	0.3 10^{-8}	27
1 Ohm Resistor	Temperature*	23.0 °C	0.010 °C	Square / B	0.5 μΩ/Ω/ °C*	0.5 10^{-8}	Infinite
	Temperature coefficients (α, β)*	0.5* μΩ/Ω / °C	<0.5 μΩ/Ω / °C	Square / B	1 μΩ/Ω/ (μΩ/Ω/°C)	<0.5 10^{-8}	Infinite
	Current dependence*	Power coefficient	<5 10^{-9} μΩ/Ω/ mW	Square / B	1	<0.5 10^{-8}	Infinite
	Pressure*	1013 hPa	0.1 hPa	Square / B	<1 10^{-9} / hPa	<0.1 10^{-8}	Infinite
	Low frequency fluctuation**	Resistance	3 10^{-9}	Square / B	1	0.3 10^{-8}	Infinite
	Measurement***	Voltage	5 10^{-9}	Normal / A	1	0.5 10^{-8}	180
	R1 Ω	1.00000				1.8 10^{-8}	ν_{eff} = 8984

The expanded uncertainty is 36 nΩ/Ω for *k*=2.

The uncertainty budget of MIKES for 10 kΩ

Table II Error budget in determination of 10 kΩ from the 100 Ω by ac CCC bridge.

Source	Input Quantity X_i	Estimate x_i	Standard Uncertainty of input $u(x_i)$	Probability Distribution / method of evaluation (A,B)	Sensitivity Coefficient c_i ($\Delta R_x / \Delta X_i$)	Uncertainty Contribution $u_i(R_x) = u(x_i)^* (\Delta R_x / \Delta X_i)$	Degree of Freedom ν_i
100 Ohm	Reference resistance, Calibration from QHR, low frequency fluctuation.	100 Ω	$0.8\ 10^{-8}$	Square / B	1	$0.8\ 10^{-8}$	Infinite
CCC bridge	Ratio, compensation current error	I_{comp} / I_1	$5\ 10^{-5}$	Square / B	$2\ 10^{-5}$	$0.1\ 10^{-8}$	Infinite
	Ratio, CCC winding error, dc -1 Hz	N_1 / N_2, 16:1600	$1.0\ 10^{-9}$	Square / B	1	$0.1\ 10^{-8}$	Infinite
	Ratio, dynamic error, FB gain	I_1 / I_2	$5\ 10^{-6}$	Square / B	$5\ 10^{-4}$	$0.25\ 10^{-8}$	Infinite
	Noise rectification, squid voltage bias changes, flux jumps	Voltage	$2\ 10^{-2}$ V	Square / B	$2\ 10^{-7}$ / V	$0.4\ 10^{-8}$	Infinite
	Error due to uncompensated ac currents and leakage currents in voltage link	Voltage	$0.5\ 10^{-8}$	Square / B	1	$0.5\ 10^{-8}$	Infinite
	Zero offset in bridge	Voltage	$3.5\ 10^{-9}$	Square / B	1	$0.35\ 10^{-8}$	Infinite
	Extrapolation to dc	Ratio	$2.2\ 10^{-8}$	Square / B	1	$2.2\ 10^{-8}$	Infinite
	Contact resistance in rotary switch of I_{comp}	Resistance μΩ/Ω	$3\ 10^{-9}$	Normal / A	1	$0.3\ 10^{-8}$	27
10 kOhm Resistor	Temperature coefficients	30 °C	0.010 °C	Square / B	0.1 μΩ/Ω/ °C*	$0.1\ 10^{-8}$	Infinite
	Temperature coefficients (α, β)*	0.1 μΩ/Ω / °C	0.01 μΩ/Ω / °C	square / B	1 μΩ/Ω/ (μΩ/Ω/°C)	$0.01\ 10^{-8}$	Infinite
	Pressure*	1013 hPa	0.1 hPa	Square / B	$<1\ 10^{-9}$ / hPa	$<0.1\ 10^{-8}$	Infinite
	Low frequency fluctuation**	Resistance	$3\ 10^{-9}$	Square / B	1	$0.3\ 10^{-8}$	Infinite
	Measurement***	Voltage	$4\ 10^{-9}$	Normal / A	1	$0.4\ 10^{-8}$	180
	R10 kΩ	10 000. 0				$2.5\ 10^{-8}$	$\nu_{eff} = 8984$

The expanded uncertainty is 50 nΩ/Ω for *k*=2.

* temperature and pressure coefficients are not known and the uncertainty are estimated approximately.

** estimate of possible combined maximum influence.

*** Estimate for typical parameters in one run of measurements: current (rms) in 100 Ω – 5 mA, frequency – 0.1 Hz, time of measurement - 20 min (degree of freedom – 180).

The uncertainty budget of SP for 1 Ω

The model for the measurements is:

$$Rx = Rs \cdot (1+ \delta tsd) \cdot r \cdot (1+ \delta cwr + \delta clr + \delta csr + \delta rxl)$$

Where

Rx is the unknown 1 Ω resistor.
Rs is the 100 Ω QHR transfer resistor.
δtsd is the relative error due to the QHR 100 Ω transfer standard drift.
r is the ratio *Rx/Rs* measured by the CCC bridge.
δcwr is the relative error due to the CCC winding ratio deviation from nominal.
δclr is the relative error due the CCC leakage resistance.
δcsr is the relative error due to the error of the shunt resistor value.
δrxl is the relative error due to the error of the internal lead resistance of the shunted resistor.

The relative standard uncertainty is given by :

$$\frac{u(Rx)}{Rx} = \sqrt{\left(\frac{u(R_S)}{R_S}\right)^2 + \left(\frac{u(r)}{r}\right)^2 + \sum u(\delta_i)^2}$$

Where $\frac{u(r)}{r}$ is the relative standard deviation of the mean for the measurement results.

Uncertainty budget for R_X = 1 Ω travelling standard s/n 64179

Quan-tity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (10^{-9})	Probability distribution / method of evaluation (A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$, (nΩ/Ω)	Degree of freedom ν_i
Rs	100.0011183 Ω	13.0	Normal/A+B	1	13.0	228
δtsd	0	12.0	Normal/B	1	12.0	∝
r	0.00999987793	7.9	Normal/A	1	7.9	1
δqpl	0	0.9	Rectangular/B	1	0.9	∝
δcwr	0	0.8	Normal/A	1	0.8	4
δclr	0	0.6	Rectangular/B	1	0.6	∝
δcsr	0	1.3	Normal/B	1	1.3	∝
δrxl	0	0.1	Normal/B	1	0.1	∝
R_X	0.999998976 Ω				18.3	27

The expanded uncertainty is 39 nΩ/Ω for *k*=2.1

Uncertainty budget for R_X = 1 Ω travelling standard s/n 64187

Quan-tity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (10^{-9})	Probability distribution / method of evaluation (A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$, (nΩ/Ω)	Degree of freedom ν_i
Rs	100.0011183 Ω	13.0	Normal/A+B	1	13.0	228
δtsd	0	12.0	Normal/B	1	12.0	∝
r	0.00999982071	17.7	Normal/A	1	17.7	1
δqpl	0	0.9	Rectangular/B	1	0.9	∝
δcwr	0	0.8	Normal/A	1	0.8	4
δclr	0	0.6	Rectangular/B	1	0.6	∝
δcsr	0	1.3	Normal/B	1	1.3	∝
δrxl	0	0.1	Normal/B	1	0.1	∝
R_X	0.999993254 Ω				25.1	4

The expanded uncertainty is 72 nΩ/Ω for k=2.87

The uncertainty budget of SP for 10 kΩ

The model for the measurements is:

$$Rx = QHR \cdot (1 + \delta QHR + \delta qpl) \cdot r \cdot (1 + \delta cwr + \delta clr + \delta csr + \delta rxl)$$

Where

Rx is the unknown 10 kΩ resistor.

QHR is the realised quantum Hall resistance at plateau i=2 with the exact numerical value 12906,4035 Ω.

δQHR is the relative error of the realised Hall resistance due to imperfect quantization and effects of imperfect contacts on the Hall sample.

δqpl is the relative error due to the QHR probe leakage resistance.

r is the ratio Rx/QHR measured by the CCC bridge.

δcwr is the relative error due to the CCC winding ratio deviation from nominal.
δclr is the relative error due the CCC leakage resistance.
δcsr is the relative error due to the error of the shunt resistor value.
δrxl is the relative error due to the error of the internal lead resistance of the shunted resistor.

The relative standard uncertainty is given by :

$$\frac{u(Rx)}{Rx} = \sqrt{\left(\frac{u(r)}{r}\right)^2 + \sum u(\delta_i)^2}$$

Where $\frac{u(r)}{r}$ is the relative standard deviation of the mean for the measurement results.

Uncertainty budget for R_X = 10 kΩ travelling standard s/n SP910102

Quan-tity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (10^{-9})	Probability distribution / method of evaluation (A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$, (nΩ/Ω)	Degree of freedom ν_i
QHR	12906,4035 Ω	0.0	-	1	0.0	∝
r	0.7748038135	1.6	Normal/A	1	1.6	4
δQHR	0	12.0	Normal/B	1	12.0	∝
δqpl	0	0.9	Rectangular/B	1	0.9	∝
δcwr	0	0.8	Normal/A	1	0.8	4
δclr	0	1.1	Rectangular/B	1	1.1	∝
δcsr	0	2.0	Normal/B	1	2.0	∝
δrxl	0	0.2	Normal/B	1	0.2	∝
R_X	9999.930650 Ω				12.4	13581

The expanded uncertainty is 25 nΩ/Ω for *k*=2

Uncertainty budget for R_X = 10 kΩ travelling standard s/n SP910102

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (10^{-9})	Probability distribution / method of evaluation (A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$, (nΩ/Ω)	Degree of freedom ν_i
QHR	12906,4035 Ω	0.0	-	1	0.0	∝
r	0.7748038258	3.3	Normal/A	1	3.3	8
δ*QHR*	0	12.0	Normal/B	1	12.0	∝
δ*qpl*	0	0.9	Rectangular/B	1	0.9	∝
δ*cwr*	0	0.8	Normal/A	1	0.8	4
δ*clr*	0	1.1	Rectangular/B	1	1.1	∝
δ*csr*	0	2.0	Normal/B	1	2.0	∝
δ*rxl*	0	0.2	Normal/B	1	0.2	∝
R_X	9999.930809 Ω				12.8	1749

The expanded uncertainty is 26 nΩ/Ω for *k*=2

Budget of uncertainty of JV for 1 Ω

The model for the measurement is:

$$R_x = R_s \cdot r \cdot (1 + \delta_{wind} + \delta_{leak} + \delta_{bal} + \delta_{shunt} + \delta_{rect} + \delta_p)$$

The components are:

R_x: the unknown resistor
R_s: the 100 Ω reference standard: 100,000 636 42 Ω
r : the ratio measured by the CCC-bridge
δ_{wind}: the relative winding ratio error
δ_{leak}: the relative error due to leakage resistance
δ_{bal}: the relative error due to bridge balancing
δ_{shunt}: the relative error due to the stability and calibration of the shunt resistor
δ_{rect}: the relative error due to noise rectification
δ_p: the relative error due to change in power dissipation in the 100 reference standard

The relative standard uncertainty is then given by :

$$\frac{u(R_x)}{R_x} = \sqrt{\left(\frac{u(R_s)}{R_s}\right)^2 + \left(\frac{u(r)}{r}\right)^2 + \sum u(\delta_i)^2}$$

Which give the following uncertainty budget for R_x:

Uncertainty budget for R_X = 1 Ω travelling standard s/n 64179

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (nΩ/Ω)	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$, (nΩ/Ω)	Degree of freedom ν_i
R_s	100.000 636 42 Ω	3.6	normal	1	3.6	∝
r	0.009 999 926 398	6.0	normal	1	6.0	50
δ_{wind}	0	1	normal	1	1	∝
δ_{leak}	0	1	rectangular	1	1	∝
δ_{bal}	0	2.0	normal	1	2.0	11
δ_{shunt}	0	0.6	normal	1	0.6	∝
δ_{rect}	0	1	rectangular	1	1	∝
δ_p	0	15	rectangular	1	15	∝
R_X	0.999 999 004 Ω				17	281

The expanded uncertainty is 34 nΩ/Ω for k=2.

Budget of uncertainty of JV for 10 kΩ

The model for the measurement is:

$$R_x = R_s \cdot r \cdot (1 + \delta_{wind} + \delta_{leak} + \delta_{bal} + \delta_{shunt} + \delta_{rect})$$

The components are:

R_x: the unknown resistor
R_s: the 100 Ω reference standard: 100.000 636 42 Ω
r : the ratio measured by the CCC-bridge
δ_{wind}: the relative winding ratio error
δ_{leak}: the relative error due to leakage resistance
δ_{bal}: the relative error due to bridge balancing
δ_{shunt}: the relative error due to the stability and calibration of the shunt resistor
δ_{rect}: the relative error due to noise rectification

The relative standard uncertainty is then given by :

$$\frac{u(R_x)}{R_x} = \sqrt{\left(\frac{u(R_s)}{R_s}\right)^2 + \left(\frac{u(r)}{r}\right)^2 + \sum u(\delta_i)^2}$$

Which give the following uncertainty budget for R_x:

Uncertainty budget for R_X = 10 kΩ travelling standard s/n SP910102

Quantity X_i	Estimate x_i	Relative standard uncertainty $u(x_i)$, (nΩ/Ω)	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(R_x)$, (nΩ/Ω)	Degree of freedom ν_i
R_s	100.000 636 42 Ω	3.6	normal	1	3.6	∝
r	99.998 669 97	0.5	normal	1	0.5	50
δ_{wind}	0	1	normal	1	1	∝
δ_{leak}	0	1	rectangular	1	1	∝
δ_{bal}	0	2.0	normal	1	2.0	11
δ_{shunt}	0	0.6	normal	1	0.6	∝
δ_{rect}	0	1	rectangular	1	1	∝
R_X	9 999.930 638 Ω				4.5	281

The expanded uncertainty is 9.0 nΩ/Ω for *k*=2.

The uncertainty budget of DFM for 1 Ω

The model for the measurement is:

$$\varepsilon_X = \varepsilon_S + \delta\varepsilon_{SP} + \delta_{T,S} + \left(r + \delta r_S + \delta r_C\right) - \delta_{T,X}$$

where ε_X is the relative deviation from nominal (RDN) of the unknown resistor, ε_S is the RDN of the reference resistor. The $\delta\varepsilon_{SP}$ is the series-parallel transfer error of the Hamon transfer device. The term $\delta_{T,S}$ is the temperature correction of the reference, *r* is the ratio as measured, δr_S is the specification of the current comparator bridge, δr_C is the correction of the bridge error, and $\delta_{T,X}$ is the temperature correction of the unknown resistor.

The uncertainty budget becomes:

i	**Quantity (unit)**	**Distribution**	x_i	$U(x_i)$	n_i	c_i	$u_i(y)$
1	ε_S, rel.dev. from nominal of standard (10^{-6})	Normal	3.562	0,247	25	1	0,150
2	$\delta\varepsilon_{SP}$, Series-parallel transfer error (10^{-6})	Rectangular	0,000	0,058	infinity	1	0,058
3	$\delta_{T,S}$, Temp deviation standard ±0,1°C (10^{-6})	Rectangular	0,000	0,080	infinity	1	0,080
4	*r*, Ratio deviation as determined (10^{-6})	Normal	-4,5427	0,020	16	1	0,005
5	δr_S, Specification for Guildline bridge (10^{-6})	Rectangular	0,000	0,058	infinity	1	0,058
	δr_C, Correction of bridge error (10^{-6})	Normal	0,000	0,003	22	1	0,001
y	ε_X, Rel. dev. From nom. of unknown (10^{-6})	Normal	-0,9757	0,273	37		

The expanded uncertainty is 540 nΩ/Ω for *k*=2.

The uncertainty budget of DFM for 10 kΩ

The model for the measurement is:

$$\varepsilon_X = \underbrace{\varepsilon_{ref,A} + \varepsilon_{ref,B} + \delta\varepsilon}_{\varepsilon_S} + \delta_{T,S} + \left(r + \delta r_S + \delta r_C\right) - \delta_{T,X}$$

where ε_X is the relative deviation from nominal (RDN) of the unknown resistor, ε_S is the RDN of the reference resistor, containing the value obtained from BIPM (type-A and type-B components) and the drift since calibration. The term $\delta_{T,S}$ is the temperature correction of the reference, *r* is the ratio as measured, δr_S is the specification of the current comparator bridge, δr_C is the correction of the bridge error, and $\delta_{T,X}$ is the temperature correction of the unknown resistor.

i	**Quantity (unit)**	**Distribution**	x_i	$u(x_i)$	n_i	c_i	$u_i(y)$
1	ε_S, rel.dev. from nominal of standard (10^{-6})	Normal	0,668	0,015	7	1	0,0151
2	$\delta_{T,S}$, Temp deviation standard ±0,1°C (10^{-6})	Rectangular	0	0,017	infinity	1	0,0170
3	*r*, Ratio deviation as determined (10^{-6})	Normal	-7,778	0,060	43	1	0,0600
4	δr_S, Specification for Guildline bridge (10^{-6})	Rectangular	0	0,058	infinity	1	0,0577
5	δr_C, Correction of bridge error (10^{-6})	Normal	0,141	0,021	30	1	0,0210
y	ε_X, Rel. dev. From nom. of unknown (10^{-6})	Normal	-6,969	0,089	infinity		

The expanded uncertainty is 180 nΩ/Ω for *k*=2.

It is further estimated that there is a correlation of the DFM result for resistor SP910102 to the BIPM working standards of **r = 0,17** (estimated from the quoted type-B relative uncertainty of BIPM of $1{,}5 \cdot 10^{-8}$.)