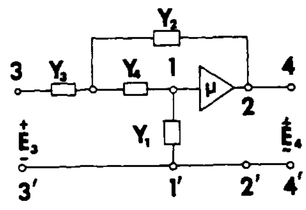

Fig. 2. Example network.

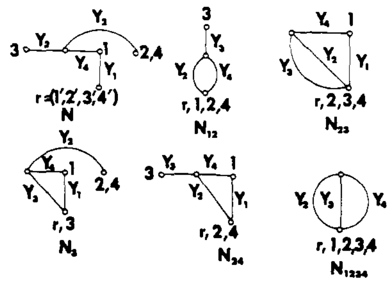

Fig. 3. Some of the subnetworks.

and

$$V_{ikl} \cap V_{jkl} = W_{ij,i'j'}{}^{(kl)} - W_{ij',i'j}{}^{(kl)}, \qquad \text{for } N_{kl} \quad (11)$$

where W denotes the sum of 2-tree-admittance products for 2-trees of the type indicated by the subscript[2] in the subnetwork specified by the superscript.

From (3), (5), and (7)–(10), the following topological formulas can be obtained: for the driving-point function (3-port network model)

$$\frac{E_3}{I_3} = \frac{V_{23} + \mu[W_{12',1'2}{}^{(3)} - W_{12,1'2'}{}^{(3)}]}{W_{2,2'} + \mu[W_{12',1'2} - W_{12,1'2'}]} \quad (12)$$

and for the voltage transfer function (4-port model)

$$\frac{E_4}{E_3} = \frac{W_{34,3'4'}{}^{(2)} - W_{34',3'4}{}^{(2)} - \mu[V_{14} \cap V_{23}]}{V_{23} + \mu[W_{12',1'2}{}^{(3)} - W_{12,1'2'}{}^{(3)}]}. \quad (13)$$

The 2-tree expression for $V_{14} \cap V_{23}$ can be derived using (4), (7), and (11) as follows:

$$\begin{aligned} V_{14} \cap V_{23} &= \Delta_{23,14} = \Delta^{-1}(\Delta_{21}\Delta_{34} - \Delta_{31}\Delta_{24}) \\ &= (V_{1234})^{-1}\{[V_{134} \cap V_{234}][V_{124} \cap V_{123}] \\ &\quad - [V_{124} \cap V_{234}][V_{134} \cap V_{123}]\} \\ &= (V_{1234})^{-1}\{[W_{12,1'2'}{}^{(34)} - W_{12',1'2}{}^{(34)}] \\ &\quad \cdot [W_{34,3'4'}{}^{(12)} - W_{34',3'4}{}^{(12)}] \\ &\quad - [W_{13,1'3'}{}^{(24)} - W_{13',1'3}{}^{(24)}] \\ &\quad \cdot [W_{24,2'4'}{}^{(13)} - W_{24',2'4}{}^{(13)}]\}. \end{aligned} \quad (14)$$

Topological formulas for networks with an infinite-gain operational amplifier is self-evident from (6), (12), and (13).

A most significant implication of the present results is that the topological formulas (12) and (13) are given as the bilinear functions of amplication factor μ, with which it is possible to express the return differences and sensitivities with respect to μ by the topological quantities.[4]

Due to the generality assumed regarding the locations of the node-pairs for E_1, E_2, E_3, and E_4 in Fig. 1, the present topological formulas, especially (14), appear cumbersome. However, they reduce to fairly simple expressions in most special cases, as is illustrated by the following example.

Example: Consider the network shown in Fig. 2 where the transfer voltage ratio E_4/E_3 is wanted. Since nodes 1', 2', 3', and 4' are the same (common grounded), there are no 2-trees of the type separating nodes i' from j'. In addition, $W_{34,3'4'}{}^{(12)} = 0$ since $3' = 4$ in N_{12} (see Fig. 3). Thus all the nonzero quantities that have to be evaluated in (13) and (14) are: $V_{23} = (Y_1 + Y_4)(Y_2 + Y_3) + Y_1Y_4$, $W_{12,1'2'}{}^{(3)} = Y_2Y_4$, $V_{14} \cap V_{23} = -W_{13,1'3'}{}^{(24)}W_{24,2'4'}{}^{(13)}/V_{1234} = -Y_3Y_4(Y_2 + Y_3 + Y_4)/(Y_2 + Y_3 + Y_4)$. Therefore

$$\frac{E_4}{E_3} = \frac{\mu Y_3 Y_4}{(Y_1 + Y_4)(Y_2 + Y_3) + Y_1Y_4 - \mu Y_2 Y_4}.$$

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[4] T. Murata, "On topological formulas for the sensitivity study of networks with a controlled source," presented at the 1972 National Electronics Conf., Oct. 1972. Also, "Topological formulas for bilinear forms of active network functions," to be published in *IEEE Trans. Circuit Theory.*

A Precision Current Multiplier/Divider

Abstract—**A basic monolithic integrated circuit building element is presented, which can perform accurate fixed-factor current multiplication or division without the necessity for precision resistors or resistor networks. The circuit, which consists of five transistors of the same polarity, is an improved version of the differential transistor pair.**

In the design of monolithic integrated circuits the good base-emitter voltage matching between adjacent transistors has been well exploited to achieve various functions without the need for precision resistors and large capacitors, as in discrete component circuit design. This principle has not yet been successfully applied in a few remaining areas only, such as high-speed A/D and D/A converters. Such microcircuits as are available are either hybrid, making use of high-precision thin-film resistor networks, or in the case of the fully monolithic form, contain medium-precision (approximately 1 percent) resistor networks. It is, however, possible to achieve a degree of accuracy at least similar to that obtained in the latter case, using only transistors for current division or multiplication as the following discussion shows.

The basic circuit is illustrated in Fig. 1(a), and is related to the current source of Widlar[1] or the compound structure of Davis and Lin;[2] only equal-geometry transistors will be considered for the time being, although this is not a necessary restriction.

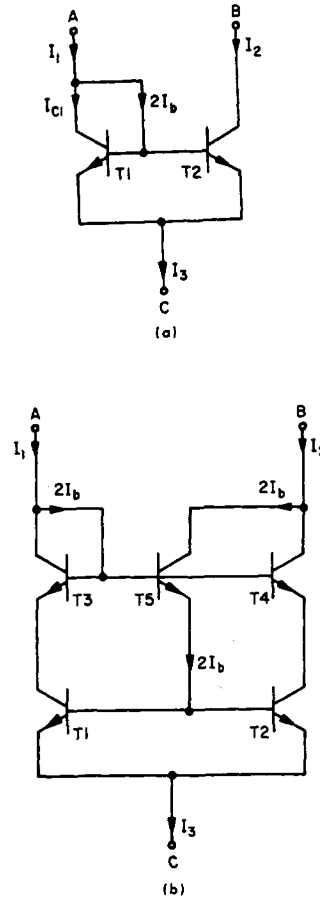

Fig. 1. (a) Basic circuit of current multiplier/divider. (b) Circuit with improved performance.

When a current source is connected to terminal A, this circuit acts as a current doubler (multiplier), such that $I_3 \simeq 2I_1$, or when a current source is connected to terminal C, it becomes a current halver (divider), giving $I_1 \simeq I_2 \simeq I_3/2$. There are three possible sources of error, which become apparent when one of the terminal current–voltage relationships[3] of a transistor is considered.

$$I_c = I_s\left[\exp\left(\frac{qV_{be}}{kT}\right) - 1\right] \cdot [1 + V_{ce}/V_A]. \quad (1)$$

The Early voltage V_A represents the effect of the finite output impedance of the transistor,[3] and can be of the order of 100 V for a planar transistor. This means that I_c changes by about 1 percent per volt change in V_{ce}. From (1) it is also easy to deduce that a V_{be} mismatch of less than 0.5 mV is required to keep the divider/multiplier error below 1 percent. Thirdly, from Fig. 1(a) it is clear that the base currents of both tran-

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[1] R. J. Widlar, "Some circuit design techniques for linear integrated circuits," *IEEE Trans. Circuit Theory*, vol. CT-12, pp. 586–590, Dec. 1965.

[2] W. R. Davis and H. C. Lin, "Compound diode-transistor structure for temperature compensation," *Proc. IEEE* (Lett.), vol. 54, pp. 1201–1202, Sept. 1966.

[3] F. A. Lindholm and D. J. Hamilton, "Incorporation of the Early effect in the Ebers–Moll model," *Proc. IEEE* (Lett.), vol. 59, pp. 1377–1378, Sept. 1971.

sistors have to be supplied from one side only.

The circuit shown in Fig. 1(b) overcomes the first and third of the above shortcomings very effectively by holding the collector voltages V_{c1} and V_{c2} of transistors T1 and T2, respectively, equal, low, and approximately constant, by the addition of the two transistors T3 and T4. This arrangement improves the effective output impedance of the T4/T2 chain, or T3/T1 chain, by some three orders of magnitude, and reduces to insignificance the error caused by a large voltage swing. Transistor T5 functions as a base current supply for T1 and T2, but draws its current from the B branch to compensate for the base current demand of T3 and T4 from the A branch. On the assumption that the four transistors T1 to T4 are gain-matched to some degree (10 percent, say), the accuracy of the end result, $I_1=I_2=I_3/2$, is determined mainly by the V_{be} match of T1/T2 and T3/T4. T5 can, but need not necessarily, be of much smaller geometry than T1 to T4.

A complete D/A converter was breadboarded, using encapsulated transistor arrays, and 7-bit accuracy was obtained, using the circuit in Fig. 1(b) as the basic current-divider element. A more complex network, but based on the same basic idea, has also been tested as an element for a high-speed tracking analog-to-digital converter requiring no time sequential logic. A more detailed report will be published at a later stage.

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42-GHz Push–Pull Gunn Oscillator

Abstract—**Two Gunn-effect diodes were placed in a waveguide resonant circuit operating in a "push–pull" mode. The circuit was designed for 42 GHz, and power as high as 260 mW with 4.0-percent efficiency was obtained. Spurious modes and other circuit problems were investigated and some techniques were developed to cope with these problems.**

Introduction

Combining more than one Gunn diode in a microwave circuit is a very attractive way to increase the output power of Gunn oscillators. Recently, a circuit has been developed utilizing two Gunn diodes in a waveguide cavity in a "push–pull" mode, at 42 GHz, for high-power parametric amplifier pump applications. This letter describes the circuit, discusses the work, and states the results of this effort.

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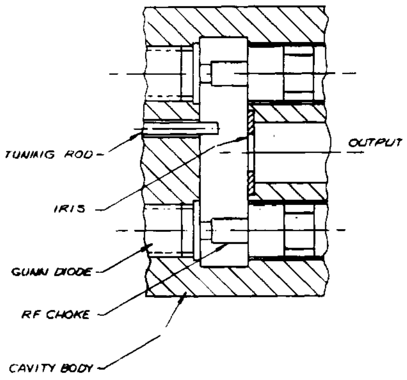

Fig. 1. Push–pull circuit layout.

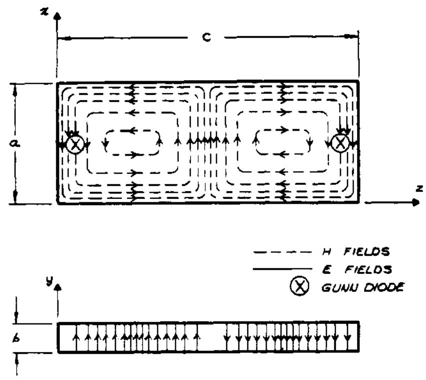

Fig. 2. Field pattern for push–pull mode.

Circuit Description

This oscillator utilizes two Gunn-effect diodes operating in what is known as a "push–pull" mode. The name "push–pull" is derived from the fact that at the load coupling point the magnetic fields add in phase, with the two diodes placed in a waveguide resonant cavity approximately one guide wavelength apart, as shown in Fig. 1. One may then inductively couple the power to an external load, thus obtaining at least double the power of a single diode. In our case we inductively coupled through an iris to rectangular waveguide.

RF isolation for the bias network is obtained by using standard choking techniques (see Fig. 1). Finally, the oscillator is tunable by means of a dielectric rod slightly off-center between the diodes to perturb the weak electric field at that point (see Fig. 1 and Fig. 2).

Discussion

One of the very basic and critical parameters in this circuit is the distance between the two Gunn diodes. Ideally, one wants this distance to be one guide wavelength for proper diode phasing to obtain power addition. However, this has to be adjusted slightly empirically to compensate for the diode package, tuning rod, etc. It is interesting to note that this diode distance rather than the distance between the cavity walls in this same direction has the largest effect on the operating frequency of the oscillator. These factors seem to put a limitation on this device for use in wide tuning applications, but we have found in the laboratory that a 5-percent tuning bandwidth does not drastically affect the performance of the oscillator.

As with the case of a single diode waveguide cavity, there are modes other than the desired mode that can couple into the push–pull circuit.

One possible mode of oscillation is at that frequency where the distance between the diode looks like one-half wavelength. We found that this mode did not seem to be present in the oscillator. It is thought that a possible reason for this is that the frequency is much lower than the transit-time frequency of the Gunn diodes, and they cannot be forced to operate at such a low frequency. It is also possible that the iris in the middle of the cavity would distort the E-fields that would be a maximum for this mode; thus the mode would be suppressed.

A very significant mode that exists in this cavity is a TEM-type mode localized around the Gunn-diode package and biasing network. It is believed that this mode was the cause of some serious turn-on and reproducibility problems we faced with this oscillator. It was determined that this mode could be tuned away from the desired operating point with the use of dielectric and metal rods placed in the cavity near the diodes. This was not a desirable solution, however, because the turn-on and other mode problems were sensitive to tuning of the operating frequency. We found that a better solution was to suppress this undesired mode by loading it down heavily. When this was done the performance of the oscillator improved markedly.

There is a significant advantage to having the diodes in parallel with the dc bias. It has been reported by other investigators [1]–[4] that when the Gunn diodes are in series with the dc bias they have to be almost perfectly matched, lest the lower current diode reach threshold first, thus acting as a current limiter. When this happens the higher current diode does not have sufficient current to reach the threshold region, unless the dc bias voltage is increased to a sometimes catastrophic level. This poses serious dc biasing problems that are not encountered with parallel biasing. One has only to match diode threshold voltages within approximately 20 percent.

The Gunn diodes used are Varian solution grown n-type epitaxial gallium arsenide. The active layer is 2.5-μ thick, grown on a tin-doped GaAs substrate which is used as an anode. The cathode contact is a gold-germanium nickel alloy contact, which has performed very well at Varian, particularly with these high-frequency diodes. An ohmic metal contact is alloyed to the substrate of the GaAs wafer. The carrier concentration of the active layer was 5.5×10^{15} cm^{-3}. The threshold voltage is approximately 1.5 V, with operating voltage about 3 times threshold. The Gunn diode is packaged in a standard Varian N34 package. This consists of a 0.030-in diameter by 0.018-in high dielectric and metal enclosure that is mounted on a metal heat-sink stud. The parameters of this package have been discussed previously by Fank and Day [5].