# EUROMET.EM-S20

# Intercomparison of a 100 mH inductance standard

# Draft B

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This draft B Report describes the organisation, equipement and results of the EUROMET.EM-S20 supplementary comparison (formerly EUROMET n. 607 intercomparison) of a 100 mH inductance standard at a frequency of 1 kHz, which took place from November 2002 to June 2003. Participants were IEN in Turin, Italy as pilot laboratory, and PTB (Germany), SP (Sweden), GUM (Poland), CMI (Czech Republic), CTU (Czech Republic), NCM (Bulgaria). Results of the comparison are reported.

<sup>\*</sup>Thanks to the participants and Vincenzo D'Elia - INRIM

# 1 Introduction

This is the Draft B report of the Euromet comparison EUROMET.EM-S20, "Intercomparison of a 100 mH inductance standard".

Metrology area, branch: Electricity and Magnetism, Inductance Description: Comparison of inductance standards

Time of measurement : 2002 - 2003 Status: Draft B

Reference(s): No references available

Measurand: Inductance: 100 mH Parameter(s): Frequency: 1000 Hz

Transfer device(s): GR1482-L 100 mH inductance standard, encased in a

thermostated enclosure

Comparison type: Supplementary comparison

Consultative Committee: CCEM (Consultative Committee for Electricity and Magnetism)

Conducted by: EUROMET (European Metrology Collaboration)

Other designation(s): EUROMET 607 (European Metrology Collaboration Project

Number 607)

The comparison has been started as EUROMET Pilot Comparison n. 607, "Intercomparison of a 100 mH inductance standard".

Text and figures refer to Istituto Elettrotecnico Nazionale Galileo Ferraris (IEN). Since January 1, 2006 the Italian National Metrology Laboratory has changed its name in Istituto Nazionale di Ricerca Metrologica (INRIM). Any use of results reported for further quoting or processing should be retagged with the new name.

# 2 Participants and organisation of the comparison

# 2.1 List of participants

IEN Istituto Elettrotecnico Nazionale Galileo Ferraris

Now Istituto Nazionale di Ricerca Metrologica - INRIM

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PTB Physikalisch-Technische Bundesanstalt

Braunschweig, Germany

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Tel.: +49 531 592 2600

SP Swedish National Testing and Research Institute

Borås, Sweden Gunnar Eklund

Box 857, 501 15 Borås Tel +46 0 33 165391

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GUM Główny Urząd Miar

Warszawa, Poland Antoni Tarłowski

**Electrical Metrology Division** 

Elektoralna 2

00-950 Warszawa P-10 Tel. +48 22 620 5970 Electricity@gum.gov.pl

CMI Czech Metrology Institute

Brno, Czech Republic

Jiri Horsky

Okruzni 31 63800 Brno Czech Republic jhorsky@cmi.cz **UME** Ulusal Metroloji Enstitüsü

Turkey

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yakup.gulmez@ume.tubitak.gov.tr

Tel:+90 262 679 5000 Fax:+90 262 679 5001

Note: the standard has been sent to UME and measured. The results are not included in the report since UME withdrew from the comparison thereafter.

**NCM National Centre of Metrology** 

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Email bohacek@feld.cvut.cz

# 2.2 Comparison schedule



Note: the standard has been sent to UME and measured. The results are not included in the report since UME withdrew from the comparison thereafter.

# 2.3 Organisation of the comparison

## 2.3.1 Circulation method

No specific circulation method has been followed. The standard returned to IEN two times during the comparison, to check possible drifts/malfunctionings since the standard was of new construction.

### 2.3.2 Timing

The standard has been under observation from Jan 2002 to Aug 2004. The circulation started Dec 2002 and ended in June 2003.

# 2.3.3 Transportation

Each participant laboratory has been responsible for the transportation to the next participant, custom charges, as well as any damage that could occur within its country. Commercial carriers have been used.

A metal enclosure with internal shock isolation has been built and employed for transportation of

the inductance standard and its accessories, shippable as freight. The dimensions of the enclosure are 600 mm height, 550 mm depth, 800 mm width: the approximate weight being 35 kg (standard and accessories included).

# 2.4 Unexpected incidents

# 2.4.1 Report of organisational and transport problems

No significant delays caused by organisational or transport problems.

# 2.4.2 Damage of standard

None.

# 3 Travelling standard and measurement instructions

# 3.1 Description of the standard

The tavelling standard is an inductance standard having the nominal value of 100 mH. It is constructed starting from a General Radio GR1482-L 100 mH inductance standard, encased in a thermostated enclosure. The external aspect of the standard (Fig. 1) is a wooden box (370 mm height, 285 mm depth, 285 mm width).

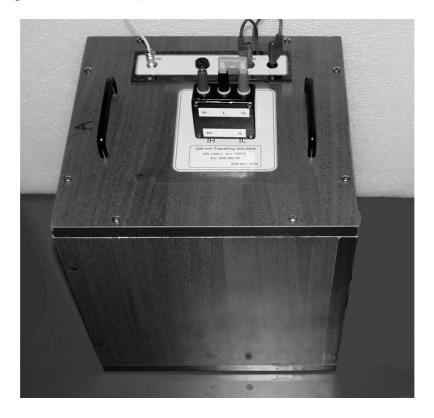


Fig. 1. External aspect of the 100 mH travelling standard.

The electrical connections to the standard consist of:

- four British Post Office MUSA connectors, for four terminal-pair electrical definition, Fig. 2.

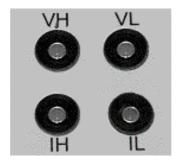


Fig. 2. Connections for the electrical definition of the standard

- a LEMO ERA.OS.304.CLL four-pole+screen connector, to access the internal PT100 thermoresistance, Fig. 3;

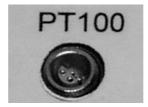


Fig. 3. LEMO connector for the internal PT100 thermoresistance

- two banana sockets, one red (+) and one black (-) for the connection of the 12V power supply to the thermostat, Fig. 4.

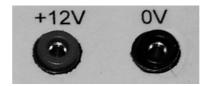


Fig. 4. Power supply connections for the thermostat

The standard is provided with the following accessories:

**4/2 adapter**: an electrical adapter to convert the four terminal-pair definition provided by the BPO connectors to two-terminal binding posts definition, Fig. 5.

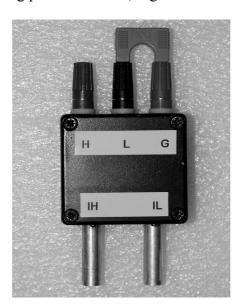


Fig. 5. 4/2 adapter.

The adapter contains a shorting link to be considered an integral part of the adapter.

The electrical schematics of the adapter is reported in Fig. 6.

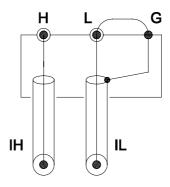


Fig. 6. Electrical schematics of the adapter.

The adapter converts four terminal-pair definition of the standard to two-terminal definition, and should be inserted on the  $I_H$ - $I_L$  ports, as shown in Fig. 7. PostOfficeCaps (see below) cover the unused VH and VL connectors.



Fig. 7. 4/2 adapter mounted on the standard.

**PT100cable**: for connection to the LEMO ERA.OS.304.CLL connector, having four free wires and screen on the other side

The connections are the following:

Red = high-current terminal,  $I_H$ ;

Yellow = high-voltage terminal,  $V_H$ ;

Green = low-voltage terminal,  $V_L$ ;

Black = low-current termina,  $I_L$ ;

**12V supply:** a 220/240 V - 50 Hz ac to 12 V dc electrical power supply to power the thermostat, Fig. 8.



Fig. 8. 12V power supply.

The supply is terminated with a 220/240V power line input socket and two banana sockets, one red (+) and one black (-).

It is provided with a green LED lamp which shows the power-up condition, an on/off switch, one fuse holder containing 315 mA fuse.

**12Vcable:** two single-wire cables, one red and one black, for connection of the 12Vsupply to the standard.

**PostOfficeCaps**: two non-shorting caps for post-office connectors, used to cover VH and VL terminals when the 4/2 adapter is used.

# 3.2 Quantities to be measured, and conditions of measurement

# 3.2.1 Description of each quantity and parameters

The quantities to be measured are:

- $L_s$  and  $R_s$ : the equivalent series inductance and resistance of the standard, defined by the configuration chosen (four terminal-pair or two-terminal);
- *f*: the measurement frequency;
- $R_{PT100}$ : the four-terminal resistance of the PT100 internal sensor;
- $T_{ext}$ : the temperature (°C) of the environment where the standard is measured;
- $H_{ext}$ : the external humidity (%) of the environment where the standard is measured.

### 3.2.2 Conditions of the measurement

The measurement has to be carried out with the following excitation:

- ac sinewave signal;
- frequency 1000 Hz;
- current 1 mA<sub>rms</sub>;
- THD+N (total harmonic distortion+noise) less than 0.01%

# 3.2.3 Aimed accuracy and/or limit for partecipation

No limit on accuracy. Total time for measurement has been limited to 2 weeks.

### 3.3 Measurement instructions

# Setup of the standard

The standard must be positioned with the face having the electrical connections in horizontal position.

Before and during the measurement, the standard must be energized with a +12.0(5) V, 500 mA dc low-noise power supply, connected to the banana sockets (+ red, -black). 12Vsupply accessory is suited for the purpose but its employment is not mandatory.

At power-up, the thermostat green LED lamp lits, indicating thermostat ON cycle. After some hours, the lamp goes off, then on again, with an approximate period of 30 minutes.

The standard reaches its operating temperature, around 28 °C, in 24 hours. Tentative measurements on the standard can be carried out before this period, but cannot be considered reliable for the comparison. DO NOT consider the reaching of a plateau for  $R_{PT100}$  the signal of a temperature stabilization of the standard. If the thermostat power supply is disconnected for any reason, the user must wait again 24 hours before measurement.

### Electrical definition of the standard

Two electrical definitions of the standard are possible:

- four terminal-pair:
- two-terminal using the given adapter.

# 3.3.1 Tests before measurement

No specific tests to be carried on the standard.

# 3.3.2 Measurement constraints

No specific constraints apart fulfillment of the measurement conditions.

# 3.4 Deviations from the protocol

No deviations reported.

# 4 Methods of measurement

### IEN Method

Three-voltage method

### **Definition of the standard**

Two-terminal and four terminal-pair

### **Traceability**

Ac resistance, from dc measurements and ac-dc corrections. Frequency.

### References

L. Callegaro, V. D'Elia, "Automated system for inductance realization traceable to AC resistance with a three-voltmeter method", *IEEE Trans. Instr. Meas.*, vol. 50, n. 6, pp. 1630-1633 (Dec. 2001).

# PTB Method

Maxwell-Wien bridge

### **Definition of the standard**

Two-terminal

### **Traceability**

Capacitance, ac resistance

### References

Hanke, R.; Kölling, A.; Melcher J.: Inductance Calibration in the Frequency Range from 50 Hz to 1 MHz at PTB. In: MĀPAN-J. of Metrology Society of India, 18 (2003), pp.255-259.

# SP Method

Resonance method for calibration of a 200 mH high-Q inductor; Transformer ratio bridge for inductance scaling

# **Definition of the standard**

Two-terminal

# **Traceability**

Capacitance, frequency, ac resistance.

### References

No reference given in the report

# **GUM** Method

Combined transformer bridge

# **Definition of the standard**

Two-terminal

# **Traceability**

Capacitance

### References

A. Muciek, "A combined transformer bridge for precise comparison of inductance with capacitance", IEEE Trans. Instr. Meas. Vol. IM-32, pp. 419-423, 1987.

### CMI Method

Transformer bridge

### **Definition of the standard**

Two-terminal

# **Traceability**

Capacitance, ac resistance

### References

Horsky, J. "High Precision Inductance Measurement and its validation", Metrology and Metrology Assurance '2004, p 27 to 31, Sozopol, Bulgaria, 2004

Horsky J., Horska J., Semenov, Y. "Inductance Calibration in Czech Metrology Institute", MAPAN, Journal of Metrology Society of India 2003, Vol 18, No 3, 2003, p 161-168

Ripper L., Horsky J, Horska J, "Resonant Method of Inductance Measurement with Inductive Divider Calibration Set", 11th International Metrology Congress, Toulon, France, October 2003

### NCM Method

1:1 substitution with a commercial inductance bridge

### **Definition of the standard**

Two-terminal

# **Traceability**

Inductance (from PTB)

### References

No reference given in the report

# CTU Method

Double balance LC bridge

### **Definition of the standard**

Four-terminal pair

### **Traceability**

Capacitance

### References

No reference given in the report

Refer to Appendix B for details on the measurement methods.

# 5 Repeated measurements of the pilot institute, behaviour of the travelling standard

Repeated measurements of the pilot laboratory have been performed before, during and after the end of the comparison. Since the aim of the measurements was primarily to evaluate possible drifts of the series inductance, only such estimate is reported. Results are given in Fig. 5.1 and Tab 5.1. Temperature drifts between different measurements can be considered negligible.

Date	<b>L</b> <sub>S</sub> <b>2T</b>	<b>L</b> s <b>4P</b>
	Н	Н
2002-10-07	0.1000529	0.10001148
2002-11-20	0.1000526	0.10001140
2003-01-13	0.1000524	0.10001145
2003-04-13	0.1000526	0.10001140
2003-06-27	0.1000530	0.10001128
2003-07-01	0.1000531	0.10001136

Tab 5.1. Repeated measurements of the pilot institute, 2T and 4P configurations.

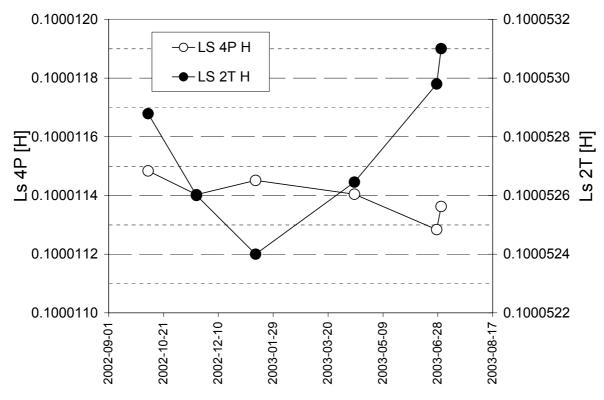


Fig 5.1. Repeated measurements of the pilot institute, 2T and 4P configurations.

The temperature coefficient of the travelling standard has been measured by IEN by trimming the set point of the thermostat around the working temperature. The result is a temperature coefficient  $dL/dR_{PT100} = 7.51 \ \mu H/\Omega$ .

#### 6 **Measurement results**

#### Results of the participating institutes 6.1

In Tab. 6.1 values and corresponding standard uncertainty, ambient conditions, are given. A graphical representation of inductance  $L_{\rm S}$  data are shown in Fig. 6.1. Refer to Appendix C for a complete uncertainty budget of each laboratory.

2T me	asurements
Lab	Date

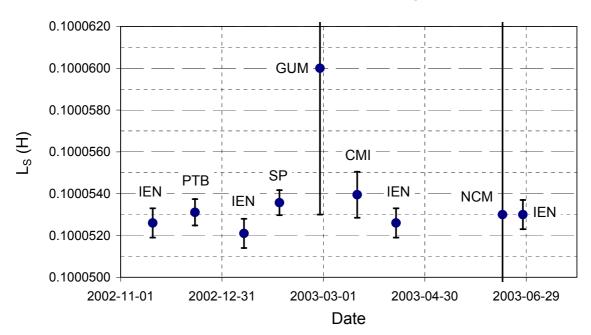
Lab	Date	Ls	U <sub>95</sub> (L <sub>S</sub> )	Rs	U <sub>95</sub> (R <sub>S</sub> )	f	U 95(f)	R <sub>PT100</sub>	U 95(R PT100)	T ext	H <sub>ext</sub>
		Н	Н	Ω	Ω	Hz	Hz	Ω	Ω	°C	%RH
IEN	2002-11-20	0.1000526	7×10 <sup>-7</sup>	82.91	0.013	1000.0000	5×10 <sup>-4</sup>	110.678	2×10 <sup>-3</sup>	22.7÷23.0	35
PTB	2002-12-15	0.10005312	6.3×10 <sup>-7</sup>	82.88	0.44	1000.5	0.6	110.6737	1.8×10 <sup>-3</sup>	22.89	16.0
IEN	2003-01-13	0.1000524	7×10 <sup>-7</sup>	82.91	0.013	1000.0000	5×10 <sup>-4</sup>	110.676	2×10 <sup>-3</sup>	22.7÷23.0	38
SP	2003-02-03	0.10005357	6.0×10 <sup>-7</sup>	82.894	0.025	1000.00	1×10 <sup>-2</sup>	110.6711	4.4×10 <sup>-3</sup>	23.0	40
GUM	2003-02-27	0.100060	7×10 <sup>-6</sup>	82.887	0.025	1000.0000	4×10 <sup>-4</sup>	110.658÷110.678	6×10 <sup>-3</sup>	22.4÷23.7	32÷38
CMI	2003-03-21	0.10005395	1.1×10 <sup>-6</sup>	82.907	0.015	1000.0000	1×10 <sup>-4</sup>			22.9	48
IEN	2003-04-13	0.100052645	7×10 <sup>-7</sup>	82.91	0.013	1000.0000	5×10 <sup>-4</sup>	110.676	2×10 <sup>-3</sup>	22.7÷23.0	56
NCM	2003-06-15	0.100053	1.3×10 <sup>-5</sup>	82.24	0.41	999÷1001	0.6	110.667÷110.673	1.7×10 <sup>-4</sup>	22.2÷22.9	39÷46
IEN	2003-06-27	0.10005298	7×10 <sup>-7</sup>	82.91	0.013	1000.0000	5×10 <sup>-4</sup>	110.677	2×10 <sup>-3</sup>	22.7÷23.0	60

### 4TP measurements

Lab	Date	Ls	U <sub>95</sub> (L <sub>S</sub> )	Rs	U <sub>95</sub> (R <sub>S</sub> )	f	U 95(f)	R <sub>PT100</sub>	U <sub>95</sub> (R <sub>PT100</sub> )	T ext	H <sub>ext</sub>
		Н	Н	Ω	Ω	Hz	Hz	Ω	Ω	°C	%RH
IEN	2003-01-13	0.10001145	7×10 <sup>-7</sup>	82.91	0.013	1000.0000	5×10 <sup>-4</sup>	110.676	2×10 <sup>-3</sup>	22.7÷23.0	38
CTU	2003-03-13	0.1000089	6.23×10 <sup>-6</sup>	82.874	0.0593	1000		110.675		23.0	26
IEN	2003-04-13	0.10001140	7×10 <sup>-7</sup>	82.91	0.013	1000.0000	5×10 <sup>-4</sup>	110.676	2×10 <sup>-3</sup>	22.7÷23.0	38

Tab. 6.1. Results given by the participating laboratories.

# 2T Series inductance L<sub>S</sub>



# 4TP Series inductance L<sub>S</sub>

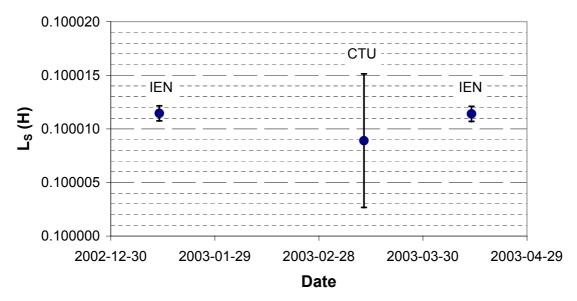


Fig. 6.1. Results given by the participating laboratories, with associated expandeded uncertainties, for the series inductance  $L_{\rm S}$ . (upper) 2T measurements. (lower) 4TP measurements.

### 6.2 Normalization of the results

## 6.2.1 Temperature dependence

The inductance values given by the participants have been corrected for the temperature, represented by the measured  $R_{\rm PT100}$  (when an interval has been given, the center of the interval is taken as  $R_{\rm PT100}$ ). The uncertainty on  $R_{\rm PT100}$ , and the uncertainty of the coefficient has not been considered significant to enlarge the reported uncertainty of L, which is left as stated.

# 6.2.2 Humidity dependence

The dependence of the inductance of the standard upon humidity has not been studied and therefore data cannot be corrected for local humidity.

### 6.2.3 Drift correction

From measurements reported in Chap. 5, the drift of the standard is considered negligible during the time of the comparison.

### 6.2.4 Normalized results

- a) IEN L and  $R_{PT100}$  measured values have been condensed in a single one, by averaging. Uncertainty is kept equal to those of a single reported value.
- b) Conventionally, all reported L values have been corrected taking as reference IEN average value,  $R_{\rm PT100} = 110.6768 \,\Omega$ .

Normalized results are reported in Tab. 6.2. for the 2T measurements.

# 6.3 Calculation of the reference value and its uncertainty

The calculation of the reference value has been conducted by following the procedure stated in M. G. Cox, "The evaluation of key comparison data", *Metrologia* 2002, 39, 589-595. Data does not completely fulfill the conditions, in particular NCM is traceable to PTB. However, a *posteriori* analysis revealed that including or excluding NCM data from calculations give irrelevant changes in the outcome

The reference value has been calculated for 2T measurements only. Reasons are:

- 4TP measurements have been conducted by only two laboratories (IEN and CTU);
- values are consistent but CTU uncertainty is much larger than IEN. Hence, Procedure A would give IEN value as reference value;
- CTU is not a National Metrology Laboratory and has no CMC claims to support. The comparison has been carried out for scientific purposes.

Normalized results pass a chi-square test. Hence, Procedure A (weighted mean) can be applied for the calculation of the reference value

The uncertainty of the reference value has been computed by adding two terms: the standard deviation of the weighed mean and the standard deviation of the mean of the four IEN values (as representative of drifts of the travelling standard).

Differences between normalized values and reference value are given in Tab. 6.2, and in graphical form in Fig. 6.2.

2T me	asurements		
Lab	Ls	$U_{95}(L_S)$	ΔĹ
	Н	Н	μН
Ref	0.10005328	4.2×10 <sup>-7</sup>	
IEN	0.10005266	7×10 <sup>-7</sup>	-0.62
PTB	0.10005314	6.3×10 <sup>-7</sup>	-0.14
SP	0.10005361	6.0×10 <sup>-7</sup>	0.33
GUM	0.10006007	7×10 <sup>-6</sup>	6.79
CMI	0.10005395	1.1×10 <sup>-6</sup>	0.67
NCM	0.10005305	1.3×10 <sup>-5</sup>	-0.23

Tab. 6.2. Normalized values of the series inductance Ls; calculated reference value; differences between normalized values and reference value, for 2T measurements.

# 2T series inductance differences $\Delta L$

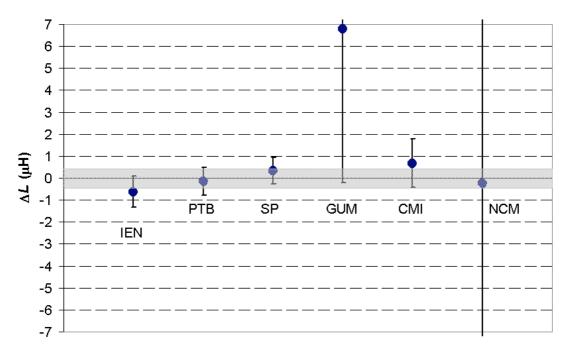


Fig. 6.2. Data of Tab. 6.2, in graphical form.

# 6.4 Degrees of Equivalence

Not requested for a supplementary comparison.

# 6.5 Link to the CCEM Key Comparison and degrees of equivalence

Not requested for a supplementary comparison.

# 6.6 Impact of the comparisons on the calibration and measurement capabilities of a participating laboratory (CMCs)

Declarations from the laboratories are here reported. See also 7: Withdrawal or changes to results.

IEN 2004.08.08	CMC claims are supported by the results given in the report.
PTB 2005.07.18	CMC claims are supported by the results given in the report. [Agreed to
	report among proposed phrases. Declaration explicited by the pilot].
SP 2005.04.12	CMC claims are supported by the result given in the report.
GUM 2005.04.16	CMC claims are supported by the result given in the report.
CMI 2005.04.15	CMC claims are supported by the result given in the report.
UME 2004.18.10	We withdrawn from the pilot study. We want to partcipate EUROMET
	supplementary comparison [EUROMET 816]. We already discussed at
	TCEM meeting.
NCM 2005.04.15	CMC claims are supported by the results given in the report.
CTU	[No CMC]

# 7 Withdrawal or changes of results

The standard has been sent to UME and measured. UME submitted to the pilot the measurement report. UME results showed a significant discrepancy with data from other laboratories. The pilot informed UME of the discrepancy. In 2004.18.10 (before Draft A publication), UME retired from the comparison. Hence, results from UME are not reported in this Draft.

After TCEM meeting in Sep 2006, UME modified its CMC declarations and will participate in the EUROMET 816 project "Intercomparison of 100 mH inductance standards", follow-up of EUROMET 607 project.

# 8 Requests for follow-up bilateral comparisons

UME decided to withdraw the results from the comparison in 2004.18.10 and will participate in a follow-up comparison.

# 9 Summary and conclusions

The comparison started as an Euromet (607) comparison and has been upgraded to a EUROMET.EM supplementary comparison (all participants agreed to this upgrading). Seven participants measured the standard and gave results. After suggestion from the pilot, one laboratory withdrew its results from the comparison.

The comparison started in Nov 2002 and ended in June 2003. The pilot laboratory measured the standard four times: two at the beginning and end of the comparison, and two times in between. No damage to the standard, either mechanical or electrical, has been identified.

No participant deviated from the comparison protocol and reported the significant information required. UME decided to withdraw from the comparison.

The standard is configured to be measured both as two-terminal (2T) and four terminal-pair (4TP). Most participants measured the standard as 2T. Only the pilot and CTU measured it as 4TP. Since CTU is not a National Metrology Laboratory and has no CMC to support, further data processing to obtain a reference value has been conducted on 2T measurements only. 2T results have been corrected for temperature dependence and reference value evaluated with a weighed mean.

Each laboratory has checked the results of the comparison on their CMC claims.

Luca Callegaro August 2006

# [Appendix A. Degrees of equivalence]

Not requested for a supplementary comparison

# **Appendix B.** Methods of measurement

# IEN Measurement method

The principle of the method is illustrated in Fig. 1. The standard impedance  $Z_s$  and the unknown impedance  $Z_x$  are connected in series. The current I flows into both, and voltage drops  $U_s = Z_s I$ ,  $U_x = Z_x I$ , and  $U = (Z_s + Z_x) I$  develop on the mesh. The rms values  $U_s$ ,  $U_x$  and U of these voltages are measured with a high-accuracy ac voltmeter. It is then possible to compute  $Z_x$  as a function of  $Z_s$  and ratios of  $U_s$ ,  $U_x$  and U.

To avoid the large common voltage in the measurement of U, it is possible instead to measure the rms voltage  $U_m$  between the common point of the impedances and the 0.5 tap of an inductive voltage divider. A diagram of the vectors involved is shown in Fig. 2. The equations corresponding to the vectors are reported in Fig. 3.

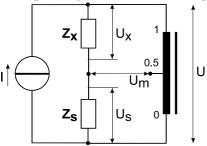


Fig. 1. Principle of the method.

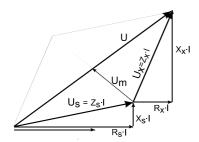


Fig. 2. Vector diagram of the voltages involved.

The fundamental relation between the components of  $Z_1$  and  $Z_2$  established by the comparison method can be expressed as

$$\begin{bmatrix} R_1 \\ X_1 \end{bmatrix} = \begin{bmatrix} \alpha_B & -\alpha_A \\ \alpha_A & \alpha_B \end{bmatrix} \cdot \begin{bmatrix} R_2 \\ X_2 \end{bmatrix}$$

where  $R_{1,2}$  and  $X_{1,2}$  are the real and imaginary parts of the two impedances and the coefficients  $\alpha_{A,B}$  are functions of the two measured voltage ratios  $\rho_1=U_1/U_2$  and  $\rho_3=U_3/U_2$ . Namely it results

$$\alpha_{A} = \frac{1}{2} \sqrt{\left[1 - \left(\rho_{3} - \rho_{1}\right)^{2}\right] \left[\left(\rho_{3} + \rho_{1}\right)^{2} - 1\right]}$$

$$\alpha_{B} = \frac{1}{2} \left(\rho_{3}^{2} - \rho_{1}^{2} - 1\right)$$

Fig.3. Equations.

# PTB 1. Description of the measurement principle

Inductance measurements at PTB are carried out with a Maxwell-Wien bridge. This bridge has the advantage that the first order bridge equation (1) is independent of frequency. Measurements at a frequency of 1 kHz require an investigation of higher order effects.

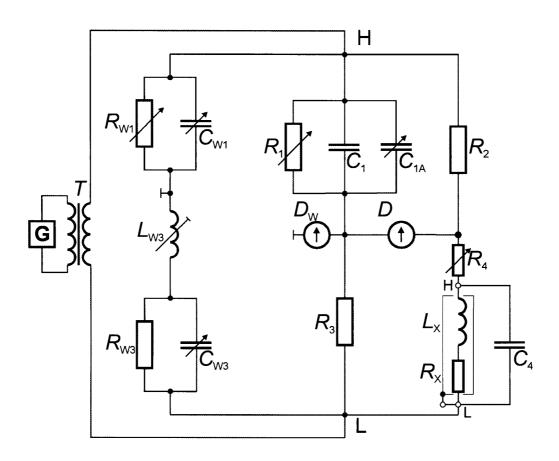


Fig. 1: Principal circuit diagram of the Maxwell-Wien bridge at PTB

The main arm of the bridge contains besides the DUT, represented by the elements  $L_X$  and  $R_X$ , the fixed capacitor  $C_{1,1}$  the variable capacitor  $C_{1,2}$ , the two fixed resistors  $R_2$  and  $R_3$  and the variable resistor  $R_1$ .

The main bridge balance (3) is achieved with components  $C_{1A}$  and  $R_{1.}$ 

The bridge is adapted to the value of inductor  $L_X$  by exchanging  $C_1$ ,  $R_2$  and  $R_3$ .

$$L_{x} \approx C_{1} \cdot R_{2} \cdot R_{3} \tag{1}$$

The impedance of the resistors can be characterized by

$$Z_n \approx R_n(f) \cdot (1 + j\omega \tau_n)$$
 (2)

With the impedances  $Z_2$  and  $Z_3$  the bridge equation leads to

$$R_X + j\omega L_X = \frac{Z_2 Z_3 (1 + j\omega (C_1 + C_{1A}) Z_1)}{Z_1 - j\omega C_4 Z_2 Z_3 (1 + j\omega (C_1 + C_{1A}) Z_1}$$
(3)

The capacitance  $C_4$  characterizes only the bridge terminals. The inherent parallel capacitance of the inductor is not included in  $C_4$ .

To eliminate the main effects of the time constants  $\tau_n$ , a zero-substitution method is employed: Inductor  $L_X$  is replaced by a relatively small inductor  $L_{X0}$ , and  $C_1$  is removed. The value  $C_{1A0}$  for  $C_{1A}$  is obtained to balance the bridge.

This procedure results in the model equation that approximates  $L_X$  within the uncertainties of the calibration

$$L_{X} = (C_{1} + C_{1A})R_{2}R_{3} - C_{1A0}R_{2}R_{3} + L_{X0}$$

$$-\omega^{2}R_{2}R_{3}C_{1}\tau_{2}\tau_{3}$$

$$-\omega^{2}R_{2}^{2}R_{3}^{2}(C_{4H}(C_{1} + C_{1A})^{2} - C_{40}C_{1A0}^{2})$$

$$+(C_{4H} - C_{40})\frac{R_{2}^{2}R_{3}^{2}}{R_{4}^{2}}$$
(4)

# 2. Configuration chosen for measurement

The Maxwell-Wien bridge of PTB is designed to measure two- or three-terminal inductance standards. The 4/2 adapter was therefore used to measure the travelling standard in a two-terminal configuration.

# 3. Details of experimental setup

When the travelling standard arrived at PTB, we started with the measurement of  $R_{PT100}$  and  $R_{L,DC}$  (dc resistance of standard). The thermostat powered by the 12-V dc supply was turned on the second day. It was run until the shipment of the standard.

## Determination of equivalent series inductance $L_s$

After four days we started the inductance measurement with the Maxwell-Wien bridge. During the measurements the thermostat was powered by a 12-V lead storage battery. During this time no resistance measurements of PT100 and coil resistance were carried out.

The standard was connected with the 4/2 adapter to the bridge without using resistor R<sub>4</sub> (see Fig. 1). Capacitor  $C_1$  is a GR1404 standard of 1 nF or a parallel connection of two GR1404 standards of 1 nF. Resistors  $R_2$  is a 10-k $\Omega$  or a 5-k $\Omega$  Vishay resistors and  $R_3$  is a 10-k $\Omega$  Vishay resistor with very low temperature coefficients. This part of the measurement, is called main measurement. The main measurement with  $C_1$  = 1 nF was repeated five times during four days. On the following four days, the main measurement with  $C_1 = 2$  nF was carried out five times.

In the second part of the measurement (zero-substitution), the standard was replaced by a small air coil ( $L_{X0}$ ) and the variable resistor  $R_4$  was inserted. The 4/2 adapter was not used. Following this the inductance  $L_{x0}$  was measured with an LCR meter.

Finally we got ten single results for the travelling standard.

### Determination of the parallel capacitance C4

The capacitance  $C_4$  was measured with the LCR meter Agilent 4284A. For the determination of  $C_{4H}$  (main measurement) the 4/2 adapter was added to the bridge terminals. The LCR meter was connected in place of the DUT. The open and short corrections were performed without the 4/2 adapter.

The capacitance  $C_{40}$  (zero-substitution) has almost no influence on the result. Here the LCR meter was connected instead of  $L_{\times 0}$ .

## Determination of equivalent series resistance $R_S$

For the determination of resistance R<sub>S</sub>, we used the LCR meter Agilent 4284A. After each measurement of L<sub>S</sub> with the Maxwell-Wien bridge, we measured the resistance with the same current. The connection between the four terminal-pair input connectors of the LCR meter and the 4/2 adapter of the standard consisted of a secondary 4/2 adapter from our laboratory.

The main part of the uncertainty budget is determined by the measurement accuracy  $A_{R_s}$  of the LCR meter.

$$A_{R_s} = X_x (D_e + \theta_{cal}) \tag{5}$$

Specifications from the Agilent 4284A manual

quantity	value
X <sub>x</sub>	628,32 Ω
D <sub>e</sub>	5 · 10 <sup>-4</sup>
$ heta_{cal}$	1 · 10 <sup>-4</sup>

Statistic analysis of the

observations

Average of the individual values observed  $r = 82,880 \Omega$ 

$$r = 82.880 \, \Omega$$

 $r = 82,880 \Omega$ Standard uncertainty of r $u_r = 5,7 \cdot 10^{-3} \Omega$ 

$$u_r = 5.7 \cdot 10^{-3} \,\Omega$$

$$R_s = r + k_R$$
  
 $k_R = 0$  (6)  
 $u_{k_R} = \frac{A_{R_s}}{\sqrt{3}} = 218 \cdot 10^{-3} \Omega$ 

# Instruments for measurement of quantities of secondary importance

f universal counter R<sub>PT100</sub> precision multimeter

 $T_{ext}$ electronic thermometer with NTC sensor electronic hygrometer with capacitive sensor  $H_{ext}$ 

# **Description of the measurement method**

The realisation of the inductance unit, the Henry, is performed by a parallel resonance method. The realised inductance unit is traceable from the realised and maintained units of frequency, capacitance and resistance at SP.

### **Inductance realisation**

In the case of 1 kHz measurement frequency a 200 mH inductor (Sullivan type R1942 s.n. 9360/1963) with a Q-value of the order of 20 is connected in parallel with a variable capacitor. The parallel connected inductor and capacitor are balanced against a resistance standard in an AC resistance bridge which forms a resonance bridge. With a fixed measurement frequency the bridge can be balanced by adjustment of the variable capacitor. At resonance the bridge balance determines the resonance conductance of the parallel connected inductor L and capacitor Cr from the resistance standard  $R_2$ .

After the resonance detection the inductor L is disconnected and the bridge is connected as a capacitance bridge and the resonance capacitance and loss angle values of Cr are determined from the capacitance standard C2. The two bridge principles are given in figure 1. The Wagner components Cw and Rw balance the shunt admittances of the cables to reduce the IVD errors and have no other effect on the bridge balance.

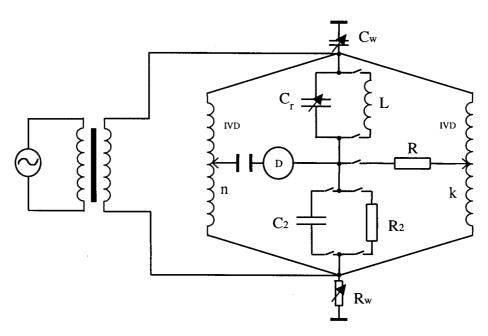


Figure 1: Inductance realisation set-up. The connection of L and R2 forms the resonance bridge and the connection of C2 and R forms the capacitance bridge

From the two bridge measurements and the measured resonance frequency fr ( $\omega r = 2\pi fr$ ) the series inductance and resistance of the inductor L can be found by calculations. The method gives a realisation of the equivalent parallel inductance by equation 1. The simultaneous determination of the Q-value (Rr) of the inductor gives the equivalent series inductance by equations 2 and 3. Appendix 1 gives the used equation (4) in the realisation with includes influences of lead inductance and Cr dissipation factor.

$$Lp = (\omega r^2 Cr)^{-1}$$
 (1)

$$Ls = Lp(1+Q^{-2})^{-1}$$
 (2)

$$Q = Rr(\omega r Lp)^{-1}$$
 (3)

The bridge in figure 1 has a coaxial construction with current equalisers in each mesh and two calibrated eight dial two stage inductive voltage dividers. The resistor R is used to balance the loss angle difference when the bridge is used as a capacitance bridge and is disconnected when the bridge is connected as a resonance bridge.

# Inductance comparison bridge

The 200 mH inductor used in the 1 kHz realisation is compared in an inductance comparison bridge to transfer the realised inductance unit to other inductance standards with 1:1 to 1:10 ratios. In the case of the 100 mH intercomparison standard the ratio 1:2 is used.

The bridge principle is given in figure 2. The bridge has a coaxial construction. The IVD is a calibrated eight dial two stage inductive voltage divider. The measurement consists of two balances to determine the lead inductance and a final balance to determine the ratio between the inductors L<sub>1</sub> and L<sub>2</sub>. The change of the inductance for different settings of the variable resistor R is determined separately. The slidewire r has a calculable inductance variation as a function of the setting.

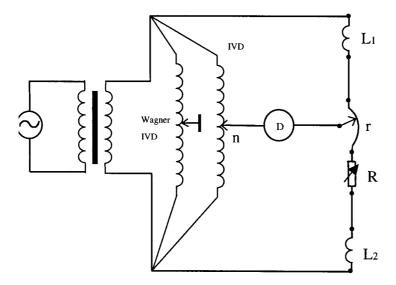


Figure 2: Inductance comparison bridge with the balance equation  $L'_1 = (n^{-1}-1) L'_2$  where L'<sub>1</sub> and L'<sub>2</sub> is the total equivalent series inductance on each side of the bridge

# GUM I. The measurement principle

The measurements were carried out by comparison of the 100 mH inductance standard with the 10 nF capacitance standard in the Combined Transformer Bridge (CTB). This instrument has been developed by Prof. Andrzej Muciek and is described in the article: "A combined transformer bridge for precise comparison of inductance with capacitance", IEEE Trans. Instrum. Meas., vol. IM-32, pp. 419-423, 1983. The short description of the measurement principle, prepared by Prof. Andrzej Muciek, is enclosed to this report in item V.

Date of measurements: from 24.02.1997 to 1.03.2003

### II. The configuration of the measurement

Measurements were carried out at two-terminal configuration of the inductance standard using the given adapter and at covered VH and VL terminal by given two non-shorting caps.

### CMI Measurement method

Our method is based on voltage ratio determination at measured coil (its voltage) to voltage at capacitance standard on transformer bridge PIM ( TESLA made). The coil and the capacitance are connected in series as in Fig. 1. Substitution method is used to decrease the  $U_R$  voltage source inaccuracy influence. In this method the measured coil is replaced with a resistor, which time constant is known and the resistance is similar to the resistance of measured coil.  $U_L / U_C$  voltage ratio is made using inductive voltage dividers and voltage ratio transformers. The  $U_R / U_C$  voltage amplitude ratio is made as well in the same way.  $90^0$  phase shift of  $U_R$  voltage is created with an electronic phase shifter and is checked with an auxiliary comparison of an precision capacitor with a resistor which time constant is known.

Used transformer bridge PIM (TESLA made) is intended for very accurate impedance and admittance comparison of resistance, capacitance and inductance standards in low frequency area.

For inductance PIM bridge measurement (Fig.1) the measured coil impedance is compared to capacitance standard impedance. For 100 mH inductance measurement at 1 kHz, 100 pF comparing capacitance is used. This capacitance is connected to the B-terminal with voltage of significance 2,533030.

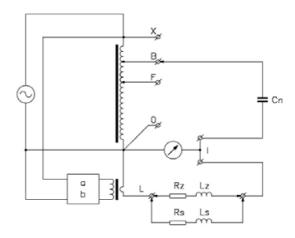


Fig.1

PIM transformer bridge is supplied with AC voltage at 1 kHz frequency from HP 33122A synthesizerAs a capacitance standard for object inductance determination the invar capacitor 100 pF GR 1404B has been used. Capacitance value and loss angle is expressed with cables influence by measurement at capacitance bridge Andeen-Hagerling model 2500E.

The significance of B and L terminals (e.g. voltage ratio to reference F terminal voltage) has been expressed using inductance divider measurement system CMI. Substitution resistor have the same value as the value of measured coil loss resistance with relative deviation up to 1 %. The value of resistance of substitution element was determined by comparison with 100  $\Omega$  bifilar standard resistor, type AC/DC No. 07/02 – 2002.with calculable frequency characteristic.using an inductance divider measurement system.

### INDUCTANCE STANDARD MEASUREMENT

Measured object in 2-terminals arrangement is connected with a special triaxial cable to the L and terminals of the PIM bridge. For that connection an outer cable screening is connected to the L terminal and the other end of this screening to the measured coil terminal, are connected to an object cover. An inner cable screening is grounded and works as a central wire screening. The central wire connects "live H terminal" of measured coil to the null indicator input I terminal.

Capacitance standard used for comparison (for 100 mH inductance measurement has nominal value 100 pF) is connected to the B and I terminals. The bridge is balanced for zero reading of null indicator by suitable adjustment or two 6-decade dividers with **a** and **b** reading. After that the capacitance supplying cable is reconnected from B-terminal to a terminal with significance "zero" (E-terminal is used for this purpose with switch set to zero). After bridge balancing the readings  $a_2$ ,  $b_2$  are recorded.

In the next step the measured object is replaced with substitution resistor that has the resistance value approximately the same as the measured coil loss resistance is. Capacitor cable is reconnected to the B-terminal. After bridge balancing the readings  $a_3$ ,  $b_3$  are recorded. In the fourth step the capacitor is reconnected back to E = 0 terminal and measured  $a_4$ ,  $b_4$ .

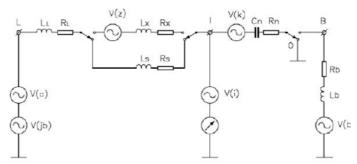
$$\alpha = a_1 - a_2 - a_3 + a_4 \tag{1}$$

$$\beta = b_1 - b_2 - b_3 + b_4 \tag{2}$$

Measured object leads influence and spurious voltage (inducted into measuring circuit parts) influence is excluded in this way.

It is possible to prove the correctness of described process mathematically with an equivalent measuring circuit scheme shown in Fig.3

This method excludes spurious inducted voltages influence, self-impedance of L-terminal influence and measured object leads impedance as well.



V(z), V(k), V(i) - PARASITIC VOLTAGE

Fig. 2

The results of inductance standard measurement in PIM bridge are evaluated by means of the equations (3) and (4).

$$L_1 = L_S + [-\alpha - Re(\gamma)] \cdot [1 + Re\delta(a) - \omega^2 \cdot L_1 \cdot dC_0] / [\omega^2 \cdot C_N \cdot B \cdot (1 + \omega^2 \cdot L_B(C_N + C_{NO})]$$
 (3)

$$R_1 = R_S + [\beta + Im(\gamma) + Im(B) + Im(B) \cdot \omega^2 L_1 (C_N + C_{NO})] \cdot (1 - 2\omega^2 \cdot L_1 \cdot dC_O) / (B \cdot \omega \cdot C_N)$$
 (4)

### Where

L<sub>1</sub> = measured coil inductance

 $L_S$  = substitution resistor inductance (less than 0,1  $\mu$ H)

R<sub>1</sub> = loss resistance of measured coil

R<sub>S</sub> = substitution resistor resistance

 $\alpha = (a_1 - a_2 - a_3 + a_4)$  result of progressive measurement in a-channel

 $\beta = (b'_1 - b_2 - b'_3 + b_4)$  result of progressive measurement in b-channel

γ = correction for residual voltage in channel a

C<sub>N</sub> = capacitance of comparative C<sub>N</sub> standard

C<sub>N0</sub> = capacitance against ground connected

 $dC_0$  = increasing of object terminal mutual capacitance after connection of measuring system

 $L_B$  = B-terminal output inductance ( $\mu H$ )

 $R_e\delta(a)$  = real part of <u>a</u> channel significance relative deviation from nominal value (affected by object spurious capacitance against ground as well)

 $I_m(a)$  = quadrature (phase) deviation of  $\underline{a}$  channel significance from nominal value

B = B-terminal significance

 $I_m(b)$  = quadrature (phase) deviation of B-terminal significance

**NCM** 

- 1. The value of the travelling standard is determined by substitution using standard inductance measure model GR 1482 L (reference standard 100 mH) indent. Nº 18163. The reference standard is put in an air thermostat at a temperature of 22,9  $\pm$  0,2 °C. The reference standard is calibrated NCM cal.sertificat 020-AV/31.03.2003 and is traceable to PTB. The two standards are measured in turn by bridge model GR 1632 A in two-wire connection (with adapter 4/2 turn on) using five decades of the bridge. The last digit of the measurements (missing in the uncertainty budget table) is estimated by debalancing the bridge with a step of the last decade in plus and in minus. Before every pair of measurements the generator frequency is fine adjusted at 1000 Hz. The measuring current is 1 mA.
- 2. The value  $L_X$  of the traveling standard is obtained from the relationship:

$$L_X = (L_S + \delta L_D + \delta L_{TS})l_K l - \delta L_{TX}$$

where:

 $L_{S}$  – inductance value of the reference standard,

 $\delta L_{\mathcal{D}}$  drift of the inductance of the reference standard since it's last calibration,

 $\delta L_{TS}$  – temperature related inductance variation of the reference,

 $I = L_{jX}/L_{jS}$  - ratio of the indicated inductance for the travelling and reference standards (index j means the number of the measurement),

 $I_K$  – correction factor for parasitic voltages, temperature dependent of the bridge and error of reading using the bridge galvanometer,

 $\delta L_{TX}$  – temperature related inductance variation of the travelling standard.

**3. Reference standard** ( $L_s$ ): The value of the inductance of reference standard from the calibration certificate is  $100,031(1 \pm 1,2\times10^{-4})$  mH (coverage factor k=2) at the specified reference temperature of 22,9 °C.

# CTU 1. Measurement principle

A double-balance L-bridge was used to compare the travelling standard with a 1 nF reference air capacitor. Principle of this bridge is evident from Fig. 1, where  $L_{\rm p}$  and  $R_{\rm p}$  are parallel equivalent inductance and resistance of the travelling standard, C is the reference capacitance, R is a variable resistance and  $R_1$ ,  $R_2$  are resistances of the bridge ratio arms, which are formed by a 10 000  $\Omega$  and a 100 000  $\Omega$  thermostatted precision metal foil resistor.

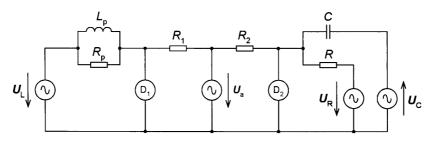


Fig. 1 Principle of the L-bridge

Zero readings of detectors  $D_1$  and  $D_2$  can be achieved by properly adjusting the voltages  $U_L$ ,  $U_C$ ,  $U_R$  and  $U_a$ . Adjustable in-phase voltages  $U_L$ ,  $U_C$  and  $U_R$  are derived from the output voltage of a single-channel generator by means of the following inductive ratio devices:

- a) A precision ratio transformer having two separate cores and two 10-section secondary windings connected in parallel. Both secondary windings have the same number of turns and one of them (the auxiliary one), envelops one core only. The other secondary winding (the ratio one) and the primary winding envelop both cores.
- b) Three eight-decade two-stage inductive voltage dividers. Exciting windings of these dividers being connected to appropriate taps on the auxiliary winding of the ratio transformer, their ratio windings are connected to the corresponding taps on the ratio winding of the ratio transformer, which are practically equipotential with those to which the exciting windings are connected.

An RC phase shifter supplied from the auxiliary winding of the ratio transformer and followed by an eight-decade inductive voltage divider serves as a source of  $U_a$ .

Balance is attained when

$$L_{\mathbf{p}} = \frac{1}{\omega^2 C} \frac{R_1}{R_2} \frac{U_{\mathbf{L}}}{U_{\mathbf{C}}} \tag{1}$$

and

$$R_{\rm p} = R \, \frac{R_1}{R_2} \, \frac{U_{\rm L}}{U_{\rm R}} \ . \tag{2}$$

Series equivalent inductance and resistance are

$$L_{\rm S} = \frac{L_{\rm p}}{1 + \tan^2 \delta_{\rm L}} \tag{3}$$

and

$$R_{\rm s} = \frac{L_{\rm s}}{CR} \frac{U_{\rm R}}{U_{\rm C}} \quad , \tag{4}$$

where

$$\tan \delta_{\mathbf{L}} = \frac{1}{\omega C R} \frac{U_{\mathbf{R}}}{U_{\mathbf{C}}} . \tag{5}$$

The ratio  $R_1/R_2$  has been measured by comparing the bridge resistive ratio arms with an eight-decade two-stage inductive voltage divider.

# Appendix C. Uncertainty budgets

# IEN Uncertainty budget for the measurement of R and L

Combined uncertainty and expanded uncertainty have been obtained by MonteCarlo simulations on the model

Name of the quantity	Sym bol	Uncertainty	Type	Distribution	Remarks
Standard series resistance	$R_{\rm S}$	1.6 mΩ	В	Rectangular	de calibration
Standard series reactance	$X_{\mathbb{S}}$	$1.1~\mathrm{m}\Omega$	В	Rectangular	ac-dc corrections
				_	Temperature
Voltage on standard HP	$U_{S}$	0.8 μV			Run of 50 measurements
Voltage on unknown HP	$U_{X}$	0.5 μV	A	Normal	
Third voltage	$U_{\rm m}$	0.5 μV			
Voltage on standard LP	$U_{\rm SL}$	0.3 μV	В	Rectangular	Deviation from zero
Voltage on unknown LP	$U_{\rm XL}$	0.3 μV		•	
Ratio <i>U<sub>X</sub>/U</i> <sub>S</sub>	α <sub>X</sub> ,	1.2.10-6	В	Rectangular	Voltmeter nonlinearity
Ratio $U_{\rm m}/U_{\rm S}$	$\alpha_{m}$	1.2.10-6			
Unknown series resistance	$R_{\rm X}$	40.10-6	В	Rectangular	Temperature
		$3~\mathrm{m}\Omega$	В	Rectangular	Contact resistance
Unknown series reactance	$X_{\mathbb{X}}$	1.7-10-7	В	Rectangular	Temperature
Series resistance of the travelling standard	R	Expan	ded unce	Combined uncertainty: rtainty (95% coverage):	7 mΩ <b>13 m</b> Ω
Series inductance of the travelling standard	L	Combined uncertainty: Expanded uncertainty (95% coverage):			340 nH <b>700 nH</b>

# PTB 5. Complete uncertainty budget for L<sub>s</sub>

Because of the two different bridge configurations during the measurement we have to give two different uncertainty budgets for each of them and another one for the combination.

### 5.1. Model equation

5.1.1. Model equation for single bridge configurations

$$\begin{split} L_{1/2} &= (C_{1H} - C_{1A0})R_2R_3 + L_{X0} + TypB_T \\ &- \omega^2 R_2 R_3 C_1 (k_2 + k_3 - \tau_2 \tau_3) \\ &- \omega^2 R_2^2 R_3^2 (C_{4H} C_{1H}^2 - C_{40} C_{1A0}^2) \\ &+ (C_{4H} - C_{40}) \frac{R_2^2 R_3^2}{R_1^2} \\ C_{1H} &= C_1 + C_{1A} \\ &\omega = 2\pi f \\ C_{1A} &= c_{1A} (1 + TypB_C) \\ C_{1A0} &= c_{1A0} (1 + TypB_R) \\ L_{X0} &= I_{X0} (1 + TypB_L) \end{split}$$

5.1.2. Model equation for final result

$$L_{s} = \frac{L_{1} + L_{2}}{2}$$

# 5.2. List of quantities

# 5.2.1. Definition of quantities

quantity	unit	definition				
Ls	Н	inductance of travelling standard				
L <sub>1/2</sub>	Н	interim results for travelling standard				
C <sub>1</sub>	F	apacitance of capacitor C <sub>1</sub>				
C <sub>1A</sub>	F	apacitance of capacitor C <sub>1A</sub>				
C <sub>1A</sub>	F	observations of capacitor $C_{1A}$				
C <sub>1A0</sub>	F	entire capacitance of zero-substitution				
C <sub>1A0</sub>	F	observations of capacitor $C_{1A0}$				
C <sub>1H</sub>	F	entire capacitance of main measurement				
C <sub>40</sub>	F	capacitance of bridge terminals in the zero-substitution				
C <sub>4H</sub>	F	capacitance of bridge terminals in the main measurement				
f	Hz	frequency of measurement				
<b>k</b> <sub>2</sub>	$s^2$	frequency coefficient of resistor R <sub>2</sub>				
<b>k</b> <sub>3</sub>	s <sup>2</sup>	frequency coefficient of resistor R₃				
$L_{x0}$	Η	inductance of small air coil $L_{x0}$				
I <sub>X0</sub>	Ι	observations of small air coil $L_{x_0}$				
R <sub>1</sub>	Ω	value of decade resistor $R_1$				
<b>r</b> <sub>1</sub>	Ω	observations of decade resistor R <sub>1</sub>				
R <sub>2</sub>	Ω	value of resistor R <sub>2</sub>				
<b>R</b> <sub>3</sub>	Ω	value of resistor R <sub>3</sub>				
TypB <sub>C</sub> <sup>1</sup>		takes into account the uncertainty of the capacitance meter				
TypB <sub>L</sub> <sup>1</sup>		takes into account the uncertainty of the inductance meter				
TypB <sub>R1</sub> <sup>1</sup>		takes into account the uncertainty of the decade resistor $R_1$				
TypB <sub>T</sub> ¹	Н	takes into account the uncertainty of the temperature stability of the travelling standard				
ω	s <sup>-1</sup>	radian frequency of measurement				
$ au_2$	s	time constant of resistor R <sub>2</sub>				
τ <sub>3</sub>	s	time constant of resistor R <sub>3</sub>				

# 5.2.2. Quantities of **first** bridge configuration ( $C_1 = 1 \text{ nF}$ )

quantity	type	value	half width, standard uncertainty	degrees of freedom
L <sub>1</sub>	result			
C <sub>1</sub>	type A summarized	1.0000113·10 <sup>-9</sup> F	607·10 <sup>-18</sup> F	1200
C <sub>1A</sub>	interim result			
C <sub>1A0</sub>	interim result			
C <sub>1H</sub>	interim result			
C <sub>40</sub>	type B rectangular	4.72·10 <sup>-12</sup> F	1·10 <sup>-12</sup> F	
C <sub>4H</sub>	type B rectangular	2.3·10 <sup>-13</sup> F	1·10 <sup>-12</sup> F	
f	type B rectangular	1000.5 Hz	0.5 Hz	
<b>k</b> <sub>2</sub>	type B rectangular	1·10 <sup>-16</sup> s <sup>2</sup>	1·10 <sup>-16</sup> s <sup>2</sup>	
<b>k</b> <sub>3</sub>	type B rectangular	1·10 <sup>-16</sup> s <sup>2</sup>	1·10 <sup>-16</sup> s <sup>2</sup>	
L <sub>x0</sub>	interim result			
R₁	interim result			
R <sub>2</sub>	type A combined	10000.1876 Ω	8.40·10 <sup>-3</sup> Ω	660
R <sub>3</sub>	type A combined	10000.2017 Ω	8.40·10 <sup>-3</sup> Ω	650
TypB <sub>C</sub>	type B rectangular	0	5·10 <sup>-5</sup>	
TypB∟	type B rectangular	0	2.5·10 <sup>-3</sup>	
TypB <sub>R1</sub>	type B rectangular	0	1·10 <sup>-3</sup>	
TypB⊤	type B rectangular	0 H	1·10 <sup>-8</sup> H	
ω	interim result			
$\pi$	constant	3.141592653589		
$ au_2$	type B rectangular	8·10 <sup>-10</sup> s	3.5·10 <sup>-9</sup> s	
τ <sub>3</sub>	type B rectangular	8·10 <sup>-10</sup> s	3.5·10 <sup>-9</sup> s	

# 5.2.3. Quantities of **second** bridge configuration ( $C_1 = 2 \text{ nF}$ )

quantity	type	value	half width, standard uncertainty	degrees of freedom
L <sub>2</sub>	result			
L <sub>2</sub>	type A summarized	2.0000946·10 <sup>-9</sup> F	1.00·10 <sup>-15</sup> F	50
C <sub>1A</sub>	interim result			
C <sub>1A0</sub>	interim result			
C <sub>1H</sub>	interim result			
C <sub>40</sub>	type B rectangular	4.72·10 <sup>-12</sup> F	1·10 <sup>-12</sup> F	
C <sub>4H</sub>	type B rectangular	2.3·10 <sup>-13</sup> F	1·10 <sup>-12</sup> F	
f	type B rectangular	1000.5 Hz	0.5 Hz	
k <sub>2</sub>	type B rectangular	$1.10^{-16} \text{ s}^2$	1·10 <sup>-16</sup> s <sup>2</sup>	
<b>k</b> <sub>3</sub>	type B rectangular	1·10 <sup>-16</sup> s <sup>2</sup>	1·10 <sup>-16</sup> s <sup>2</sup>	
L <sub>x0</sub>	interim result			
R <sub>1</sub>	interim result			
R <sub>2</sub>	type A combined	4999.7014 Ω	4.60·10 <sup>-3</sup> Ω	380
R <sub>3</sub>	type A combined	10000.2017 Ω	8.40·10 <sup>-3</sup> Ω	650
$TypB_C$	type B rectangular	0	5·10 <sup>-5</sup>	
TypB∟	type B rectangular	0	2.5·10 <sup>-3</sup>	
TypB <sub>R1</sub>	type B rectangular	0	1.10-3	
TypB <sub>T</sub>	type B rectangular	0 H	1·10 <sup>-8</sup> H	
ω	interim result			
$\pi$	constant	3.141592653589		
$ au_2$	type B rectangular	8·10 <sup>-10</sup> s	3.5·10 <sup>-9</sup> s	
τ <sub>3</sub>	type B rectangular	8·10 <sup>-10</sup> s	3.5·10 <sup>-9</sup> s	

# 5.3. Measurements

# 5.3.1. Measurements of **first** bridge configuration ( $C_1 = 1 \text{ nF}$ )

observation No.	C <sub>1A</sub> in F	C <sub>1A0</sub> in F	<i>I<sub>x0</sub></i> in H	$r_1$ in $\Omega$
1	7.3328·10 <sup>-13</sup>	2.9208·10 <sup>-13</sup>	3.88·10 <sup>-6</sup>	1205798.0
2	7.3334·10 <sup>-13</sup>	2.9471·10 <sup>-13</sup>	4.14·10 <sup>-6</sup>	1205816.0
3	7.3417·10 <sup>-13</sup>	2.9565·10 <sup>-13</sup>	4.13·10 <sup>-6</sup>	1205816.0
4	7.3305·10 <sup>-13</sup>	2.9443·10 <sup>-13</sup>	4.24·10 <sup>-6</sup>	1205811.0
5	7.55·10 <sup>-13</sup>	2.965·10 <sup>-13</sup>	4.12·10 <sup>-6</sup>	1205795.0
arithmetic mean	737.77·10 <sup>-15</sup> F	294.674·10 <sup>-15</sup> F	4.1020·10 <sup>-6</sup> H	1.20580720·10 <sup>6</sup> Ω
standard uncertainty	4.31·10 <sup>-15</sup> F	744·10 <sup>-18</sup> F	59.5·10 <sup>-9</sup> H	4.49 Ω
degrees of freedom	4	4	4	4

# 5.3.2. Measurements of **second** bridge configuration ( $C_1 = 2 \text{ nF}$ )

observation no.	C <sub>1A</sub> in F	С <sub>1А0</sub> in F	<i>I<sub>x0</sub></i> in H	$r_1$ in $\Omega$
1	1.32234·10 <sup>-12</sup>	3.6264·10 <sup>-13</sup>	4.24·10 <sup>-6</sup>	603070.0
2	1.32332·10 <sup>-12</sup>	3.6203·10 <sup>-13</sup>	4.19·10 <sup>-6</sup>	603071.0
3	1.32177·10 <sup>-12</sup>	3.6065·10 <sup>-13</sup>	4.20·10 <sup>-6</sup>	603065.5
4	1.32167·10 <sup>-12</sup>	3.6021·10 <sup>-13</sup>	4.21·10 <sup>-6</sup>	603067.5
5	1.32065·10 <sup>-12</sup>	3.5913·10 <sup>-13</sup>	4.20·10 <sup>-6</sup>	603067.7
arithmetic mean	1.321950·10 <sup>-12</sup> F	360.932·10 <sup>-15</sup> F	4.20800·10 <sup>-6</sup> H	603.068340·10 <sup>3</sup> Ω
standard uncertainty	438·10 <sup>-18</sup> F	631·10 <sup>-18</sup> F	8.6·10 <sup>-9</sup> H	0.975 Ω
degrees of freedom	4	4	4	4

# 5.4. Correlation coefficients

C <sub>1</sub>	1 nF	2 nF
$r(\tau_2,\tau_3)$	1	1
$r(c_{1A}, c_{1A0})$	0.63	0.86
$r(c_{1A}, I_{xO})$	0.08	-0.01
$r(c_{1A},r_1)$	-0.67	0.69
$r(c_{1A0}, I_{xO})$	0.68	0.46
$r(c_{1A0},r_1)$	0.14	0.68
$r(l_{xO},r_1)$	0.55	0.19
$r(R_2,R_3)$	1	1
r(C <sub>40</sub> ,C <sub>4H</sub> )	1	1

## 5.5. Uncertainty budget

## 5.5.1. Uncertainty budget for **first** bridge configuration ( $C_1 = 1 \text{ nF}$ )

quantity			degrees of freedom	sensitivity coefficient	uncertainty contribution	index
C <sub>1</sub>	1.000011300·10 <sup>-9</sup> F	607·10 <sup>-18</sup> F	1200	100·10 <sup>6</sup>	61·10 <sup>-9</sup> H	1.4 %
C <sub>1A</sub>	737.77·10 <sup>-15</sup> F	4.31·10 <sup>-15</sup> F				
C <sub>1A</sub>	737.77·10 <sup>-15</sup> F	4.31·10 <sup>-15</sup> F	4	100·10 <sup>6</sup>	430·10 <sup>-9</sup> H	70.6 %
C <sub>1A0</sub>	294.674·10 <sup>-15</sup> F	744·10 <sup>-18</sup> F				
C <sub>1A0</sub>	294.674·10 <sup>-15</sup> F	744·10 <sup>-18</sup> F	4	-100·10 <sup>6</sup>	-74·10 <sup>-9</sup> H	2.1 %
C <sub>1H</sub>	1.00074907·10 <sup>-9</sup> F	4.35·10 <sup>-15</sup> F				
C <sub>40</sub>	4.720·10 <sup>-12</sup> F	577·10 <sup>-15</sup> F	∞	-6900	-4.0·10 <sup>-9</sup> H	0.0 %
C <sub>4H</sub>	230·10 <sup>-15</sup> F	577·10 <sup>-15</sup> F	∞	-390·10 <sup>3</sup>	-220·10 <sup>-9</sup> H	19.1 %
F	1000.500 Hz	0.289 Hz	∞	-180·10 <sup>-12</sup>	-52·10 <sup>-12</sup> H	0.0 %
<b>k</b> <sub>2</sub>	100.0·10 <sup>-18</sup> s <sup>2</sup>	57.7·10 <sup>-18</sup> s <sup>2</sup>	∞	4.0·10 <sup>6</sup>	230·10 <sup>-12</sup> H	0.0 %
<b>k</b> <sub>3</sub>	100.0·10 <sup>-18</sup> s <sup>2</sup>	57.7·10 <sup>-18</sup> s <sup>2</sup>	∞	4.0·10 <sup>6</sup>	230·10 <sup>-12</sup> H	0.0 %
L <sub>x0</sub>	4.1020·10 <sup>-6</sup> H	59.8·10 <sup>-9</sup> H				
I <sub>xO</sub>	4.1020·10 <sup>-6</sup> H	59.5·10 <sup>-9</sup> H	4	1.0	60·10 <sup>-9</sup> H	1.3 %
R <sub>1</sub>	1.205807·10 <sup>6</sup> Ω	696 Ω				
<i>r</i> <sub>1</sub>	1.20580720·10 <sup>6</sup> Ω	4.49 Ω	4	0.0	0.0 H	0.0 %
R <sub>2</sub>	10000.1876 Ω	8.40·10 <sup>-3</sup> Ω	660	10·10 <sup>-6</sup>	84·10 <sup>-9</sup> H	2.7 %
R <sub>3</sub>	10000.2017 Ω	8.40·10 <sup>-3</sup> Ω	650	10·10 <sup>-6</sup>	84·10 <sup>-9</sup> H	2.7 %
$TypB_C$	0.0	28.9·10 <sup>-6</sup>	∞	44·10 <sup>-6</sup>	1.3·10 <sup>-9</sup> H	0.0 %
TypB∟	0.0	1.44·10 <sup>-3</sup>	8	4.1·10 <sup>-6</sup>	5.9·10 <sup>-9</sup> H	0.0 %
TypB <sub>R1</sub>	0.0	577·10 <sup>-6</sup>	8	62·10 <sup>-9</sup>	36·10 <sup>-12</sup> H	0.0 %
TypB <sub>₹</sub>	0.0 H	5.77·10 <sup>-9</sup> H	∞	1.0	5.8·10 <sup>-9</sup> H	0.0 %
ω	6286.33 s <sup>-1</sup>	1.81 s <sup>-1</sup>				
π	3.1415926535898					
$ au_2$	800·10 <sup>-12</sup> s	2.02·10 <sup>-9</sup> s	8	-3.2·10 <sup>-3</sup>	-6.4·10 <sup>-12</sup> H	0.0 %
$ au_3$	800·10 <sup>-12</sup> s	2.02·10 <sup>-9</sup> s	∞	-3.2·10 <sup>-3</sup>	-6.4·10 <sup>-12</sup> H	0.0 %
L <sub>1</sub>	0.10005332 H	487·10 <sup>-9</sup> H	7			

#### 5.5.2. Uncertainty budget of **second** bridge configuration ( $C_1 = 2 \text{ nF}$ )

		-111				
quantity	value	standard uncertainty	degrees of freedom	sensitivity coefficient	uncertainty contribution	index
C <sub>1</sub>	2.00009460·10 <sup>-9</sup> F	1.00·10 <sup>-15</sup> F	50	50·10 <sup>6</sup>	50·10 <sup>-9</sup> H	3.6 %
C <sub>1A</sub>	1.321950·10 <sup>-12</sup> F	439·10 <sup>-18</sup> F				
C <sub>1A</sub>	1.321950·10 <sup>-12</sup> F	438·10 <sup>-18</sup> F	4	50·10 <sup>6</sup>	22·10 <sup>-9</sup> H	0.7 %
C <sub>1A0</sub>	360.932·10 <sup>-15</sup> F	631·10 <sup>-18</sup> F				
C <sub>1A0</sub>	360.932·10 <sup>-15</sup> F	631·10 <sup>-18</sup> F	4	-50·10 <sup>6</sup>	-32·10 <sup>-9</sup> H	1.4 %
C <sub>1H</sub>	2.00141655·10 <sup>-9</sup> F	1.09·10 <sup>-15</sup> F				
C <sub>40</sub>	4.720·10 <sup>-12</sup> F	577·10 <sup>-15</sup> F	8	-6900	-4.0·10 <sup>-9</sup> H	0.0 %
C <sub>4H</sub>	230·10 <sup>-15</sup> F	577·10 <sup>-15</sup> F	∞	-390·10 <sup>3</sup>	-220·10 <sup>-9</sup> H	71.9 %
f	1000.500 Hz	0.289 Hz	8	-180·10 <sup>-12</sup>	-52·10 <sup>-12</sup> H	0.0 %
<b>k</b> <sub>2</sub>	100.0·10 <sup>-18</sup> s <sup>2</sup>	57.7·10 <sup>-18</sup> s <sup>2</sup>	8	4.0·10 <sup>6</sup>	230·10 <sup>-12</sup> H	0.0 %
<b>k</b> ₃	100.0·10 <sup>-18</sup> s <sup>2</sup>	57.7·10 <sup>-18</sup> s <sup>2</sup>	8	4.0·10 <sup>6</sup>	230·10 <sup>-12</sup> H	0.0 %
L <sub>x0</sub>	4.2080·10 <sup>-6</sup> H	10.5·10 <sup>-9</sup> H				
lxo	4.20800·10 <sup>-6</sup> H	8.60·10 <sup>-9</sup> H	4	1.0	8.6·10 <sup>-9</sup> H	0.1 %
R <sub>1</sub>	603.068·10 <sup>3</sup> Ω	<b>348</b> Ω				
r <sub>1</sub>	603.068340·10 <sup>3</sup> Ω	0.975 Ω	4	0.0	0.0 H	0.0 %
R <sub>2</sub>	4999.7014 Ω	4.60·10 <sup>-3</sup> Ω	380	20·10 <sup>-6</sup>	92·10 <sup>-9</sup> H	12.1 %
R <sub>3</sub>	10000.2017 Ω	8.40·10 <sup>-3</sup> Ω	650	10·10 <sup>-6</sup>	84·10 <sup>-9</sup> H	10.1 %
ТурВс	0.0	28.9·10 <sup>-6</sup>	8	48·10 <sup>-6</sup>	1.4·10 <sup>-9</sup> H	0.0 %
TypB∟	0.0	1.44·10 <sup>-3</sup>	8	4.2·10 <sup>-6</sup>	6.1·10 <sup>-9</sup> H	0.1 %
TypB <sub>R1</sub>	0.0	577·10 <sup>-6</sup>	8	62·10 <sup>-9</sup>	36·10 <sup>-12</sup> H	0.0 %
TypB⊤	0.0 H	5.77·10 <sup>-9</sup> H	80	1.0	5.8·10 <sup>-9</sup> H	0.0 %
ω	6286.33 s <sup>-1</sup>	1.81 s <sup>-1</sup>				
π	3.1415926535898					
$ au_2$	800·10 <sup>-12</sup> s	2.02·10 <sup>-9</sup> s	∞	-3.2·10 <sup>-3</sup>	-6.4·10 <sup>-12</sup> H	0.0 %
τ <sub>3</sub>	800·10 <sup>-12</sup> s	2.02·10 <sup>-9</sup> s	8	-3.2·10 <sup>-3</sup>	-6.4·10 <sup>-12</sup> H	0.0 %
L <sub>2</sub>	0.10005291 H	293·10 <sup>-9</sup> H	4600			

#### 5.5.3. Uncertainty budget for combination of interim results

quantity	value	standard uncertainty	degrees of freedom	sensitivity coefficient	uncertainty contribution	index
L <sub>1</sub>	0.10005332 H	490·10 <sup>-9</sup> H	7	0.5	240·10 <sup>-9</sup> H	74.1 %
L <sub>2</sub>	0.10005291 H	290·10 <sup>-9</sup> H	4600	0.5	140·10 <sup>-9</sup> H	25.9 %
Ls	0.10005312 H	285·10 <sup>-9</sup> H	12			

#### 5.6. Result

quantity	estimator	combined standard uncertainty					coverage
Ls	0.10005312 H	290·10 <sup>-9</sup> H	6.3·10 <sup>-6</sup>	2.2	t-table 95%		

$\mathbf{C}$	n
•	М
v.	L

Inductance Ls for 100 mH inductance standard s.n. 13975	standard uncertainty
Uncertainty contribution from:	(µH/H)
Resonance bridge measurement on 200 mH standard:	
Resonance frequency measurement fr ( $\omega r = 2\pi$ fr)	0.1
Resonance resistance measurement Rr	0.6
Resonance bridge lead inductance correction ls	0.1
Resonance capacitance measurement Cr Resonance capacitance loss angle measurement Dcr	2.2 0.6
•	0.0
Inductance comparison bridge measurement on 100 mH travelling standard:	
Inductance comparison bridge measurement uncertainty	1.6
Experimental standard deviation of the mean	0.8
Standard uncertainty (RSS)	3.0
Expanded uncertainty (k=2)	6.0
Resistance Rs for 100 mH inductance standard s.n. 13975	standard
Uncertainty contribution from;	uncertainty $(\mu\Omega/\Omega)$
Resonance bridge measurement on 100 mH travelling standard:	
Resonance frequency measurement fr ( $\omega r = 2\pi$ fr)	0
Resonance resistance measurement Rr	101
Resonance bridge lead resistance correction	61
Reproducibility of inductor terminals resistance Resonance capacitance measurement Cr	5
Resonance capacitance loss angle measurement Dcr	5 78
Experimental standard deviation of the mean	13
Standard uncertainty (RSS)	142
Expanded uncertainty (k=2)	290
Uncertainty budget Inductance comparison bridge measurement of Li	standard
	uncertainty
Uncertainty contribution from:	(μH/H)
IVD ratio uncertainty	0.45
L2 cable capacitance correction (δL2´´) uncertainty	1.50
L1 lead inductance (11) uncertainty	0.15
L2 lead inductance (l2) uncertainty	0.08
L <sub>1</sub> self capacitance uncertainty ( $\delta L_1$ )	0.10
L2 self capacitance uncertainty (δL2')	0.20
Bridge decade resistor inductance (8lR) uncertainty	0.25
Standard uncertainty (RSS)	1.61

#### GUM IV. Uncertainty budget for the inductance

The inductance  $L_X$  of the standard is obtained from the relationship:

$$L_{X}=L_{S}+\delta L_{K}+\delta L_{d}+\delta L_{CW}-\delta L_{TX}+\delta L_{p}$$

Where:

 $L_{\rm S}$  – conventional value of the inductance of the unknown standard,

 $\delta L_{\rm K}$  – correction for the errors of indication of the comparator,

 $\delta L_d$  – correction for resolution of the comparator,

 $\delta L_{CW}$  – correction due to standard uncertainty of measurement of the reference capacitance standard C,

 $\delta L_{\rm TX}$  – temperature-related inductance variation of the unknown inductor,

 $\delta L_p$  – correction due to influence of the cables.

#### Uncertainty budget $(L_X)$ :

		standard	probability	sensitivity	Uncertainty
quantity	estimate	uncertainty	distribution	coefficient	contribution
Xi	x <sub>i</sub>	$u(x_i)$		c <sub>i</sub>	u <sub>i</sub> (y)
$L_s$	100,060 mH	0,00051 mH	normal	1	0,00051 mH
δL <sub>K</sub>	0 mH	0,00375 mH	rectangular	1	0,00375 mH
$\delta L_{d}$	0 mH	0,00012 mH	rectangular	1	0,00012 mH
δL <sub>CW</sub>	10,01287 nF	0,00005 nF	normal	-10 mHnF <sup>-1</sup>	0,0005 mH
$\delta L_{TW}$	0 mH	0,00037 mH	rectangular	1	0,00037 mH
$\delta L_{p}$	0 mH	0,00029 mH	rectangular	1	0,00029 mH
L <sub>x</sub>	100,060 mH				0,00385 mH

Expanded uncertainty -

 $U = k \cdot u(L_X) = 1.71 \cdot 0.00385 \text{ mH} = 0.0066 \text{ mH} \approx 0.007 \text{ mH}$ 

Reported result:

 $L_{\rm X} = (100,060 \pm 0,007) \text{ mH}$ 

The reported expanded uncertainty of measurement is stated with the level of confidence of 95 %.

#### CMI UNCERTAINTY BUDGET

TYPE A	VALUE	UNCERT	UNITS	COR	SENS	DIST	DEG	DIV	STD UNC
alfa Repeatability	1.00E-03	5.40E-09	N/A	N/A	100	t	29	2.09	2.58E-07
B Repeatability	5.30E-13	4.00E-06	N/A	N/A	0.04	t	29	2.09	7.66E-08
Cn Repeatability		1.00E-16	F	N/A	1.00E+09	t	29	2.09	4.78E-08
dCo Repeatability		1.00E-13	F	N/A	4.00E+05	t	29	2.09	1.91E-08
TYPE B		UNCERT	UNITS	COR	SENS	DIST	DEG	DIV	STD UNC
Ls:		6.00E-08	Н	N/A	1	t	>100	2	3.00E-08
Red a		8.00E-06	N/A	N/A	0.1	t	50	2.051	3.90E-07
В		4.60E-07	N/A	N/A	0.04	t	50	2.051	8.97E-09
f: Stab. of 1 kHz		1.00E-04	Hz	N/A	2.00E-04	t	>100	2	1.00E-08
Cn		4.00E-16	F	N/A	1.00E+09	t	>100	2	2.00E-07
Cno		1.00E-10	F	N/A	64	t	8	2.366	2.71E-09
LB		2.00E-05	Н	N/A	1.00E-03	t	8	2.366	8.45E-09

RESULTS TYPE A UNCERT TYPE A STD UNC TYPE A VAR	4.00E-06 2.74E-07 7.53E-14
TYPE B UNCERT	1.02E-04
TYPE B STD UNC	4.40E-07
TYPE B VAR	1.93E-13
COMBINED UNC	1.02E-04
COMBINED STD UNC	5.18E-07
COMBINED VAR	2.69E-13
EFFECTIVE DEGREES OF FREEDOM	>100
COVERAGE FACTOR (k)	2
EXPANDED UNCERTAINTY	1.04E-06

Result

#### $L_{PIM} = 0.100\ 053\ 95\ H \pm 0.000\ 0011\ H$

NCM 2. The value  $L_X$  of the traveling standard is obtained from the relationship:

$$L_X = (L_S + \delta L_D + \delta L_{TS})l_K l - \delta L_{TX}$$

where:

 $L_{S}$  – inductance value of the reference standard,

 $\delta L_{D^{\prime}}$  drift of the inductance of the reference standard since it's last calibration,

 $\delta L_{TS}$  – temperature related inductance variation of the reference,

 $I = L_{jx}/L_{jS}$  - ratio of the indicated inductance for the travelling and reference standards (index j means the number of the measurement),

 $I_K$  – correction factor for parasitic voltages, temperature dependent of the bridge and error of reading using the bridge galvanometer,

 $\delta L_{TX}$  – temperature related inductance variation of the travelling standard.

#### 9. Uncertainty budget (L<sub>X</sub>)

Quan- tity	Estimate		Standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution
Xi	Xi		u(xi)		Ci	$u_i(L_X)$
Ls	100,031	mΗ	6,00E-05	normal	1,0	6,00E-05
$dL_D$	0,000	mΗ	5,77E-06	rectangular	1,0	5,77E-06
dLTs	0,000	mΗ	1,73E-06	rectangular	1,0	1,73E-06
LK	1,00000		2,04E-05	triangular	1,0	2,04E-05
L	1,00022		7,45E-07	normal	1,0	7,45E-07
$dLT_X$	0,00000		3,46E-07	rectangular	-1,0	-3,46E-07
Lχ	100,053	mΗ				6,37 <b>E-</b> 05

Expended relative uncertainty is 1,3E-04.

The reported relative expanded uncertainty of measurement is stated with the level of confidence of 95 % and k=2.

# CTU

Uncertainty component $u(x_i) / x_i$	Value of $u(x_i) / x_i$	Sensitivity coefficient $c_i$	$u_{i}(L_{s})/L_{s} =  c_{i}  u(x_{i})/x_{i}$	Degrees of freedom vi
$u(L_{\rm smean}) / L_{\rm smean}$	2.1 × 10 <sup>-6</sup>	1	2.1 × 10 <sup>-6</sup>	9
u(ω) / ω	$2.1 \times 10^{-6}$	-2	4.2 × 10 <sup>-6</sup>	12
u(C) / C	$2.02 \times 10^{-5}$	-1	$2.02 \times 10^{-5}$	6
$u(R_1/R_2) / (R_1/R_2)$	$6.1 \times 10^{-6}$	1	6.1 × 10 <sup>-6</sup>	12
$u(U_{\rm L})$ / $U_{\rm L}$	$1.38 \times 10^{-5}$	1	$1.38 \times 10^{-5}$	12
u(U <sub>c</sub> ) / U <sub>c</sub>	5.5 × 10 <sup>-6</sup>	-1	$5.5 \times 10^{-6}$	12
$u(1 + \tan^2 \delta) / (1 + \tan^2 \delta)$	1.48 × 10 <sup>-5</sup>	-1	$1.48 \times 10^{-5}$	12

$\sum (u_i(L_s)/L_s)^2$	$9.07 \times 10^{-10}$
$u(L_{\rm s}) / L_{\rm s}$	3.01 × 10 <sup>-5</sup>
Effective degrees of freedom $v_{\text{eff}}$	23.5
$t_{0.95}(v_{\mathrm{eff}})$	2.07
$U_{0.95}(L_{\rm s}) / L_{\rm s}$	$6.23 \times 10^{-5}$
$U_{0.95}(L_{\rm s})$	$6.23 \times 10^{-3} \text{ mH}$
$L_{\rm s}$	100.008 9 mH
$L_{\rm s}$ - $U_{0.95}(L_{\rm s})$	100.002 7 mH
$L_{\rm s} + U_{0.95}(L_{\rm s})$	100.015 2 mH

# **Appendix D.** Optional measurements

No optional measurements have been proposed in the comparison protocol.

## **Appendix E.** Comparison protocol



# Istituto Elettrotecnico Nazionale Galileo Ferraris

Torino, Italy

# Euromet Project 607:

Intercomparison of a 100 mH inductance standard

**Draft 28 November 2002** 

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## 1 Description of the travelling standard

The tavelling standard is an inductance standard having the nominal value of 100 mH. It is constructed starting from a General Radio GR1482-L 100 mH inductance standard, encased in a thermostated enclosure.

#### 1.1 General aspect

The external aspect of the standard (Fig. 1) is a wooden box (370 mm height, 285 mm depth, 285 mm width);

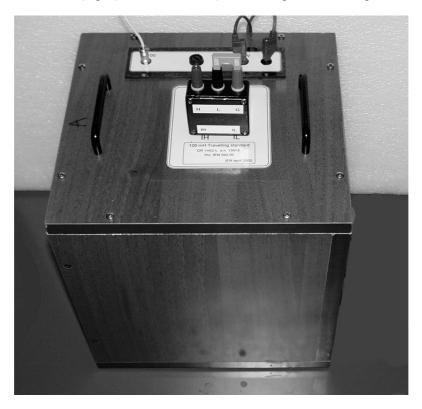


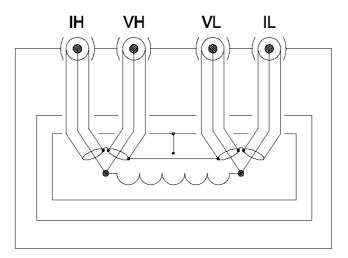
Fig. 1. External aspect of the 100 mH travelling standard.

The top surface (Fig. 2) provides all electrical connections to the standard, and two handles for transport.



Fig. 2. Top surface of the travelling standard.

#### 1.2 <u>Electrical connections</u>



The electrical connections to the standard consist of:

- four British Post Office MUSA connectors, for four terminal-pair electrical definition, Fig. 3a.

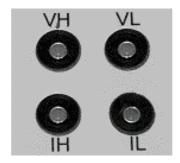


Fig. 3a. Connections for the electrical definition of the standard

- a LEMO ERA.OS.304.CLL four-pole+screen connector, to access the internal PT100 thermoresistance, Fig. 3b;



Fig. 3b. LEMO connector for the internal PT100 thermoresistance

- two banana sockets, one red (+) and one black (-) for the connection of the 12V power supply to the thermostat, Fig. 3c.

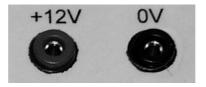


Fig. 3c. Power supply connections for the thermostat

#### 1.3 Accessories provided

The standard is provided with the following accessories:

**4/2 adapter**: an electrical adapter to convert the four terminal-pair definition provided by the BPO connectors to two-terminal binding posts definition, Fig. 4.

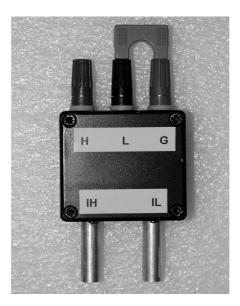


Fig. 4. 4/2 adapter.

The adapter contains a shorting link to be considered an integral part of the adapter; do NOT report on any measurement made without this link.

The electrical schematics of the adapter is reported in Fig. 5.

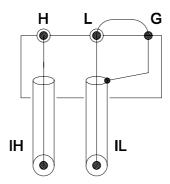


Fig. 5. Electrical schematics of the adapter.

The adapter converts four terminal-pair definition of the standard to two-terminal definition, and should be inserted on the  $I_{H}$ - $I_{L}$  ports, as shown in Fig. 6. PostOfficeCaps (see below) cover the unused VH and VL connectors.



Fig. 6. 4/2 adapter mounted on the standard.

PT100cable: for connection to the LEMO ERA.OS.304.CLL connector, having four free wires and screen on the other side

The connections are the following:

Red = high-current terminal,  $I_H$ ;

Yellow = high-voltage terminal,  $V_H$ ;

Green = low-voltage terminal,  $V_L$ ;

Black = low-current termina,  $I_L$ ;

12V supply: a 220/240 V – 50 Hz ac to 12 V dc electrical power supply to power the thermostat, Fig. 7.



Fig. 7. 12V power supply.

The supply is terminated with a 220/240V power line input socket and two banana sockets, one red (+) and one black (-).

It is provided with a green LED lamp which shows the power-up condition, an on/off switch, one fuse holder containing 315 mA fuse.

12V cable: two single-wire cables, one red and one black, for connection of the 12V supply to the standard.

**PostOfficeCaps**: two non-shorting caps for post-office connectors, used to cover VH and VL terminals when the 4/2 adapter is used, see Fig. 2.

#### 2. What to measure

The quantities to be measured are:

- $L_s$  and  $R_s$ : the equivalent series inductance and resistance of the standard, defined by the configuration chosen (four terminal-pair or two-terminal);
- f: the measurement frequency;
- $R_{PT100}$ : the four-terminal resistance of the PT100 internal sensor;
- $T_{ext}$ : the temperature (°C) of the environment where the standard is measured;
- $H_{ext}$ : the external humidity (%) of the environment where the standard is measured.

#### 3. Measurement instructions

#### 3.1 Set-up of the standard

The standard must be positioned with the face having the electrical connections in horizontal position.

Before and during the measurement, the standard must be energized with a +12.0(5) V, 500 mA dc low-noise power supply, connected to the banana sockets (+ red, -black). 12Vsupply accessory is suited for the purpose but its employment is not mandatory.

At power-up, the thermostat green LED lamp lits, indicating thermostat ON cycle. After some hours, the lamp goes off, then on again, with an approximate period of 30 minutes.

The standard reaches its operating temperature, around 28 °C, in 24 hours. Tentative measurements on the standard can be carried out before this period, but cannot be considered reliable for the comparison. DO NOT consider the reaching of a plateau for  $R_{PTI00}$  the signal of a temperature stabilization of the standard. If the thermostat power supply is disconnected for any reason, the user must wait again 24 hours before measurement.

#### 3.2 Electrical connections to the standard

Two electrical definitions of the standard are possible:

- four terminal-pair:
- two-terminal using the given adapter.

#### 3.3 Measurement parameters

The measurement has to be carried out with the following excitation:

- ac sinewave signal;
- frequency 1000 Hz;
- current 1 mA<sub>rms</sub>;
- THD+N (total harmonic distortion+noise) less than 0.01%

#### 4 Report

After measurement, the laboratory must report:

- the measurement principle and details of the experimental setup;
- the configuration chosen for the measurement (two-terminal or four terminal-pair);
- the measured quantities corresponding to the list of Chapter 2.

For each quantity listed in §2, the user must report:

- the estimator;
- the estimated standard uncertainty  $(1\sigma)$
- the estimated 95% confidence interval.

For the inductance, a complete uncertainty budget must also be reported, in a format compliant to the Guide on Uncertainty in Measurement (GUM), ISO, 1995.

#### 5 Scheduling

The following laboratories are engaged in the comparison:

Lab	Contact
IEN	Mr. Vincenzo D'Elia Istituto Elettrotecnico Nazionale Galileo Ferraris Strada delle Cacce, 91 I-10135 Torino, Italy Tel. +39 011 3919437 Fax. +39 011 346384 email delia@ien.it
РТВ	Dr. Juergen Melcher Physikalisch-Technische Bundesanstalt Section 2.11 Alternating Current Quantities Bundesallee 100 D-38023 BRAUNSCHWEIG Germany
	Tel. +49 531 592 2010 (D) Fax. +49 531 592 2015 e-mail Juergen.Melcher@ptb.de

SP	Mr. Gunnar Eklund SP Swedish National Testing and Research Institute Electricity and Time, MTe Brinellgatan 4 SE-504 62 Borås Sweden  Tel. +46 33 16 53 87 (D) +46 33 16 50 00 (O) Fax. +46 33 12 50 38 e-mail gunnar.eklund@sp.se
GUM	Mr. Antoni Tarłowski GŁÓWNY URZĄD MIAR (Central Office of Measures) ZAKŁAD METROLOGII ELEKTRYCZNEJ (Electricity Division) Inductance & Capacitance Standards Lab ul. Elektoralna 2 00-139 WARSZAWA POLAND Tel: 0-048-22-6205970 Fax: 0-048-22-6208378 electricity@gum.gov.pl
СМІ	Mr. Jiri Horsky Czech Metrological Institute Okruzni 31 63800 BRNO Czech Republic  Tel. +420 5 45 222 727 (O) Fax. +420 5 45 222 183 e-mail jhorsky@cmi.cz
UME	Mr. Yakup Gulmez TUBITAK - Ulusal Metroloji Enstitüsü ANİBAL CAD. MAM KAMPÜSÜ 41470 GEBZE - KOCAELI Turkey Tel. +90 262 646 63 55 (ext. 470) Fax. +90 262 646 5914 e-mail yakup.gulmez@ume.tubitak.gov.tr
NCM	Mrs. Petya Aladzhem National Centre of Metrology (NCM) 52B G.M.Dimitrov Blvd. BG - 1797 Sofia Bulgaria  Tel. +359 2 71 02 37 Fax +359 2 71 70 50 email ncm@sasm.orbitel.bg

Each laboratory has a measurement time of two weeks for the measurements. The travelling and measurement sequence is reported in Fig. 1.

Dates	IEN	PTB	SP	GUM	CMI	UME	NC
11 nov - 17 nov							
18 nov -24 nov							
25 nov - 1 dec							
2 dec - 8 dec							
9 dec - 15 dec							
16 dec - 22 dec							
23 dec - 29 dec							
30 dec - 5 gen							
20 gen - 26 gen							
27 gen - 2 feb							
3 feb -9 feb							
10 feb - 16 feb							
24 feb - 2 mar							
3 mar - 9 mar							
10 mar - 16 mar							
17 mar - 23 mar							
24 mar - 30 mar							
31 mar - 6 apr							
19 mag - 25 mag							
26 mag - 1 giu							
<u> </u>							
		Holidays					
		Measurement					
	11 nov - 17 nov 18 nov -24 nov 25 nov - 1 dec 2 dec - 8 dec 9 dec - 15 dec 16 dec - 22 dec 23 dec - 29 dec 30 dec - 5 gen 6 gen - 12 gen 13 gen - 19 gen 20 gen - 26 gen 27 gen - 2 feb 3 feb - 9 feb 10 feb - 16 feb 17 feb - 23 feb 24 feb - 2 mar 3 mar - 9 mar 10 mar - 16 mar 17 mar - 23 mar	11 nov - 17 nov  18 nov -24 nov  25 nov - 1 dec  2 dec - 8 dec  9 dec - 15 dec  16 dec - 22 dec  23 dec - 29 dec  30 dec - 5 gen  6 gen - 12 gen  13 gen - 19 gen  20 gen - 26 gen  27 gen - 2 feb  3 feb -9 feb  10 feb - 16 feb  17 feb - 23 feb  24 feb - 2 mar  3 mar - 9 mar  10 mar - 16 mar  17 mar - 23 mar  24 mar - 30 mar  31 mar - 6 apr  7 apr - 13 apr  14 apr - 20 apr  28 apr - 4 mag  5 mag - 11 mag  12 mag - 18 mag  19 mag - 25 mag	11 nov - 17 nov  18 nov -24 nov  25 nov - 1 dec  2 dec - 8 dec  9 dec - 15 dec  16 dec - 22 dec  23 dec - 29 dec  30 dec - 5 gen  6 gen - 12 gen  13 gen - 19 gen  20 gen - 26 gen  27 gen - 2 feb  3 feb -9 feb  10 feb - 16 feb  17 feb - 23 feb  24 feb - 2 mar  3 mar - 9 mar  10 mar - 16 mar  17 mar - 23 mar  24 mar - 30 mar  31 mar - 6 apr  7 apr - 13 apr  14 apr - 20 apr  21 apr - 27 apr  28 apr - 4 mag  5 mag - 19 iu  Holidays  Not available	11 nov - 17 nov 18 nov -24 nov 25 nov - 1 dec 2 dec - 8 dec 9 dec - 15 dec 16 dec - 22 dec 23 dec - 29 dec 30 dec - 5 gen 6 gen - 12 gen 13 gen - 19 gen 20 gen - 26 gen 27 gen - 2 feb 3 feb - 9 feb 10 feb - 16 feb 17 feb - 23 feb 24 feb - 2 mar 3 mar - 9 mar 10 mar - 16 mar 17 mar - 23 mar 24 mar - 30 mar 31 mar - 6 apr 7 apr - 13 apr 14 apr - 20 apr 28 apr - 4 mag 5 mag - 11 mag 12 mag - 18 mag 19 mag - 25 mag 26 mag - 1 giu  Holidays Not available Measurement	11 nov - 17 nov  18 nov -24 nov  25 nov - 1 dec  2 dec - 8 dec  9 dec - 15 dec  16 dec - 22 dec  23 dec - 29 dec  30 dec - 5 gen  6 gen - 12 gen  13 gen - 19 gen  20 gen - 26 gen  27 gen - 2 feb  3 feb - 9 feb  10 feb - 16 feb  17 feb - 23 feb  24 feb - 2 mar  3 mar - 9 mar  10 mar - 16 mar  17 mar - 23 mar  24 mar - 30 mar  31 mar - 6 apr  7 apr - 13 apr  14 apr - 20 apr  21 apr - 27 apr  28 apr - 4 mag  5 mag - 11 mag  19 mag - 25 mag  19 mag - 25 mag  Holidays  Not available  Measurement	11 nov - 17 nov  18 nov -24 nov  25 nov - 1 dec  2 dec - 8 dec  9 dec - 15 dec  16 dec - 22 dec  23 dec - 29 dec  30 dec - 5 gen  6 gen - 12 gen  13 gen - 19 gen  20 gen - 26 gen  27 gen - 2 feb  3 feb - 9 feb  10 feb - 16 feb  17 feb - 23 feb  24 feb - 2 mar  3 mar - 9 mar  10 mar - 16 mar  17 mar - 23 mar  24 mar - 30 mar  31 mar - 6 apr  7 apr - 13 apr  14 apr - 20 apr  28 apr - 4 mag  5 mag - 11 mag  12 mag - 18 mag  19 mag - 25 mag  26 mag - 1 giu  Holidays  Not available  Measurement	11 nov - 17 nov  18 nov -24 nov  25 nov - 1 dec  2 dec - 8 dec  9 dec - 15 dec  16 dec - 22 dec  30 dec - 29 dec  30 dec - 5 gen  6 gen - 12 gen  13 gen - 19 gen  20 gen - 26 gen  27 gen - 2 feb  3 feb - 9 feb  10 feb - 16 feb  17 feb - 23 feb  24 feb - 2 mar  3 mar - 9 mar  10 mar - 16 mar  17 mar - 23 mar  24 mar - 30 mar  31 mar - 6 apr  7 apr - 13 apr  14 apr - 20 apr  21 apr - 27 apr  28 apr - 4 mag  5 mag - 11 giu  Holidays  Not available  Measurement

Tab. 1. Measurement scheduling.

#### **Transportation**

Each participant laboratory is responsible for its own costs for the measurement, transportation and any custom charges, as well as any damage that may occur within its country.

An enclosure is provided for the inductance standard and its accessories, shippable as freight.

The dimensions of the enclosure are 600 mm height, 550 mm depth, 800 mm width: the approximate weight being 35 kg (standard and accessories included).

#### After arrival:

- an email should be immediately sent (delia@ien.it) to the pilot laboratory and the shipping laboratory;
- the thermostat should be energized and at least 24 hours should pass before the beginning of the measurements;

#### After measurement;

- the standard should be repacked ensuring that all accessories are included in the enclosure;

#### After departure;

- an email should be immediately sent (delia@ien.it) to the pilot laboratory and the receiving laboratory.