

Final Report

EUROMET project No. 846

COMPARISON ON 1.018 V AND 10 V DC VOLTAGES

**KCDB identifier: EUROMET.EM.BIPM-K11.6
Key Comparison of DC voltages**

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Abstract: This report presents the results of a trilateral comparison of measurements on a Zener voltage standard within the framework of EUROMET. The participants are the National Metrology Institutes of Romania (INM), Turkey (UME) and Austria (BEV) which acts as the pilot laboratory. The comparison was performed in order to link the National Institute of Metrology Bucharest (INM) to the BIPM key comparisons.

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Introduction

The objective of this comparison was to link the National Institute of Metrology Bucharest (INM) to the key comparisons BIPM.EM-K11.a and BIPM.EM-K11.b¹.

The primary standard for DC voltage is formed by the Josephson Array Voltage Standard (JAVS). This type of standard is used by UME and BEV. The national standard of INM is a Zener standard, calibrated at BIPM.

The main targets of this comparison are:

- to demonstrate equivalence of metrological practice
- to contribute to acceptance of INM in EUROMET
- to confirm the proposed CMC values of INM in the field of DC voltage
- to check the correctness of the calibration results
- to check the correct traceability of the standards

Participants

The participating institutes are listed in the following table.

| Country | Institute | Acronym | Date |
|---------|--|---------|-------------------------|
| Turkey | Ulusal Metroloji Enstitüsü | UME | 14. – 18. June 2005 |
| Romania | Institutul National de Metrologie | INM | 20. June – 8. July 2005 |
| Austria | Bundesamt für Eich- und Vermessungswesen | BEV | 11. – 15. July 2005 |

Table 1: Participating laboratories

The comparison was organized in a loop of the participating laboratories. The circulation of the standard started in June 2005 and was finished in July 2005. The detailed time schedule for the comparison is given in table 1. Due to difficulties in organizing the time schedule it was decided that the pilot laboratory BEV was doing the measurements at the end of the loop.

The participants were asked to follow their usual measurement procedure to achieve their best measurement capabilities.

Travelling standard

The standard used was a Fluke 732 B electronic DC reference standard with s/n 8008001 provided by INM. The Fluke 732 B electronic DC reference standard, henceforth denoted by the standard, has two output voltages, nominally 1.018 V and 10 V respectively.

Measurements

The quantities to be measured were

- 1.018 V and 10 V outputs
- resistance of the oven temperature thermistor
- ambient temperature, humidity and pressure.

No correction for the influence of thermistor temperature, air pressure and humidity was applied. The internal thermistor resistance was measured by applying a current $\leq 10 \mu\text{A}$. Before and after these comparison measurements additional measurements were performed at INM to check the stability of the standard. These measurements show a linear time dependence of the Zener, typically for solid state voltage standards.

The device under test (DUT) was connected continuously to the AC line power except during the measurements, where at least 2 hours between disconnecting the main power and the start of the measurements elapsed. During the measurements the front panel GUARD binding post was connected to the guard of the measuring system and to the front panel CHASSIS binding post. In one point of the measuring system the guard was connected to ground.

The measurement conditions at the different laboratories are listed in the following table.

| Participant | Room temperature [°C] | Thermistor resistance [kΩ] | Humidity [%] | Air pressure [hPa] | Measurement method |
|-------------|-----------------------|----------------------------|--------------|--------------------|---|
| UME | 23.0 ± 1 | 38.73 ± 0.01 | 57 ± 10 | 995 ± 3 | comparison with JAVS voltage |
| INM | 23.5 ± 0.4 | 38.65 ± 0.05 | 60 ± 7 | 1007 ± 5 | comparison with a calibrated Zener standard |
| BEV | 23.0 ± 0.2 | 38.74 ± 0.04 | 48 ± 5 | 992 ± 2 | comparison with JAVS voltage |

Table 2: Reference conditions in the laboratories during the measurements

Results

Each of the laboratories gave one value for the output voltage at 1.018 V and 10 V, the corresponding measurement uncertainty for a confidence level of 95 % and the mean measurement date for every set of measurements, respectively. The uncertainty budget of each laboratory was presented in the form of a table according to chapter 4 of the EA-4/02 document 'Expression of the Uncertainty of Measurement in Calibration' (see Annex 2).

| Participant | Date | V_{meas} [V] | $U(V_{\text{meas}})$ [μV] | V_{meas} [V] | $U(V_{\text{meas}})$ [μV] |
|-------------|----------|-----------------------|---------------------------|-----------------------|---------------------------|
| INM | 08.06.05 | 9.999 997 4 | 4.2 | 1.018 075 7 | 0.5 |
| UME | 16.06.05 | 9.999 994 96 | 0.44 | 1.018 075 76 | 0.22 |
| INM | 30.06.05 | 9.999 998 3 | 5.0 | 1.018 075 8 | 0.6 |
| BEV | 13.07.05 | 9.999 994 82 | 0.70 | 1.018 075 48 | 0.10 |
| INM | 20.07.05 | 9.999 998 9 | 5.9 | 1.018 075 9 | 0.6 |

Table 3: Measurement results and uncertainties ($k = 2$) of the participating laboratories

The expected value $V_{\text{exp,INM}}$ of the Zener standard at the time of measurement at INM was calculated by the weighted mean of the measurement results obtained by UME and BEV. This expected value was also used to calculate the degree of equivalence. With this procedure the well known drift effect of the output voltage of Zener standards was taken into account.

The degree of equivalence D_{INM} with the corresponding expected value V_{exp} was calculated by subtracting the expected value from the laboratory result V_{meas} according to

$$D_{INM} = V_{meas,INM} - V_{exp,INM}$$

with an associated uncertainty given by

$$U(D_{INM}) = \sqrt{U^2(V_{meas,INM}) + U^2(V_{exp,INM})}.$$

No correlations between the different measurements were taken into account.

The degree of equivalence and the uncertainty for the measurements performed at INM are stated in the following table.

| Evaluation of measurements for INM | | | | | | |
|---|---------------------|----------------------|-----------------------|---------------------------|-------------------|----------------------|
| | V_{exp} [V] | $U(V_{exp})$ [μV] | $V_{meas,INM}$ [V] | $U(V_{meas,INM})$ [μV] | D_{INM} [μV] | $U(D_{INM})$ [μV] |
| 1.018 V | 1.018 075 53 | 0.09 | 1.018 075 8 | 0.6 | 0.3 | 0.6 |
| 10 V | 9.999 994 92 | 0.37 | 9.999 998 3 | 5.0 | 3.4 | 5.1 |

Table 4: Calculated expected value at the mean measurement date at INM (30.06.05) and degree of equivalence with the associated uncertainty ($k = 2$) of the result with respect to the expected value

The measurement results and the estimation of the expected value at the mean measurement dates are also shown in figure 1 and 2.

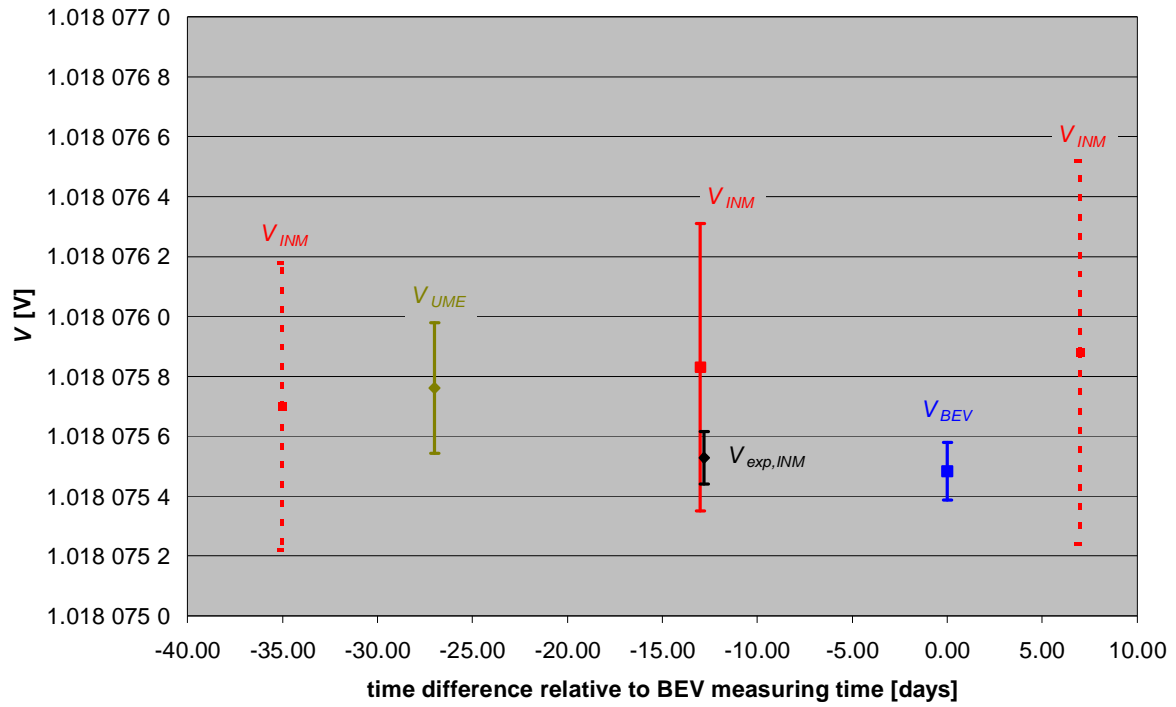


Figure 1: Measurement results and expected value for 1.018 V at the time of INM measurement (uncertainty bars for $k = 2$)

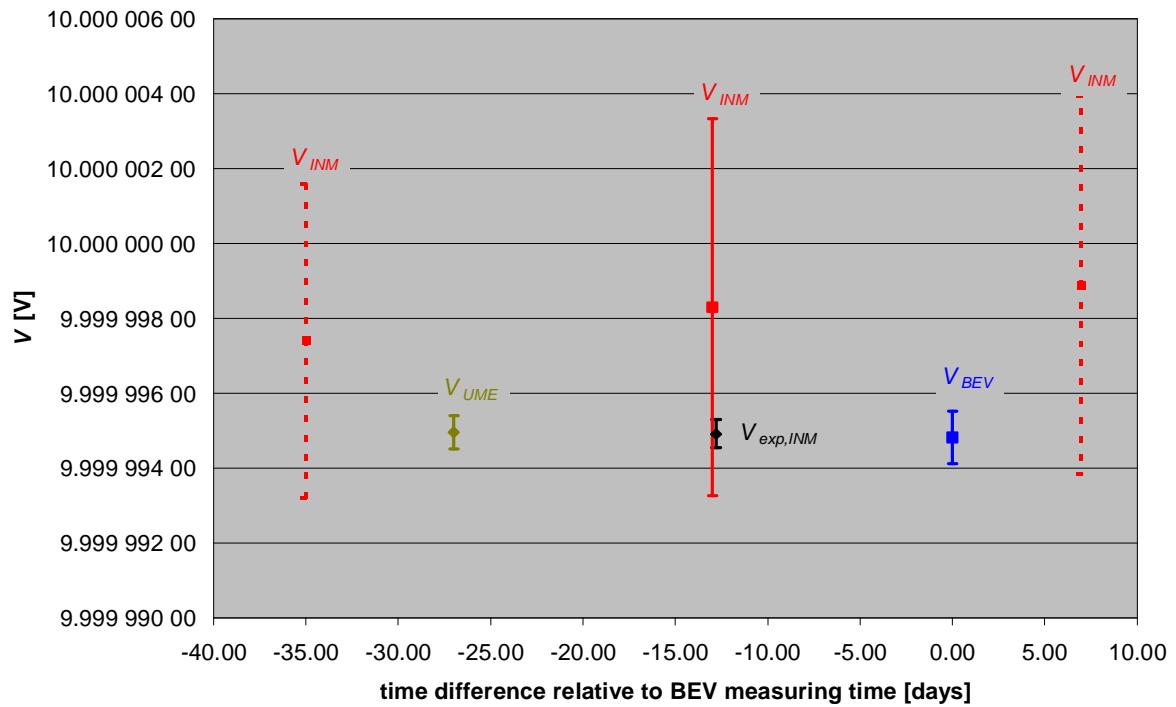


Figure 2: Measurement results and expected value for 10 V at the time of INM measurement (uncertainty bars for $k = 2$)

Evaluation of degrees of equivalence linked to BIPM.EM-K11.a and BIPM.EM-K11.b comparisons

For the INM measurements of DC voltage a link is given to the comparisons BIPM.EM-K11.a “DC voltage: 1.018 V, Zener diode”, BIPM.EM-K11.b: “DC voltage: 10 V, Zener diode”. BEV took part in both of these comparisons, UME took part in EUROMET.EM.BIPM-K11, their link to the BIPM.EM-K11.b was calculated in the BIPM document².

In the Rapport BIPM-2001/03 (April 2001)¹ the final results of the comparison are presented as the differences between the values assigned to a 1.018 V and a 10 V standard by each laboratory and stated together with the combined standard uncertainty u_c (for $k = 1$). According to the results stated in the “BIPM key comparison database” these differences are used as the degree of equivalence $D_{K11,BEV}$ and the expanded uncertainty $U(D_{K11,BEV}) = 2 \times u_c$ (for $k = 2$) of BEV as follows:

| BEV | $D_{K11,BEV}$ [μ V] | $U(D_{K11,BEV})$ [μ V] |
|---------|-----------------------------|--------------------------------|
| 1.018 V | -0.013 | 0.034 |
| 10 V | -0.04 | 0.20 |

Table 5: Degree of equivalence $D_{K11,BEV}$ and the expanded uncertainty $U(D_{K11,BEV})$ with respect to the BIPM Reference Value given in the Rapport BIPM-2001/03.

The results for UME from the EUROMET.EM.BIPM-K11 comparison and the corresponding link to the BIPM.EM-K11.b are as follows in table 6:

| UME | $D_{K11,UME}$ [μ V] | $U(D_{K11,UME})$ [μ V] |
|-----------------------------|-----------------------------|--------------------------------|
| 10 V, EUROMET.EM.BIPM-K11 | 0.097 | 0.409 |
| 10 V, Link to BIPM.EM-K11.b | -0.05 | 0.48 |

Table 6: Degree of equivalence $D_{K11,UME}$ and the expanded uncertainty $U(D_{K11,UME})$ according to the final report of EUROMET.EM.BIPM-K11 and the Link to the BIPM.EM-K11.b² which is used for the evaluation of the INM degree of equivalence.

These values are used now in this comparison for the evaluation of the degrees of equivalence linked to BIPM.EM-K11.a and BIPM.EM-K11.b comparisons for the following reasons:

- As BEV and UME used in these comparisons and in the current comparison the same Josephson system for measuring the Zener standard, the same reproducibility of these measurements can be assumed.
- No drift of the Josephson measurements has to be taken into account as the Josephson system is a primary standard.

Therefore the degree of equivalence $D_{K11.6}$ and the expanded uncertainty $U_{K11.6}$ of INM can be calculated by

$$D_{K11.6,INM} = D_{K11,BEV} + D_{INM}$$

$$D_{K11.6,INM} = D_{K11,UME} + D_{INM} , \text{ respectively.}$$

with the associated uncertainties

$$U(D_{K11.6,INM}) = \sqrt{U^2(D_{K11,BEV}) + U^2(D_{INM})} \text{ and } U(D_{K11.6,INM}) = \sqrt{U^2(D_{K11,UME}) + U^2(D_{INM})} .$$

Because both the UME and the BEV values are used for the calculation of the expected value at the mean measurement date at INM also a weighted degree of equivalence (“weighted link”) is given in table 7 for the 10 V level. For the measurements at 1.018 V only BEV is linked to the BIPM and therefore only BEV results are used in the calculation of $D_{K11.6,INM}$ and $U(D_{K11.6,INM})$ at 1.018 V.

| Evaluation of the degree of equivalence for INM | | |
|---|-------------------------|----------------------------|
| 10 V | $D_{K11.6,INM}$ [μV] | $U(D_{K11.6,INM})$ [μV] |
| Weighted Link BEV-UME: | 3.34 | 5.06 |
| 1.018 V | | |
| BEV: BIPM.EM-K11.a | 0.29 | 0.57 |

Table 7: Degree of equivalence $D_{K11.6,INM}$ and the expanded uncertainty $U_{(DK11.6,INM)}$

Conclusion

The measurements of this trilateral comparison were carried out according to the agreed time table. The INM results at the voltage level of 1.018 V as well as at 10 V show adequate agreement within the stated uncertainties.

¹ W. Waldmann, D. Reymann and T. J. Witt, "Rapport BIPM-2001/03: Bilateral Comparison of 1.018 V and 10 V Standards between the BEV, Austria and the BIPM", BIPM Publications, April 2001, published online in the *Key Comparison Data Base*: <http://kcdb.bipm.fr>.

² D. Reymann, "Link between the comparison EUROMET.EM.BIPM-K11.b and the ongoing comparison BIPM.EM-K11.b" from 30.9.2002, published online in the *Key Comparison Data Base*: <http://kcdb.bipm.fr>.

Annex 1

History of the analysis of the measurement results

After the circulation of the Draft B Report within the scope of EUROMET several comments has come to hands concerning the linking of the results to the BIPM database and the evaluation of the expected value of the Zener standard at the time of measurement.

After detailed discussion at the TC-EM Meeting in Thessaloniki, Greece, 19.-20.10.2006, about a revised evaluation procedure of the expected value of the Zener at the time of measurement and the linking to the BIPM database an approach was agreed, where the three sets of INM measurements before, at and after the comparison were used to calculate the drift of the standard and the BEV results were used to fix the absolute value of the output voltage.

After preparation of the revised Draft B1 report there were some objections by UME against this evaluation procedure.

In a meeting at BIPM in March 2007 it was suggested to determine the expected value for the INM measurements by calculation of the mean of the UME and BEV results both for the 1.018 V and the 10 V level.

The weighted mean is calculated by

$$V_{exp,INM} = \frac{\frac{V_{UME}}{U^2(V_{UME})} + \frac{V_{BEV}}{U^2(V_{BEV})}}{\frac{1}{U^2(V_{UME})} + \frac{1}{U^2(V_{BEV})}}$$

for 1.018 V and 10 V, respectively. The associated uncertainty is estimated according to

$$U(V_{exp,INM}) = \sqrt{\frac{1}{\frac{1}{U^2(V_{UME})} + \frac{1}{U^2(V_{BEV})}}}$$

With these evaluations the degree of equivalence of INM with respect to the expected value of the Zener was calculated.

In a second step the degree of equivalence with respect to the BIPM database by using the BEV and UME links as stated above were calculated for 10 V, for 1.018 V the BEV link was used.

After a comment of UME about the linking of the EUROMET.EM.BIPM-K11.b to the BIPM.EM-K11.b comparison the UME link was updated according to reference 2 as stated in the report above. The final result for the INM measurements at 10 V shows only minor changes by this evaluation procedure.

Annex 2

Uncertainty budgets of the participants

UME

UNCERTAINTY BUDGET

Model Function:
$$V_{UUT} = \frac{n}{K_{J-90}} f + V_{DVM} - dV_{leak} - dV_{gain} - dV_{offset}$$

where;

- f = The frequency of the applied microwave power.
- n = Step number to which the Josephson array is set.
- K_{J-90} = The Josephson constant, equal to 483597.9 GHz/V.
- V_{DVM} = The voltage difference between the Josephson array and the traveling standard as measured with the digital voltmeter.
- dV_{leak} = Leakage voltage drop due to the finite resistance of the leads and the leakage resistance between the leads.
- dV_{gain} = The voltage deviation due to gain error of the digital voltmeter
- dV_{offset} = Residual offset voltages due to thermal EMF's in the leads, thermal EMF's in the applied scanner/switch or rectification induced by electromagnetic interference.

Table 1. The uncertainty budget of UME for 1.018V

| Quantity X_j | Estimate x_j | Standard Uncertainty $u(x_j)$ | Probability Distribution | Sensitivity coefficient c_j | Uncertainty Contribution $u_i(y)$ | Degree of freedom ν_i |
|---------------------------------------|----------------------|-------------------------------------|-----------------------------|-------------------------------------|---|---------------------------------|
| Frequency | 74.99GHz | 8.66 Hz | Rectangular / B | 0.014 nV/Hz | 0.12 nV | 100 |
| Leakage error | 0 nV | 0.21 nV | Rectangular / B | 1 | 0.21 nV | 10000 |
| DVM gain | 0 nV | 0.12 nV | Rectangular / B | 1 | 0.12 nV | 5 |
| Uncomp. offset | 0 nV | 1.15 nV | Rectangular / B | 1 | 1.15 nV | 10 |
| Standard Deviation | | 109.29 nV | Normal /A | 1 | 109.29 nV | 4 |
| X | 1.018075761 V | | | | | |
| Combined standard uncertainty: | | | | | 109.30 nV | |
| Effective degrees of freedom: | | | | | 4 | |
| Expanded uncertainty (k=2)*: | | | | | 218.60 nV | |

Table 2. The uncertainty budget of UME for 10V

| Quantity X_i | Estimate x_i | Standard Uncertain ty $u(x_i)$ | Probability Distribution | Sensitivity coefficient c_i | Uncertainty Contribution $u_i(y)$ | Degree of freedom ν_i |
|-----------------------|----------------------|---|-----------------------------|-------------------------------------|---|---------------------------------|
| Frequency | 74.99GHz | 8.66 Hz | Rectangular / B | 0.133 nV/Hz | 1.15 nV | 100 |
| Leakage error | 0 nV | 2.02 nV | Rectangular / B | 1 | 2.02 nV | 10000 |
| DVM gain | 0 nV | 1.15 nV | Rectangular / B | 1 | 1.15 nV | 5 |
| Uncomp. offset | 0 nV | 1.15 nV | Rectangular / B | 1 | 1.15 nV | 10 |
| Standard Deviation | | 221.75 nV | Normal /A | 1 | 221.75 nV | 4 |
| X | 9.999994956 V | | | | | |
| | | Combined standard uncertainty: | | | 221.77 nV | |
| | | Effective degrees of freedom: | | | 4 | |
| | | Expanded uncertainty (k=2)*: | | | 443.54 nV | |

*: As a requirement of Technical Protocol, coverage factor was taken as 2 (k = 2).

INM

Uncertainty calculation

The uncertainty calculation was carried out according to EA-4/02. The most significant factors contributing to the uncertainty are included in the relationship. Their contributions to the standard uncertainty were evaluated as follows.

Reference standard V_s

From the BIPM calibration certificate (26/26.05.2005-BIPM):

- for the 10 V output: $V_s = 9.999\ 985\ 6\ \text{V}$ $u_c(V_s) = 0.14 \times 10^{-6}\ \text{V}$

- for the 1.018 V output: $V_s = 1.018\ 111\ 23\ \text{V}$ $u_c(V_s) = 0.014 \times 10^{-6}\ \text{V}$

It is a normal distribution: $u(V_s) = u_c(V_s)$

The uncertainty due to the transport of the reference standard to and from BIPM could not be estimated and, since it can be assumed to be small enough to be neglected, was not included in the uncertainty budget.

Drift of reference standard dV_D

The value drift of the reference voltage since its last calibration (26.05.2005) is estimated, according to the BIPM calibration certificate.

Correction due to drift in time of standard is: $dV_D = 0$

The deviation estimated is: $a = c(t-t_0)$

a constant from the BIPM calibration certificate

t average date of measurements in INM

t_0 average date of measurements in BIPM

It is a rectangular distribution. The standard uncertainty is $u(dV_D) = \frac{a}{\sqrt{3}}$

Nanovoltmeter correction dV_{SN}

It is used the correction taken from the INM calibration certificate (03.01-538/2005-INM): $dV_{SN} = 0$

It is a normal distribution. The standard uncertainty is $u(dV_{SN}) = U/k$.

Voltage measurements $\overline{\Delta V} = V_{Xi} - V_{Si}$

Both V_s and V_x were measured with the same equipment, so the uncertainty contributions are correlated and we consider only the relative difference in the voltage readings due to systematic effects. For normal distribution:

$$u(\overline{\Delta V}) = s_{\overline{\Delta V}} = \sqrt{\frac{\sum_{i=1}^n (\Delta V_i - \overline{\Delta V})^2}{n(n-1)}}$$

Nanovoltmeter resolution dV_N

Correction due to resolution displayed by the multimeter. Generally, no correction is applicable, thus $dV_N = 0$.

We estimate the modification of the multimeter indication ($\pm a$) depends on the nanovoltmeter's resolution.

It is a rectangular distribution. The uncertainty is estimated by $u(dV_N) = \frac{a}{\sqrt{3}}$.

Drift of nanovoltmeter dV_{ND}

Correction due to drift in time of standard dX_{ND} is taken into account depending on the data provided by the history of the standard used.

Based on the history of the standard, a time drift a is evaluated. The associated uncertainty is calculated as $u(dV_{ND}) = \frac{a}{\sqrt{3}}$.

When the history does not exist, the value drift of the nanovoltmeter is estimated, according to the manufacturer's specifications:

For 1 mV range: $a = 0.0050\%$ reading + 0.0020% range
 $dV_{ND} = 0$.

It is a rectangular distribution. The standard uncertainty is $u(dV_{ND}) = \frac{a}{\sqrt{3}}$.

Instabilities of the nanovoltmeter dV_P

Disturbances in the power supply line may cause instabilities of the indicated values of the nanovoltmeter. dV_P is the correction due to variations of the supply network. No correction is applicable, thus $dV_P = 0$.

We estimate the modification of the multimeter indication ($\pm a$) when there are instabilities in the supply network.

It is a triangular distribution. The uncertainty is estimated as $u(dV_P) = \frac{a}{\sqrt{6}}$.

Offset correction dV_{OFF}

No correction is applicable, thus $dV_{OFF} = 0$.

We estimate the modification of the nanovoltmeter indication ($\pm a$).

It is a rectangular distribution. The standard uncertainty is $u(dV_{ND}) = \frac{a}{\sqrt{3}}$.

Standard uncertainty

The standard uncertainty is obtained from the relationship $u_C(y) = \sqrt{\sum u_i^2(y)}$.

Expanded uncertainty

The expanded uncertainty was calculated using a coverage factor $k = 2$, which corresponds to a confidence level of $\approx 95\%$ $U = k \cdot u_C$.

Number of degrees of freedom

The degrees of freedom are obtained from the relationship $n_{eff} = \frac{u_c^4(y)}{\sum \frac{u_i^4(y_i)}{n_i}}$.

Uncertainty budget for 1.018V, period (20.06...08.07.2005)

| Quantity X_i | Estimate x_i | Standard Uncertainty $u(x_i)$ | Probability Distribution | Sensitivity coefficient c_i | Uncertainty Contribution $u_i(y)$ | Degree of freedom n_i |
|-----------------------|-------------------|-------------------------------------|-----------------------------|-------------------------------------|---|-------------------------------|
| V_S | 1.018 111 23 V | 0.014 μ V | normal | 1 | 0.014 μ V | 8 |
| dV_D | 0 | 0,174 μ V | rectangular | 1 | 0,174 μ V | - |
| dV_{SN} | 0 | 0.2 μ V | normal | 1 | 0.2 μ V | 49 |
| $\overline{\Delta V}$ | -35.397 μ V | 0.0061 μ V | normal | 1 | 0.0061 μ V | 252 |
| dV_N | 0 | 0.00003 μ V | rectangular | 1 | 0.00003 μ V | - |
| dV_{ND} | 0 | 0.013 μ V | rectangular | 1 | 0.013 μ V | - |
| dV_P | 0 | 0.078 μ V | triangular | 1 | 0.078 μ V | 2 |
| dV_{OFF} | 0 | 0.058 μ V | rectangular | 1 | 0.058 μ V | - |
| X | 1.018 075 83 V | | | | | |
| | | Combined standard uncertainty: | | | 0.28 μ V | |
| | | Effective degrees of freedom: | | | 64 | |
| | | Expanded uncertainty: | | | 0.56 μ V | |

The reported results are: $V = 1.018\ 075\ 8\ \text{V}$, $U = 6 \times 10^{-7}\ \text{V}$

Uncertainty budget for 10 V, period (20.06...08.07.2005)

| Quantity X_i | Estimate x_i | Standard Uncertainty $u(x_i)$ | Probability Distribution | Sensitivity coefficient c_i | Uncertainty Contribution $u_i(y)$ | Degree of freedom ν_i |
|-----------------------|-------------------|-------------------------------------|-----------------------------|-------------------------------------|---|---------------------------------|
| V_S | 9.999 985 59 V | 0.141 μ V | normal | 1 | 0.141 μ V | 8 |
| dV_D | 0 | 1.7 μ V | rectangular | 1 | 1.7 μ V | |
| dV_{SN} | 0 | 0.2 μ V | normal | 1 | 0.2 μ V | 49 |
| $\overline{\Delta V}$ | 12.708 μ V | 0.023 μ V | normal | 1 | 0.023 μ V | 252 |
| dV_N | 0 | 0.003 μ V | rectangular | 1 | 0.003 μ V | |
| dV_{ND} | 0 | 0.012 μ V | rectangular | 1 | 0.012 μ V | |
| dV_P | 0 | 0.64 μ V | triangular | 1 | 0.64 μ V | 2 |
| dV_{OFF} | 0 | 1.73 μ V | rectangular | 1 | 1.73 μ V | |
| X | 9.999 998 3 V | | | | | |
| | | Combined standard uncertainty: | | | 2.52 μ V | |
| | | Effective degrees of freedom: | | | 333 | |
| | | Expanded uncertainty: | | | 5.04 μ V | |

The reported results are: $V = 9.999\ 998\ \text{V}$, $U = 5 \times 10^{-6}\ \text{V}$

BEV

Type A contribution V_{meas} includes

- Instability of the voltage of the DUT and the frequency counter
- Variation of locked frequency
- Electromagnetic interference
- Noise of null detector
- Variation of thermal emf

Type B uncertainty includes

Frequency dV_f

Contribution of EIP counter. For the BEV counter a frequency error of 15 Hz is assumed according to EIP Microwave, Inc. Models 575 & 578 Source Locking Microwave Counters, Manual Part Number: 5580032-00 September 1987.

$$dV_f = (15 \text{ Hz} / 70 \text{ GHz}) = 2,14 \times 10^{-10}$$

Leakage dV_l

Component of leakage current of the JAVS wiring. For the BEV system a leakage resistance of 12.5 G Ω at 10 V and a resistance of the JAVS wiring of 5.5 Ω was measured.

$$dV_l = (5,5 \text{ } \Omega / 12,5 \text{ G}\Omega) = 4,4 \times 10^{-10}$$

Temperature dV_t

Temperature dependence of the output voltage of the DUT. According to literature a temperature coefficient of $5 \times 10^{-8}/\text{K}$ is assumed. The temperature stability of the air condition is better than $\pm 0.35 \text{ K}$

$$dV_t = \pm 5 \times 10^{-8}/\text{K} \times 0,35 \text{ K} = \pm 1,75 \times 10^{-8}$$

Air pressure dV_p

Pressure dependence of the output voltage of the DUT. According to literature a pressure coefficient of $2 \times 10^{-9}/\text{hPa}$ is assumed. The variation of pressure is smaller than 6 hPa.

$$dV_p = 2 \times 10^{-9}/\text{hPa} \times 6 \text{ hPa} = 1,2 \times 10^{-8}$$

Switch dV_{switch}

Contribution by uncompensated thermal emf at the switch

$$dV_{\text{switch}} = 5 \times 10^{-9}.$$

Repetition $dV_{\text{repetition}}$

Contribution by repetition of Zener measurements at different days.

Uncertainty budget for the 1.018 V measurements on 13.7.2005

| Quantity | Relative standard uncertainty | Type | Probability distribution | Relative uncertainty contribution |
|---|-------------------------------|------|--------------------------|-----------------------------------|
| X_i | $u(x_i)$ | | | $u_i(y)$ |
| V_{meas} | 2.81E-08 | A | normal | 2.81E-08 |
| dV_f | 2.14E-10 | B | rectangular | 1.24E-10 |
| dV_l | 4.40E-10 | B | rectangular | 2.54E-10 |
| dV_t | 1.75E-08 | B | rectangular | 1.01E-08 |
| dV_p | 1.20E-08 | B | rectangular | 6.93E-09 |
| dV_{switch} | 5.00E-09 | B | rectangular | 2.89E-09 |
| $dV_{repetition}$ | 3.63E-08 | A | normal | 3.63E-08 |
| U_{DUT} | | | | 4.76E-08 |
| Zener voltage of 1.018 075 48 V with an relative expanded uncertainty ($k=2$) of 9.53E-08 | | | | |

Uncertainty budget for the 10 V measurements on 13.7.2005

| Quantity | Relative standard uncertainty | Type | Probability distribution | Relative uncertainty contribution |
|---|-------------------------------|------|--------------------------|-----------------------------------|
| X_i | $u(x_i)$ | | | $u_i(y)$ |
| V_{meas} | 1.34E-08 | A | normal | 1.34E-08 |
| dV_f | 2.14E-10 | B | rectangular | 1.24E-10 |
| dV_l | 4.40E-10 | B | rectangular | 2.54E-10 |
| dV_t | 1.75E-08 | B | rectangular | 1.01E-08 |
| dV_p | 1.20E-08 | B | rectangular | 6.93E-09 |
| dV_{switch} | 5.00E-09 | B | rectangular | 2.89E-09 |
| $dV_{repetition}$ | 2.96E-08 | A | normal | 2.96E-08 |
| U_{DUT} | | | | 3.48E-08 |
| Zener voltage of 9.999 994 82 V with an relative expanded uncertainty ($k=2$) of 6.97E-08 | | | | |