Uncertainties of resistors temperature coefficients

E. Dudek, M. Mosiadz, M. Orzepowski

Voltage and Resistance Laboratory, Electricity Department, Central Office of Measures
Email: electricity@gum.gov.pl
ul. Elektoralna 2, 00-139 Warszawa, Poland

Abstract. *Resistors calibration gives high level of measurement accuracy. Analysis of results and uncertainty budget should consider influence of temperature changes on resistance. Article describes resistors temperature properties, influence of temperature changes for results, and influence of temperature coefficients uncertainties for uncertainty budget.*

Keywords: standard resistor, temperature coefficient, uncertainty

1. Introduction

Resistors calibration gives measurement uncertainties as good as 10^{-8}. At such accuracies taking into account the influence of temperature at resistance value is necessary. Resistance of all materials depends on temperature. Standard resistors are calibrated at 23 °C. Their have to be thermostabilized in oil bath, which provides temperature stabilization at 0,01 °C ÷ 0,001 °C. Not all resistors can be put into oil bath, but can be placed in air thermostats, which provides temperature stabilization at 0,1 °C. There are also resistors calibrated in the other temperature then 23 °C, or resistors work in other temperatures. Resistance values of such a resistors should be corrected to the value for ambient temperature by using correction coefficients – temperature coefficients. Temperature coefficients can be provided by resistor manufacturer or determined by laboratory.

2. Temperature coefficients determination

Temperature dependence of device resistance is characterized by temperature coefficients α, β [1]. Values of coefficients are used for correction of measurement result, if temperature differs from calibration temperature.

$$R(T) = R_{ref}\left[1 + \alpha(\Delta T) + \beta(\Delta T)^2\right], \tag{1}$$

$R(T)$ – resistance at temperature T, R_{ref}– resistance at calibration temperature, $\Delta T = T - T_{ref}$. Temperature coefficients are determined by temperature characteristic measurements. Measurement parameters (temperature resolution), should be chosen taking into account required accuracy of measurements and thermal properties of resistor. Firstly resistor should be calibrated at 23 °C. Next steps are resistor calibration in each chosen temperature point. Results accuracy depends depends on measurement equipment. We use measurement system for resistor calibration extended with thermostatic chamber. Fig. 1 shows measurement system. Measurement accuracy is improved by immersing standard resistor R_S into the oil bath (23°C), and stabilized at 0,01°C. Examined R_X is placed into thermostatic chamber with temperature resolution 0,1°C. For temperature coefficient determination such thermal conditions are sufficient. For measurements we use resistance comparators. This equipment gives measurement accuracies 0,01 ppm ($R \approx 1\ \Omega$) ÷ 500 ppm ($R \approx 1\ \mathrm{T}\Omega$) [2]. Fig. 2 shows measurement procedure. Results allow to determine α, β.Using measured characteristic gives possibility of determination equation of the

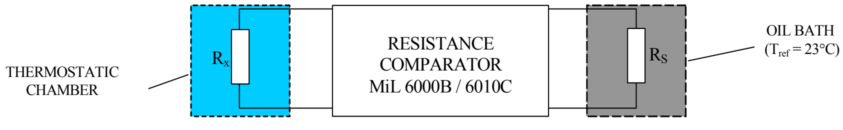

Fig. 1. Measurement system for resistance devices temperature characteristic investigations

approximation 2nd order equation, which can be written as normalized (1)

$$\frac{R(\Delta T)}{R(23)} - 1 = \alpha \cdot \Delta T + \beta \cdot (\Delta T)^2 . \qquad (2)$$

Normalized resistance $r(\Delta T) = [R(\Delta T)/R(23)]-1$ allows to simplify (2). Points ($R(\Delta T)/R_{23} - 1$; $\Delta T = T - 23$) is approximated by $y(x) = ax^2 + bx + c$ curve. Numeric calculations can be done in any mathematical software with implemented approximation algorithms [3][4]. Using binomial regression method one can write system of equations(3), where N – number of measurement points, ΔT – difference between temperature of measurement and reference temperature, α, β - 1st and 2nd order temperature coefficient Ω/K, Ω/K^2, γ - polynomial constant. System of equations (3) can be written as matrixes (4).

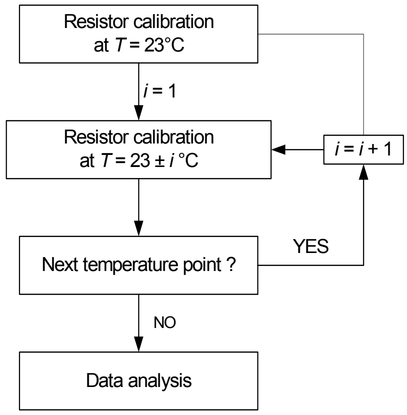

Fig. 2. Measurement procedure

$$\begin{cases} N\gamma + \alpha\sum_{n=1}^{N}\Delta T_n + \beta\sum_{n=1}^{N}\Delta T_n^2 = \sum_{n=1}^{N} r_n \\ \gamma\sum_{n=1}^{N}\Delta T_n + \alpha\sum_{n=1}^{N}\Delta T_n^2 + \beta\sum_{n=1}^{N}\Delta T_n^3 = \sum_{n=1}^{N}\Delta T_n r_n \\ \gamma\sum_{n=1}^{N}\Delta T_n^2 + \alpha\sum_{n=1}^{N}\Delta T_n^3 + \beta\sum_{n=1}^{N}\Delta T_n^4 = \sum_{n=1}^{N}\Delta T_n^2 r_n \end{cases} . \qquad (3)$$

$$T = \begin{bmatrix} N & \sum_{n=1}^{N}\Delta T_n & \sum_{n=1}^{N}\Delta T_n^2 \\ \sum_{n=1}^{N}\Delta T_n & \sum_{n=1}^{N}\Delta T_n^2 & \sum_{n=1}^{N}\Delta T_n^3 \\ \sum_{n=1}^{N}\Delta T_n^2 & \sum_{n=1}^{N}\Delta T_n^3 & \sum_{n=1}^{N}\Delta T_n^4 \end{bmatrix}, C = \begin{bmatrix} \gamma \\ \beta \\ \alpha \end{bmatrix}, Y = \begin{bmatrix} \sum_{n=1}^{N} r_n \\ \sum_{n=1}^{N}\Delta T_n r_n \\ \sum_{n=1}^{N}\Delta T_n^2 r_n \end{bmatrix}, \qquad (4)$$

where $\boldsymbol{T}$ – matrix of temperature differences (input data for regression), $\boldsymbol{C}$ – matrix of equation coefficients, $\boldsymbol{Y}$ – output data matrix. From (4) one can determine $\boldsymbol{C}$ matrix solving

$$TC = Y \Rightarrow C = T^{-1}Y . \qquad (5)$$

3. Type A uncertainty of temperature coefficients

Type A measurement uncertainty is determined from statistical analysis of experimental results - in this case it is uncertainty of results approximation with the curve described by (2) equation. Type A uncertainty u_y is connected with $\boldsymbol{Y}$ matrix elements, and can be written as

$$u_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(r_i - \gamma - \alpha\Delta T_i - \beta\Delta T_i)^2} . \qquad (6)$$

Approximation uncertainty come from matrix elements, whose factors are output data matrix elements connected with equation coefficient, which uncertainty is unknown. Uncertainties of α, β can be written as

$$u_A(\alpha) = u_y\sqrt{\frac{\left(N\sum_{n=1}^{N}\Delta T_n^4 - \left(\sum_{n=1}^{N}\Delta T_n^2\right)^2\right)}{\det T}} , \qquad (7a)$$

$$u_A(\beta) = u_y \sqrt{\frac{\left(N\sum_{n=1}^{N} \Delta T_n^2 - \left(\sum_{n=1}^{N} \Delta T_n \right)^2 \right)}{\det T}} . \tag{7b}$$

Using temperature coefficients as correction coefficients for resistor values suggests determination of uncertainties of these values. For the resistors below 1 kΩ these uncertainties are about 1-3% of coefficient value. For higher resistances uncertainties increases, but radius of curve also increases. Fig. 3.1 - 3.2 shows dependence of normalized resistance on temperature. For higher resistances type A uncertainty of β doesn't have any influence on the results. It's caused by flattening of the curve. It suggest, that type A uncertainty of β coefficient should be used only for resistors with parabolic temperature characteristic. Resistance dependence on temperature is an individual quantity of each resistor. Results of type A uncertainty calculations for 6 measurement points are show in table 1. Increasing of measurement point will results with improving approximation accuracy.

Table 1. Type A uncertainties of temperature coefficients

R Ω	$u_A(\alpha)$ 1/K	$u_A(\beta)$ 1/K
100	7,89623E-09	1,84735E-09
10^6	4,94877E-07	1,13894E-07

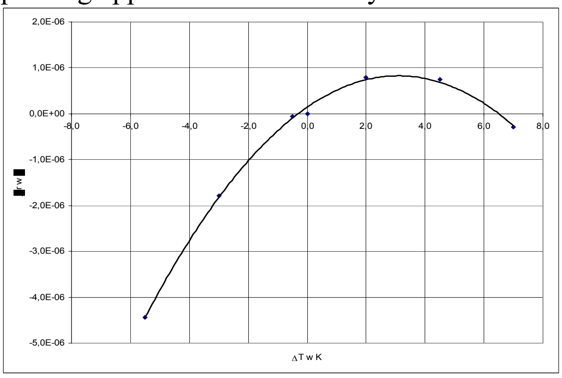

Fig. 3.2 Temperature characteristic for 100 Ω resistor

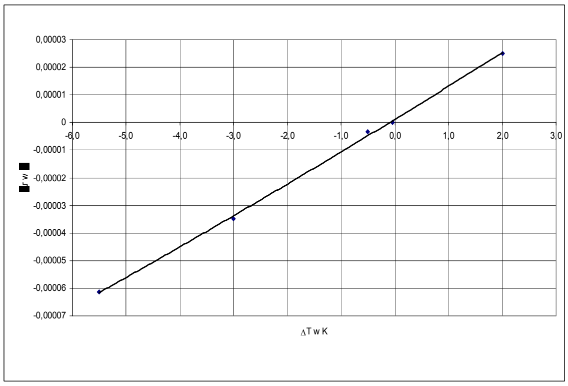

Fig. 3.2. Temperature characteristic of 1 MΩ resistor

4. Type B uncertainty of temperature coefficients

Type B uncertainty comes from non-statistical measurement analysis. The main influence for its value has accuracy of temperature and resistance measurements. From (1) one can write

$$\alpha = \frac{r - \beta \Delta T^2}{\Delta T} , \tag{8a}$$

$$\beta = \frac{r - \alpha \Delta T}{\Delta T^2} . \tag{8b}$$

Standard uncertainties of these coefficients can be written as follows

$$u_{SB}(\alpha) = \sqrt{\left(\frac{d\alpha}{d\Delta T} \right)^2 u^2(\Delta T) + \left(\frac{d\alpha}{dr} \right)^2 u^2(r)} , \tag{9a}$$

$$u_{SB}(\beta) = \sqrt{\left(\frac{d\beta}{d\Delta T} \right)^2 u^2(\Delta T) + \left(\frac{d\beta}{dr} \right)^2 u^2(r)} . \tag{9b}$$

Uncertainty of normalized resistance is the uncertainty of R(ΔT)/R(T=23):

$$u_{Rrel} = \frac{u_{R(\Delta T)}}{R(\Delta T)} , \tag{10}$$

and overall type B relative uncertainty can be written in form of sum

$$u_{rel}(r) = u_{Erel}(\Delta T) + u_{Erel}(T = 23)\,. \tag{11}$$

Absolute value of resistance measurements type B uncertainty is much lower then temperature measurements uncertainty. For each temperature point we have

$$u_B(\alpha, \beta) = \sqrt{\frac{\sum_{i=1}^{N} u_{SB,i}(\alpha,\beta)}{N}}\,, \tag{12}$$

Results shows, value of u_B is determined by temperature measurement uncertainty. In the case of temperature measurements with accuracy of ± 0,1 K influence of uncertainty of resistance measurements isn't observed. When temperature measurements are done with accuracy better then 10 mK, consideration of resistance measurement uncertainty influence is necessary.

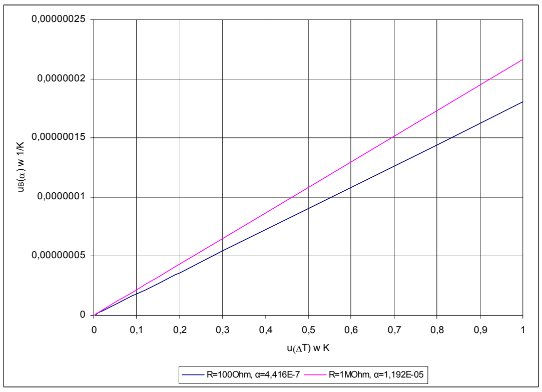

Fig. 4.1. Type B uncertainty of α coefficient dependence on temperature measurement accuracy

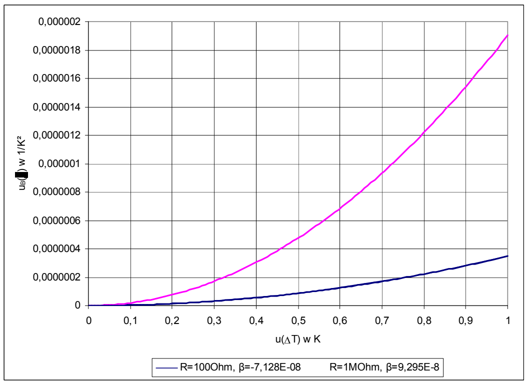

Fig. 4.2. Type B uncertainty of β coefficient dependence on temperature measurement accuracy

Fig. 4 illustrates u_B uncertainty of α, β dependence on accuracy of temperature measurements. Its visible, that α coefficient uncertainty linear arises with arising of temperature measurement uncertainty. In the case of β coefficient this dependence is quadrature.

5. Conclusions

Overall uncertainty of temperature coefficients depends on two main elements:

1. Statistic uncertainty of regression. In the case of resistors above 1 kΩ this method causes overestimating of overall uncertainty. Characteristics of them are closer to linear equation then binomial one, so β coefficients aren't useful. For α value determination we used linear regression. In the case of other resistors, with parabolic characteristic, both coefficients should be used.
2. Uncertainties of resistance and temperature measurements, which determines α, β uncertainties. For the resistance measurements with accuracy at few ppms the influence of resistance measurements uncertainty on uncertainty of temperature coefficients wasn't observe, in the case of temperature accuracy not better then ±10 mK. When measurements are done with equipment provides better thermostabilization, the influence of resistance measurement uncertainty should be calculated. Omitting it results with underestimating of temperature coefficient uncertainty budget.

Our results shows, determination of uncertainty of resistor temperature coefficient is possible, but these values are omitted in resistor calibration uncertainty budget. Calculations show, the contribution of temperature coefficients uncertainty to overall resistor calibration budget can be omitted without any influence for overall calibration accuracy.

References

[1] Chwaleba A., Poninski M., Siedlecki M.: Metrologia elektryczna. WNT, Warszawa, 1998.

[2] Measurement International: Automatic DC Resistance Bridge - Technical Manual. Canada 2006.

[3] MathSoft: Mathcad 8 – User's Guide. USA 1998.

[4] Numerical Recipes in C: The art of scientific computing. Cambridge Unversity Press, USA, 1992.