Report on Bilateral Comparison

P1-APMP.EM.BIPM-K11.1

Bilateral Comparison of dc voltage

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1 Introduction and general conditions of the comparison

In April 2002 the National Measurement Laboratory (NML) of Australia, now known as the National Measurement Institute of Australia, NMIA, and henceforth referred to as the NMIA, and the Standards and Calibration Laboratory (SCL) of Hong Kong embarked on a bilateral comparison of their standards of dc voltage. Both laboratories maintain 10 V dc Josephson-array standards, which also directly provide standards at lower dc voltages. Both laboratories use the internationally accepted numerical value $K_{J-90} = 483597.9$ GHz V^{-1} for the Josephson constant K_J. In charge of the measurements at the two laboratories were R B (Bob) Frenkel of NMIA and D W K (Dennis) Lee of SCL. The main decisions regarding the comparison were that:

(a) NMIA would be the pilot laboratory.

(b) The dc voltage standard artefact, to be transported from one laboratory to the other as the transfer standard, would be a Fluke 732B Zener-based instrument, serial number 5875101, owned by SCL.

(c) Since the two outputs of the Fluke are nominally 10 V and 1.018 V respectively, these would therefore be the dc voltages undergoing comparison.

(d) Before the comparison commenced, the temperature coefficient, humidity coefficient (and any accompanying time-constant) and pressure coefficient of the Fluke would be measured, this 'characterisation' of the Fluke being essential to compensate for differences in the respective ambients at NMIA and SCL.

(e) This characterisation would make use of facilities already available at NMIA, namely a temperature/humidity chamber and a pressure chamber.

(f) After the conclusion of the characterisation, the bilateral comparison would proceed in the sequence NMIA-SCL-NMIA.

(g) Since the major predictable difference in ambient between NMIA and SCL is the difference in ambient temperatures, 20° C at NMIA and 23° C at SCL, the Fluke when at NMIA would be placed in the temperature/humidity chamber set at 23° C, for the comparison measurements that would follow the characterisation.

(h) Together with the Fluke, a continuously operating temperature/humidity recorder (Testo 171) would be transported.

2 The dc voltage standard systems at the laboratories

The equipment used at NMIA and at SCL for maintaining a standard of dc voltage includes the following:

(1) At NMIA:

(a) PTB niobium-based 10 V Josephson-junction array;

(b) Hewlett-Packard HP5071A cesium-clock frequency standard;

(c) EIP 578B source-locking counter with remote sensor;

(d) Millitech Gunn-diode with integral isolator, operating near 74 GHz;

(e) WR-12 waveguide and other millimeter-wave components (Millitech and Custom Microwave);

(f) RF filter box (in-house manufacture);

(g) 12mm diameter fibreglass-covered stainless-steel waveguide (in-house manufacture);

(h) Wessington CH60 liquid-helium dewar;

(i) Gunn-diode driver and stabiliser circuit (in-house manufacture);

(j) Array bias unit (in-house manufacture);

(k) Low-thermal switching circuit (in-house manufacture);

(l) Hewlett-Packard 130C oscilloscope;

(m) Hewlett-Packard HP3458A digital multimeter.

(2) At SCL:

(a) Hypres 10V Josephson-junction array of 20208 niobium-aluminium oxide-niobium junctions;

(b) Millitech Gunn-Diode with integral isolator, operating near 75 GHz;

(c) RMC WR12 dielectric waveguide;

(d) Hewlett-Packard 5061B cesium frequency standard;

(e) EIP 578B source-locking counter;

(f) Astro Endyne JBS500 Josephson Bias Source;

(g) Guildline 9145A5 low-thermal selector switch;

(h) Tektronix 2225 oscilloscope;

(i) Hewlett-Packard HP3458A digital multimeter.

The measurements at NMIA were made manually, whereas at SCL the measurements were under computer control, using NISTVolt5.2 software. Both laboratories carried out the measurements of the Fluke with the Fluke disconnected from mains power and therefore running on its internal battery. The Fluke was disconnected from mains power at least one hour before the commencement of measurements. The Fluke guard terminal was connected to system ground and the Fluke chassis terminal was left floating.

3 Main dates of the comparison, and extension of the comparison

After initial discussion between the parties in July-August 2001 and subsequently, the Fluke was sent to NMIA for characterisation, which commenced on 16 November 2001. The characterisation was completed on 14 March 2002. The comparison measurements then took place on the following dates:

NMIA measurements (stage 1): 2 April 2002 to 15 April 2002.

SCL measurements: 24 April 2002 to 8 May 2002.

NMIA measurements (stage 2): 17 May 2002 to 7 June 2002.

After the results were considered, it was agreed that two further stages of measurements, one at each laboratory, would be desirable. The extended sequence is accordingly NMIA-SCL-NMIA-SCL-NMIA, with the SCL measurements from 24 April 2002 to 8 May 2002 now referred to as SCL stage 1. The subsequent dates were:

SCL measurements (stage 2): 24 July 2002 to 9 August 2002.

NMIA measurements (stage 3): 9 September 2002 to 20 September 2002.

4 Equipment for characterising the Fluke, and results of characterisation

The Fluke was characterised using the following facilities at NMIA:

Temperature and humidity coefficients: a commercial Thermoline cabinet, model RH-460, GD-710-D, serial number 15121. Temperature range: 15° C to 50° C and relative humidity range: 20% to 90%.

Pressure coefficient: custom-built chamber by Oil-Free Air Co., Australia, with pressure settable between −500 hPa to +500 hPa relative to atmospheric.

The measured dependence of voltage on temperature, humidity and pressure is expressed by linear terms only. No higher-order dependence was observed. In any case the differences in ambient at NMIA and at SCL were small, particularly in view of the condition stated in section 1(g) above. All observed voltage values were corrected to 'standard' values of ambient, namely: 23° C temperature, 45% relative humidity and 1013.25 hPa pressure.

The standard uncertainties of the measurements of temperature, humidity and pressure were respectively estimated as follows at NMIA and at SCL:

At NMIA: 0.05° C, 0.25% and 0.05 hPa;

At SCL: 0.1° C, 0.45% and 0.1 hPa.

(1) Temperature coefficient of Fluke.

The cabinet was set at temperatures ranging from 17° C to 27° C (nominal), with at least 24 hours settling time allowed at each temperature before the start of voltage measurements. Although the Fluke contains a built-in thermistor that would have provided extra information about its internal temperature, unfortunately a power surge at NMIA during an electrical storm on the weekend ending Sunday 18 November 2001 caused the thermistor to open-circuit.

The results were:

10 V output: -8.5 nV/V ($^\circ$ C)$^{-1}$, standard uncertainty 4.7 nV/V ($^\circ$ C)$^{-1}$.

1.018 V output: -13.5 nV/V ($^\circ$ C)$^{-1}$, standard uncertainty 6.1 nV/V ($^\circ$ C)$^{-1}$.

(2) Humidity coefficient of Fluke.

Each of the two voltage outputs was measured at 35%, 70% and again at 35% relative humidity. The temperature was set at 20° C and the duration of measurements at each humidity setting was between 6 and 8 days, with the voltage measured and recorded automatically every hour over this period. For these automatic measurements, a second Zener (owned by NMIA) was used as a reference at constant relative humidity.

The results were:

10 V output: $+0.56$ nV/V (1%)$^{-1}$, standard uncertainty 0.07 nV/V (1%)$^{-1}$, with a time-constant of approximately 1.5 days. Thus for a change in relative humidity of (for example) 45% to 46%, the voltage rises by 0.56 nV/V.

1.018 V output: any response to humidity change was masked by the noise of this output, expressed by a relative standard deviation of about 87 nV/V. By contrast, the relative standard deviation of the 10 V output was about 38 nV/V. These two measures of noise include the noise of the NMIA reference Zener mentioned above.

(3) Pressure coefficient of Fluke.

The response of the Fluke to a change in pressure was immediate, with no observable settling time.

10 V output: $+1.6$ nV/V (hPa)$^{-1}$, standard uncertainty 0.2 nV/V (hPa)$^{-1}$.

1.018 V output: $+1.7$ nV/V (hPa)$^{-1}$, standard uncertainty 0.2 nV/V (hPa)$^{-1}$.

5 Observed variation of voltage values with time

Figs. 1 and 2 display respectively the voltages at the 10 V and 1.018 V levels, for the complete sequence NMIA-SCL-NMIA-SCL-NMIA. The uncertainties (error bars) are not given in Figs. 1 and 2, but will be discussed in the following section. All values in Figs. 1 and 2 have been corrected to the standard values of ambient. One reading is assigned to each day, this reading being calculated as the mean of several readings obtained on that day. The time of this mean reading is the mean time of the several readings on that day.

At the 10 V level (Fig. 1), the number of resulting readings were: NMIA stage 1: 10; SCL stage 1: 10; NMIA stage 2: 10; SCL stage 2: 13; NMIA stage 3: 9.

At the 1.018 V level (Fig. 2), the number of resulting readings were: NMIA stage 1: 10; SCL stage 1: 10; NMIA stage 2: 10; SCL stage 2: 13; NMIA stage 3: 8.

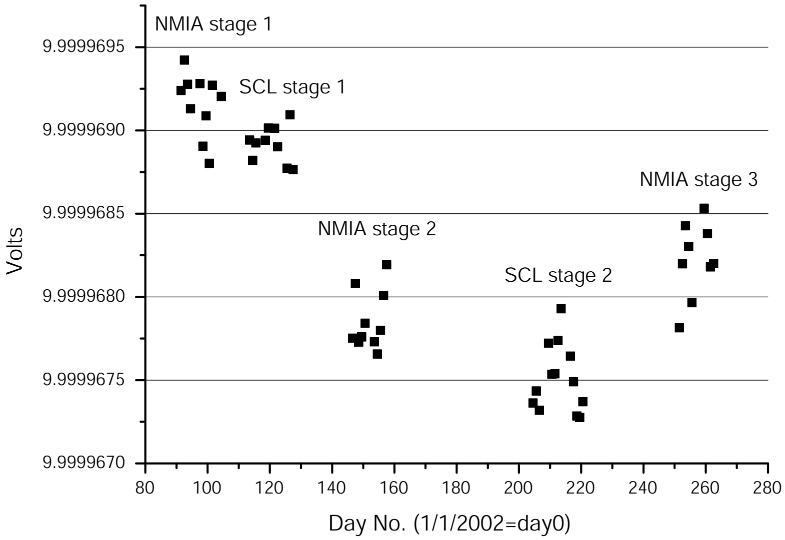

Fig. 1: 10 V results

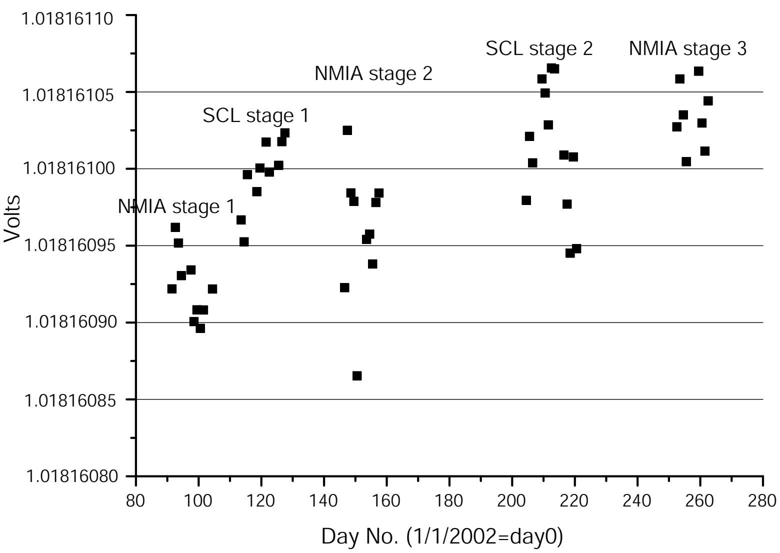

Fig. 2: 1.018 V results

6 Uncertainty Analysis

The estimated uncertainties assigned by each laboratory to its measurements are as follows. Standard uncertainty is abbreviated as St. Unc. and degrees of freedom as D.F. The component 'ambient coefficient' refers to the net uncertainty resulting from uncertainties in the temperature, humidity and pressure coefficients.

(1) At NMIA:

The 'budgets' as itemised below refer to the mean of several measurements of the Fluke against the Josephson array. As one consequence, the distribution of errors due to DMM rounding is normal, rather than rectangular.

NMIA 10 V level:

Uncertainty source	Type	Distribution	St. Unc. (nV)	D.F.
Frequency offset	B	Normal	1	4
Frequency variation	A	Normal	1	4
Leakage resistance	B	Normal	0.6	2
Uncancelled thermals	B	Normal	20	6
DMM gain	B	Normal	2	6
DMM rounding	A	Normal	0.83	Large
Noise Floor	B	Normal	70	4
Ambient coefficient	A	Normal	14	4
Combined st. unc.			74	5.0

The estimated relative standard uncertainty at NMIA of the 10 V measurements is therefore 7.4 nV/V.

NMIA 1.018 V level:

Uncertainty source	Type	Distribution	St. Unc. (nV)	D.F.
Frequency offset	B	Normal	0.1	4
Frequency variation	A	Normal	0.1	4
Leakage resistance	B	Normal	0.06	2
Uncancelled thermals	B	Normal	20	6
DMM gain	B	Normal	2	6
DMM rounding	A	Normal	0.83	Large
Random noise	A	Normal	14	6
Ambient coefficient	A	Normal	2	4
Combined st. unc.			25	11

The estimated relative standard uncertainty at NMIA of the 1.018 V measurements is therefore 25 nV/V.

(2) At SCL:

SCL 10 V level:

Uncertainty source	Type	Distribution	St. Unc. (nV)	D.F.
Frequency	B	Normal	1	200
Leakage	B	Normal	3	32
Offset	B	Normal	65	12
Null Voltage	B	Normal	10	200
Noise Floor	B	Normal	70	4
Ambient coefficient	A	Normal	23	4
Combined st. unc.			99	13

The estimated relative standard uncertainty at SCL of the 10 V measurements is therefore 9.9 nV/V.

SCL 1.018 V level:

Uncertainty source	Type	Distribution	St. Unc. (nV)	D.F.
Frequency	B	Normal	0	200
Leakage	B	Normal	1	32
Offset	B	Normal	41	12
Null Voltage	B	Normal	10	200
Random	A	Normal	20	39
Ambient coefficient	A	Normal	2	4
Combined st. unc.			47	20

The estimated relative standard uncertainty at SCL of the 1.018 V measurements is therefore 46 nV/V.

Note: It will be seen that the stochastic noise component in each of the four tables is termed 'noise floor' for the 10 V level, but 'random' for the 1.018 V level. The reason is that the $1/f$ noise floor level for the Zener is taken as the currently accepted proportional value of approximately 7 nV/V generic to most Zeners (see, for example, the report Euromet project no. 429, KCDB identifier EUROMET.EM.BIPM-K11). For the 10 V level, therefore, the noise floor is 70 nV, and this is entered into the tables since the random component reported by the participants was less than this value. Since the noise floor was not itself measured for the particular Zener used here, only 4 degrees of freedom are assigned (implying a high, around 35%, uncertainty in the $1/f$ amplitude). By contrast, the observed noise for the 1.018 V level reported by both participants was greater than 7 nV, and so the higher reported value has been retained in the tables. The uncertainty type for noise floor has been quoted as Type B in the tables, although a case for viewing it as Type A can be made.

Each of the points in Figs. 1 and 2 has an associated standard uncertainty whose value is stated as 'combined standard uncertainty' in the tables above.

7 Analysis of measurement results

7.1 Estimation of difference in realised unit of voltage between NMIA and SCL

Let $m(V_i)$ denote the difference $V_i^{(NMIA)} - V_i^{(SCL)}$, where $V_i^{(NMIA)}$ and $V_i^{(SCL)}$ are the respective numerical values of voltage assigned by NMIA and SCL to the nominal output voltage level i of the Fluke.

In view of the erratic changes and high scatter evident in Figs. 1 and 2 over some portions of the total period of the comparison, a robust analysis was performed for estimating $m(V_{10})$ and $m(V_{1.018})$. The median value of the Fluke was calculated for all the pooled NMIA results and for all the pooled SCL results, at both voltage levels. The standard uncertainty of the median was also calculated, using the formula (involving the median of absolute deviations) given by Müller [1].

We note that among the uncertainty sources tabulated in section 6, the errors due to frequency offset and leakage resistance in the NMIA tables are the most likely to have remained constant over all the NMIA measurements. The same assumption can be made for the uncertainty sources named 'frequency' and 'leakage' in the SCL measurements. In view of the likely high correlation of these uncertainty sources (strictly speaking, the likely high correlation of the associated errors), the combined standard uncertainties were recalculated omitting these correlated uncertainty sources. These recalculated combined standard uncertainties were then root-sum-squared with the standard uncertainties of the medians and with the standard uncertainties of the correlated sources. This procedure avoids multiple counting of these correlated uncertainties. In practice, because these uncertainties are very small, the same results would have been obtained if the combined standard uncertainties, as tabulated in section 6, were simply root-sum-squared with the standard uncertainties of the medians.

The results were as follows. The median is stated and then the overall standard uncertainty and its associated degrees of freedom. The mean date of the period of comparison is 23 June 2002.

10 V level:

NMIA: 9.99996820 V, 0.17 μV, 33 d.f.

SCL: 9.99996774 V, 0.21 μV, 33 d.f.

With u denoting the standard uncertainty, we obtain for the 10 V level and with a high number of d.f.:

$m(V_{10}) = +0.46\ \mu$V, $u(m(V_{10})) = 0.27\ \mu$V.

1.018 V level:

NMIA: 1.018160960 V, 0.029 μV, 19 d.f.

SCL: 1.018161004 V, 0.048 μV, 22 d.f.

We obtain for the 1.018 V level with a high number of d.f.:

$m(V_{1.018}) = -0.044\ \mu$V, $u(m(V_{1.018})) = 0.056\ \mu$V.

7.2 Degrees of equivalence of SCL relative to NMIA

Degrees of equivalence, D, are as follows, with U (upper-case U) denoting the *expanded uncertainty* of D, calculated from the standard uncertainty using a coverage factor $k = 1.96 \sim 2$ in view of the high accumulated number of d.f.:

10 V level:

$D_{10}(SCL - NMIA) = -0.46\ \mu\text{V}$, $U\{D_{10}(SCL - NMIA)\} = 0.54\ \mu\text{V}$.

1.018 V level:

$D_{1.018}(SCL - NMIA) = +0.044\ \mu\text{V}$, $U\{D_{1.018}(SCL - NMIA)\} = 0.112\ \mu\text{V}$.

8 Link to bilateral comparison between BIPM and NMIA

This bilateral comparison took place in the period October-December 2003 (as part of the ongoing BIPM key comparison in dc voltage BIPM.EM-K11a,b), and gave the following results [2].

$$D_{10}(NMIA - BIPM) = V_{10}^{(NMIA)} - V_{10}^{(BIPM)} = +0.13\ \mu\text{V} \tag{1}$$

$$u\left\{V_{10}^{(NMIA)} - V_{10}^{(BIPM)}\right\} = 0.14\ \mu\text{V} \tag{2}$$

$$U\{D_{10}(NMIA - BIPM)\} = 0.28\ \mu\text{V}. \tag{3}$$

and

$$D_{1.018}(NMIA - BIPM) = V_{1.018}^{(NMIA)} - V_{1.018}^{(BIPM)} = +0.028\ \mu\text{V} \tag{4}$$

$$u\left\{V_{1.018}^{(NMIA)} - V_{1.018}^{(BIPM)}\right\} = 0.026\ \mu\text{V} \tag{5}$$

and

$$U\{D_{1.018}(NMIA - BIPM)\} = 0.052\ \mu\text{V} \tag{6}$$

8.1 Linking of SCL results at 10 V level

We have

$$V_{10}^{(NMIA)} - V_{10}^{(SCL)} = +0.46\ \mu\text{V} \tag{7}$$

$$u\left\{V_{10}^{(NMIA)} - V_{10}^{(SCL)}\right\} = 0.27\ \mu\text{V}. \tag{8}$$

Therefore

$$V_{10}^{(SCL)} - V_{10}^{(BIPM)} = (V_{10}^{(SCL)} - V_{10}^{(NMIA)}) - (V_{10}^{(BIPM)} - V_{10}^{(NMIA)}) = (-0.46 + 0.13)\ \mu\text{V} = -0.33\ \mu\text{V}, \tag{9}$$

and

$$u\left\{V_{10}^{(SCL)} - V_{10}^{(BIPM)}\right\} = (0.27^2 + 0.14^2)^{\frac{1}{2}}\ \mu\text{V} = 0.30\ \mu\text{V}. \tag{10}$$

8.2 Linking of SCL results at 1.018 V level

We have

$$V_{1.018}^{(NMIA)} - V_{1.018}^{(SCL)} = -0.044\ \mu\text{V} \tag{11}$$

$$u\left\{V_{1.018}^{(NMIA)} - V_{1.018}^{(SCL)}\right\} = 0.056\ \mu\text{V}. \tag{12}$$

Therefore

$$V_{1.018}^{(SCL)} - V_{1.018}^{(BIPM)} = (V_{1.018}^{(SCL)} - V_{1.018}^{(NMIA)}) - (V_{1.018}^{(BIPM)} - V_{1.018}^{(NMIA)}) = (+0.044 + 0.028)\ \mu\text{V} = +0.072\ \mu\text{V} \tag{13}$$

and

$$u\left\{V_{1.018}^{(SCL)} - V_{1.018}^{(BIPM)}\right\} = (0.056^2 + 0.026^2)^{\frac{1}{2}}\ \mu\text{V} = 0.062\ \mu\text{V}. \tag{14}$$

In (10) and (14) it has been assumed that transportation instability in the Zener(s) effectively makes negligible the correlation that would otherwise be significant between $V^{(SCL)} - V^{(NMIA)}$ and $V^{(BIPM)} - V^{(NMIA)}$, at both voltage levels.

9 Summary of SCL link to BIPM

$$D_{10}(SCL - BIPM) = V_{10}^{(SCL)} - V_{10}^{(BIPM)} = -0.33\ \mu\text{V} \tag{15}$$

$$u\left\{V_{10}^{(SCL)} - V_{1.018}^{(BIPM)}\right\} = 0.30\ \mu\text{V} \tag{16}$$

$$U\left\{D_{10}(SCL - BIPM)\right\} = 0.60\ \mu\text{V} \tag{17}$$

and

$$D_{1.018}(SCL - BIPM) = V_{1.018}^{(SCL)} - V_{1.018}^{(BIPM)} = +0.072\ \mu\text{V}, \tag{18}$$

$$u\left\{V_{1.018}^{(SCL)} - V_{1.018}^{(BIPM)}\right\} = 0.062\ \mu\text{V} \tag{19}$$

$$U\left\{D_{1.018}(SCL - BIPM)\right\} = 0.124\ \mu\text{V}. \tag{20}$$

References:

1. J. W. Müller, 'Possible Advantages of a Robust Evaluation of Comparisons', J. Research NIST, **105**, No. 4, July-August 2000.

2. R. Frenkel, D. Reymann and T.J. Witt, 'Bilateral Comparison of 1.018 V and 10 V Standards between the NML/CSIRO (Australia) and the BIPM, October to December 2003 (part of ongoing BIPM key comparisons BIPM.EM-K11a and b)', Rapport BIPM-2004/.

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