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Calibration of the Absolute Linearity of Lock-In Amplifiers

David Corminboeuf

***Abstract*— This paper describes the development of a system capable to calibrate the absolute linearity of lock-in amplifiers in their 1-μV range for frequencies between 20 Hz and 1 kHz. The system is based on cascaded inductive voltage dividers that have been especially designed to maximize their input impedance and to have minimal influence on the previous stage. The cascaded topology reduces the uncertainty for very large ratios (>1000:1) and reaches measurement uncertainties below 1 nV at 10 μV.**

***Index Terms*— Inductive voltage divider (IVD), linearity, lock-in amplifier, synchronous sampling, transformer, very low ac voltage, voltage ratio.**

I. Introduction

LOCK-IN amplifiers are very sensitive and low noise instruments, well known in the metrology. In particular, they are often used as null detectors. When the amplitude of the signal is close to 0, the nonlinearity of the instrument can be neglected. However, in most real cases, the detector always measures a residual voltage, which is used to calculate a correction. Therefore, by knowing the nonlinearity or the gain error of the detector, the uncertainty on this correction can be reduced. In addition, knowing precisely the deviation of the linearity of a lock-in amplifier can also improve the speed of convergence in the case of an automatic bridge balancing. In fact, the balancing procedure can stop earlier and accept a larger residual voltage because the uncertainty of this measurement will be lower.

On the other hand, lock-in amplifiers can also be used as measuring instruments, for example, in the sensor area, where low ac voltages are measured via resistance ratios. In this case, the linearity calibration of the lock-in is extremely important.

A method for the calibration of lock-in amplifiers is described in [1], but it is based on optical components. One of the simplest systems to generate very small ac voltages consists in a voltage source coupled to a voltage divider which can be either resistive [2] or inductive [3]. Although a single IVD can produce an output voltage down to 1 μV or less, the uncertainty on the output voltage will be too high for high precision calibration. For example, if an IVD ratio of 100 000:1 is used with a ratio uncertainty of 1×10^{-7} [4], the relative uncertainty on the output voltage is 1%. Furthermore, the IVD is loaded by the input impedance of the lock-in amplifier, introducing an additional error in the calibration. To solve these two problems, several IVDs are connected in cascade. This cascaded topology can only work properly if the input impedance of each IVD is sufficiently large to prevent any loading on the output of the previous stage. To meet these loading conditions, a buffer amplifier has been added to the primary stage of each divider, reducing the loading error. With this scheme, the measurement uncertainty was strongly reduced and the objective of an uncertainty of 1 nV at 10 μV for a frequency of 20 Hz was achieved. This uncertainty level makes possible to calibrate not only the linearity of a lock-in amplifier but also the gain error in the microvolt ranges, which is defined in this paper as the absolute linearity. Preliminary results are presented in [5].

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II. Calibration Setup

A. First Attempt

The first attempt to successfully generate a voltage of 10 μV and to calibrate a lock-in amplifier (type AMETEK 7265) was based on two IVDs: an 8 decades reference IVD (IVDref) and a second homemade IVD (IVDdiff) [6]. In this configuration, the maximum achievable ratio is 25 000:1, 25:1 for IVDdiff, and 1000:1 for IVDref. To calibrate the linearity of the 10-μV range of the lock-in, several points between 10% and 100% of the full scale have to be measured between 1 and 10 μV. Under these conditions, the voltage source was set to 25 mV (25 000 × 1 μV) with an uncertainty of less than 100 μV/V, as measured with a calibrated HP 3458A multimeter operating in digitizing mode. With this system, the first calibration of the linearity of a lock-in amplifier in the range 10 μV could be carried out. Although this system works well, it has absolutely no flexibility since it is limited to the ratio 25 000:1 with an input voltage of 25 mV.

Therefore, a new system has been developed with the aim to achieve greater flexibility, a better uncertainty and an extension of the range to 1 μV.

B. Principle

In order to maintain the small uncertainty, the principle of cascaded IVDs is kept in the improved setup (Fig. 1). The new system consists of four dividers, three new transformers with ratios of 10:1 and the IVDref. The main improvement is the addition of an operational amplifier on the primary winding of the divider. This leads to an increase in its input impedance and, thus, prevents any measurable loading of the

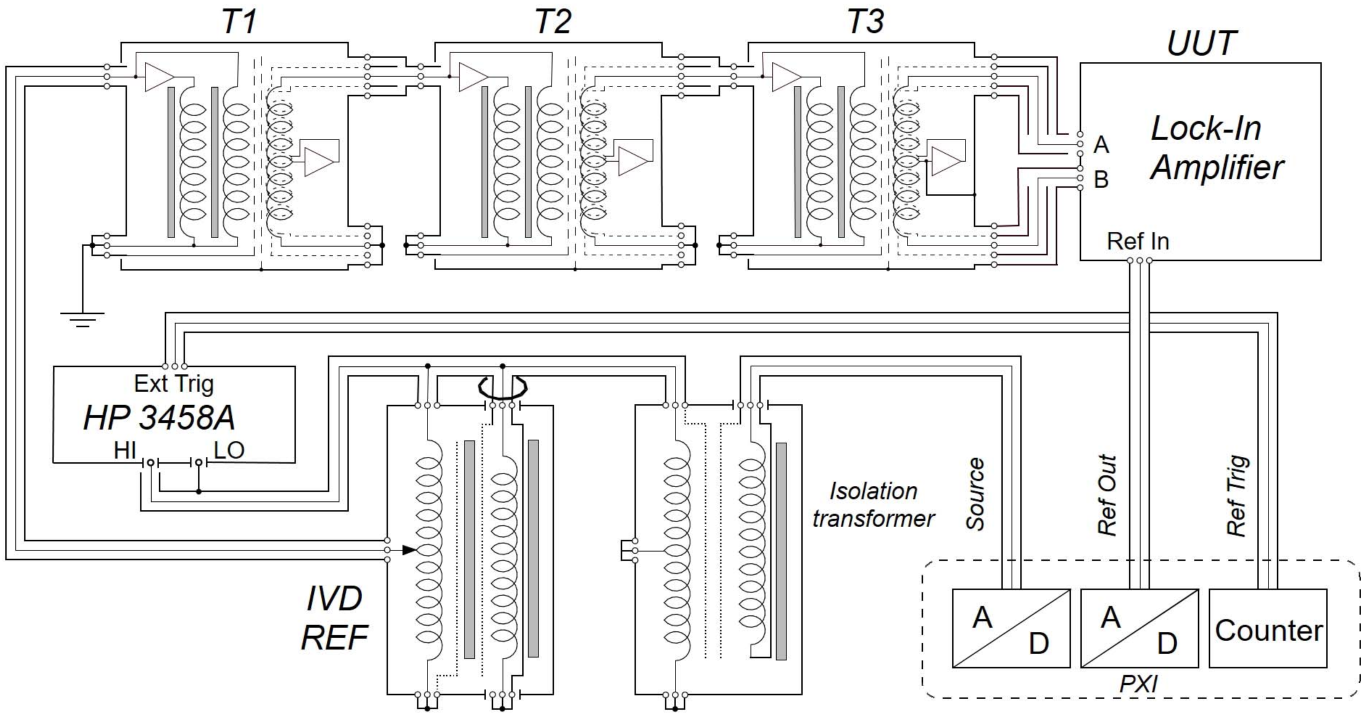

Fig. 1. Schematic of the calibration setup with, the PXI chassis and its three output signals “Source,” “Ref Out,” and “Ref Trig,” the four cascaded dividers IVDref, T1, T2, and T3, the multimeter 3458A used in digitizing mode and the lock-in amplifier as UUT.

previous stage. With this principle, it is possible to cascade N dividers and to know the ratio deviation of each stage.

A PXI chassis with two boards gives the “Source,” the “Ref Out,” and the “Ref Trig” signals. The “Source” is the signal that is measured by the lock-in amplifier after being divided successively by IVDref and the three active transformers. Its amplitude is measured by the multimeter 3458A at the input of IVDref. The “Ref Out” signal is a sine wave used as reference by the lock-in amplifier. The “Ref Trig” signal is a square wave used as sampling clock by the multimeter 3458A.

III. System Description

A. Active Transformers

The active transformer is a two-stage, double-shielded transformer, which consists of two windings wound around two magnetic toroidal cores. The first winding (magnetizing winding) is wound around the first core, and the second winding (ratio winding) is wound around both the cores [7]. These two windings have a copper overlaying electrical screen to avoid parasitic effects between the primary and secondary windings. The magnetizing winding is powered by a very low-noise JFET operational amplifier with an input impedance of 10^{13} Ω and 9 pF. This amplifier is mounted as a buffer; it increases the input impedance of the two-stage transformer, thus avoiding an excessive load that would degrade the ratio of the previous stage.

The secondary winding is realized with a coaxial cable. To cancel the leakage current between the center conductor and the shield, the shield of this winding is fixed to the same potential as the center conductor by a second JFET operational amplifier. Each of the three windings is made with antiprogression turn to reduce sensitivity to external magnetic fields. A second electrical copper screen is placed between the first screen and the secondary winding. This screen can be fixed to a potential that allows the transformer to be independent on the common mode. Finally, the transformer is encased in a mu-metal box to avoid external magnetic field perturbations.

Each active transformer has a 10:1 ratio with 100 turns for the primary winding and 10 turns for the secondary winding and works at frequencies from 20 Hz to 10 kHz at a maximum voltage of 0.08 V/Hz.

These three active transformers are mounted into a 19″ rack powered by the 220-V network (Fig. 2). All the JFET operational amplifiers are powered by a METRON ±15-V power supply, which has very high performances in terms of noise and insulation. This is mandatory to reach a sub-microvolt signal level with a minimum environmental noise.

The active transformers were characterized by using the IVD comparison method described in [4], and the results are presented in Sections III-A.1 and III-A.2.

1) Frequency and Voltage Dependence: The in-phase ratio deviation of the active transformers decreases when the frequency increases as can be seen in Fig. 3. This behavior is caused by the inductive component of the load. However, this dependence is consistent between each of the active transformers T1, T2, and T3.

No voltage dependence can be identified according to the measurements of the in-phase ratio deviation with a voltage from 100 mV to 5 V (Fig. 4). The uncertainty is very low

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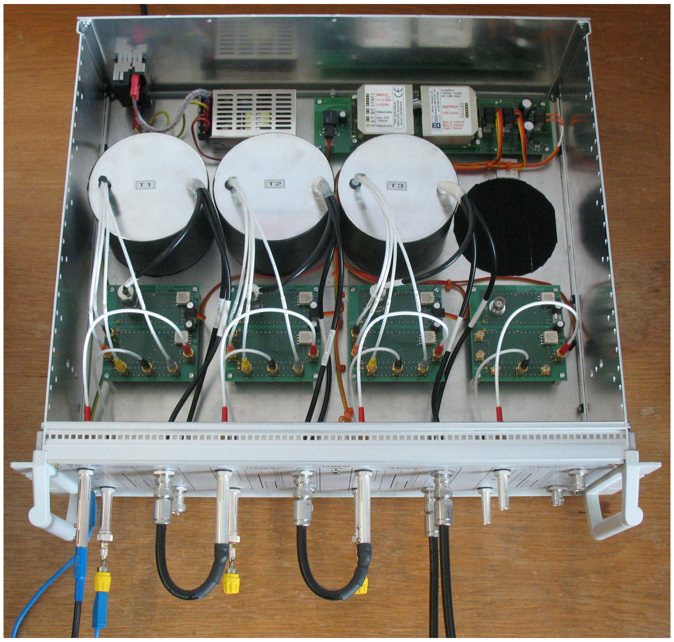

Fig. 2. Photograph of the three active transformers and the power supply at the back of the 19″ rack.

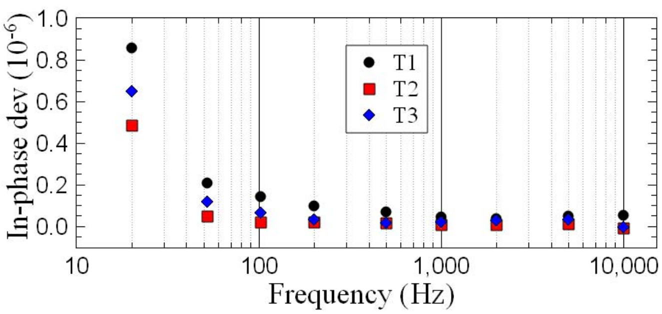

Fig. 3. Frequency dependence of the in-phase ratio deviation of each active transformer T1, T2, and T3.

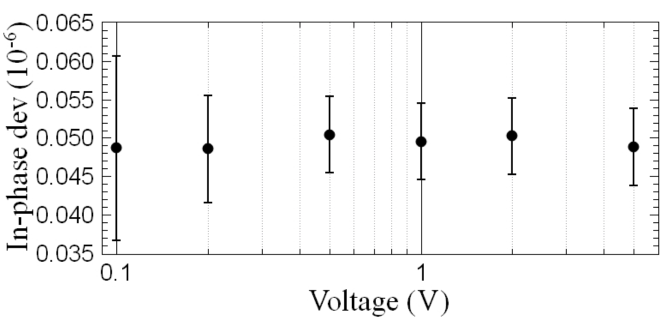

Fig. 4. Voltage dependence of the in-phase ratio deviation of the active transformer T1.

because the setup has not changed between each measurement. Therefore, only type-A uncertainty is taken into account.

2) Load Dependence: To check if the results must be corrected for the input impedance of the unit under test (UUT), the dependence of the active transformers on the load was measured. The UUT was either a Stanford Research Systems SR850 or an AMETEK 7265 lock-in amplifier, both with an

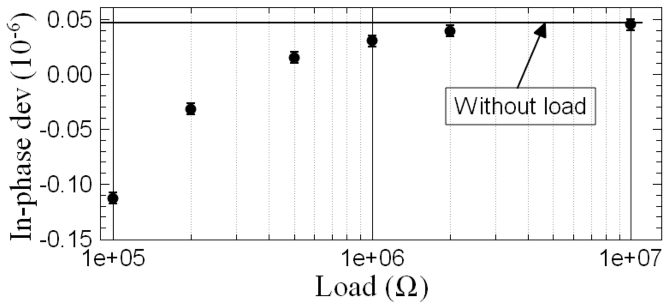

Fig. 5. In-phase ratio deviation of the T1 active transformer with six resistive loads between 100 kΩ and 10 MΩ connected to its output.

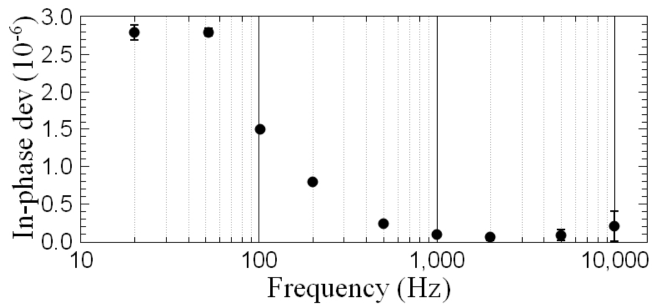

Fig. 6. In-phase ratio deviation of the T1 active transformer with T2 as load connected to its output.

input impedance of 10 MΩ. As shown in Fig. 5, no correction is required since the error due to the load is only a few parts in 10^9 and affects only the last stage (ratio 10:1). Therefore, the error relative to the output is a fraction of a part in 10^6, which is negligible in this paper.

The load of the active transformers in the cascaded topology must also be considered. Fig. 6 shows that this load dependence is not constant over the whole frequency range. This result is clearly explained by the active component of the JFET operational amplifier at the input of the transformer. The maximal deviation is 2.8 parts in 10^6 at 20 Hz.

The corrections for the three possible configurations can be calculated as follows.

The transfer functions for the ratios 10:1 and 100:1 are

$$\frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN10}}} = R_1 + \delta_1 \quad \text{and} \quad \frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN100}}} = (R_1 + \delta_1)(R_2 + \delta_2) \qquad (1)$$

where R_i is the ratio of the i active transformer, and δ_i is the ratio deviation of the i active transformer.

Assuming $R_1 = R_2 = R$ and neglecting second-order corrections, the ratio becomes

$$\frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN100}}} = R(R + (\delta_1 + \delta_2)) \qquad (2)$$

Equation (2) means that the deviation of two cascaded dividers is the sum of each deviation relative to the output.

At 100 Hz, the deviation relative to the output of one active transformer is 1.4 parts in 10^6 without load and 15 parts in 10^6 loaded by another active transformer. Table I shows the three configurations.

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TABLE I
DEVIATION RELATIVE TO THE OUTPUT OF THE THREE ACTIVE TRANS FORMERS CONFIGURATIONS AT 100 Hz

Active transformers	Ratio 10:1 deviation	Ratio 100:1 deviation	Ratio 1000:1 deviation
T1	1.4	15	15
T2	-	0.2	15
T3	-	-	0.7
Total dev in 10^{-6}	**1.4**	**15.2**	**30.7**

According to (2), the uncertainty relative to the output becomes $1 \times u$ for ratio 10:1, $2 \times u$ for ratio 100:1, and $3 \times u$ for ratio 1000:1, where u is the measured relative uncertainty referred to the output.

At 20 Hz, the measured relative uncertainty referred to the input is 0.1 parts in 10^6, corresponding to an uncertainty relative to the output of 1 parts in 10^6 for a 10:1 ratio. Three cascaded transformers giving a ratio of 1000:1 have an uncertainty relative to the output of 3 parts in 10^6, which is an uncertainty relative to the input of only 3 parts in 10^9.

3) Differential Output: The secondary of the active transformer is not connected to the primary, like in an isolation transformer, making a differential measurement possible. This is particularly interesting when the two inputs of the UUT must be calibrated in the differential mode. Indeed, the calibration can be carried out in a single simultaneous measurement. In this configuration, it is important to take care of the cable shield to avoid ground loops.

B. Reference IVD

The reference divider is a two-stage, single-shielded divider with 8 decades. The unloaded divider was calibrated in the whole frequency range, from 20 Hz to 10 kHz. Its in-phase relative deviation on the output is between -6 and -11 parts in 10^6 (worst case at 20 Hz) for the first decade. In the cascaded setup, IVDref is loaded by the input of the first active transformer. With the active transformer connected to its output, the relative deviation on the output of IVDref varies from -1 to -7 parts in 10^6 at 20 Hz. The deviation decreases because the amplifier of the active transformer gives current which appears as a negative resistance. Due to its inductive and capacitive components, this load also influences the quadrature part by less than 2 μrad, its contribution to the in-phase part being negligible.

C. Voltage Source

The voltage source was realized by one of the two 24-bit digital-to-analog converters of a PXI board, type NI-PXI4461. The second output, perfectly synchronous to the first one, is used to generate the reference signal of the lock-in amplifier with a constant signal amplitude of 1 V. Another NI board, the NI-PXI6220, is used as a clock generator for the voltmeter HP 3458A to ensure synchronization between the voltmeter and the source signal. The PXI chassis is controlled by a computer connected via fiber optic. This computer also controls the settings and data acquisition of the voltmeter and the lock-in amplifier with a homemade software developed in LabVIEW.

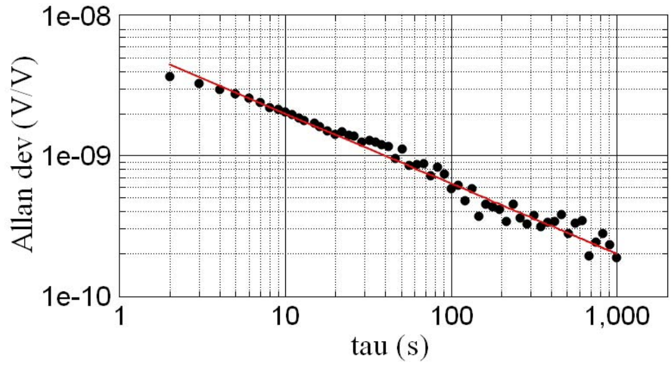

Fig. 7. Allan deviation of the in-phase signal at 103 Hz. The fit gives a noise spectral density of 6 nV/$\sqrt{}$Hz.

D. Voltmeter

The voltmeter is an HP 3458A used in digitizing mode [8]. In this mode, the specifications given by the manufacturer are better than in the ac mode and can reach 10 μV/V in the 100-mV range. Due to the input low-pass filter of the voltmeter, a frequency correction [9] is applied to the measured value. The verification of this correction factor was carried out by measuring a sine-wave voltage generated with a calibrated calibrator. The frequencies 40 Hz, 500 Hz, and 1 kHz were tested at 1 V, and the differences were smaller than 5 parts in 10^6. The amplitude of the signal was calculated with a fast Fourier transform algorithm. To verify the accuracy of this function and its implementation, a second algorithm was used. A sinus fit function was applied to the same data set, and the results were compared. The two different methods show a difference of less than 2 parts in 10^6.

The sampling clock provided by the NI-PXI6220 board and the internal clock of the voltmeter are not synchronized, introducing an uncertainty on the amplitude of the measured signal. This uncertainty is similar to the jitter and can be calculated according to [10]. In the frequency range from 20 Hz to 1 kHz and with 1000 data samples, this uncertainty is less than 1 part in 10^6.

IV. MEASUREMENTS AND UNCERTAINTIES

Measurements were carried out in the ranges 1, 10, and 100 μV at frequencies between 20 Hz and 1 kHz with the setup presented in Fig. 1.

A. Noise

The noise of the SR850 was measured by applying an input signal of 1 μV at 103 Hz in the 1-μV range and calculating the Allan deviation. The result, displayed in Fig. 7, shows that the noise is white up to 1000 s with a noise spectral density of 6 nV/$\sqrt{}$Hz. The measured noise is exactly the input noise declared by the manufacturer [11].

The deviation from white noise observed at small observation time τ is due to the low-pass filter of the lock-in, which

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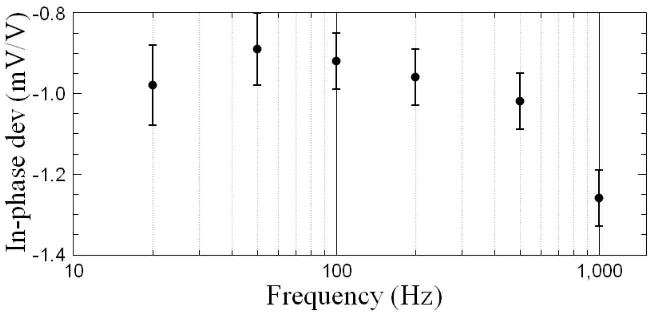

Fig. 8. In-phase deviation of a 10-μV sine wave measured by the lock-in amplifier AMETEK 7265 in frequency range from 20 Hz to 1 kHz.

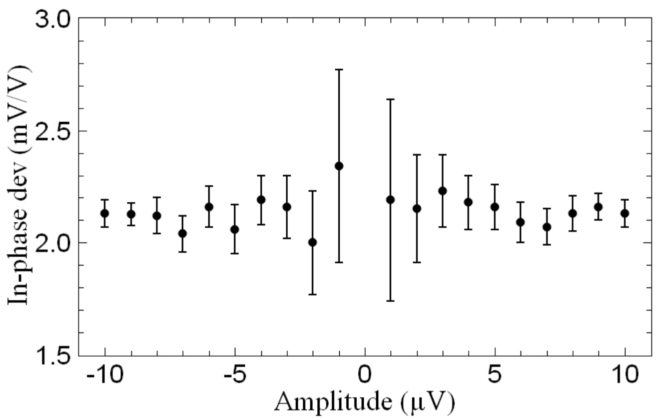

Fig. 9. In-phase deviation of a sine wave measured by the lock-in amplifier SR850 in the 10-μV range at 103 Hz.

is controlled by the "time constants" parameter. For these measurements, it was set to 300 ms. To verify that the input signal does not increase the noise level, the intrinsic noise of the instrument was measured by putting two short-circuits on the inputs A and B. No significant difference between these two measurement results was observed.

B. Lock-In Linearity

The first linearity measurements in the 10-μV range at 20 Hz were carried out with the AMETEK 7265. During the calibration, one must verify that the "line frequency rejection filter" (notch filter) of the lock-in is disabled otherwise erroneous data can be obtained around 50 and 100 Hz. Particular attention should be paid to the settings of the lock-in amplifier like notch filter, time constants, single-ended, or differential; because they can greatly influence the results, especially with small signals.

In order to test the system and the lock-in over the whole frequency range, six signals with frequencies from 20 Hz to 1 kHz and an amplitude of 10 μV were measured (Fig. 8). The in-phase deviation of the lock-in at full scale of the 10-μV range of is not constant over this frequency range.

Fig. 9 shows the linearity of the lock-in amplifier SR850 in the range 10 μV at 103 Hz which gives a deviation around 2100 parts in 10^6 from -10 to 10 μV. The autophase function was performed at the beginning of the measurements with a full-scale signal. For negative values, the input signal was

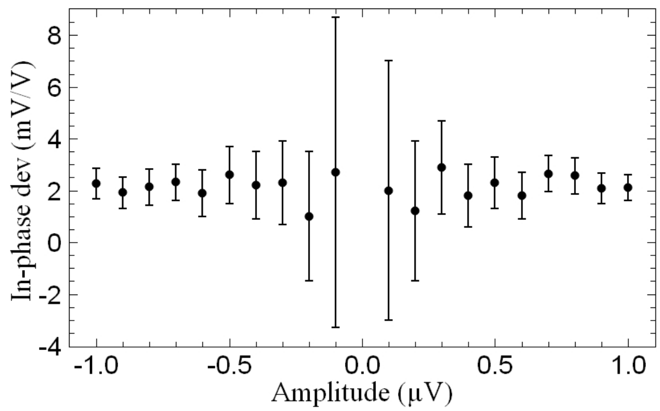

Fig. 10. In-phase deviation of a sine wave measured by the lock-in amplifier SR850 in the 1-μV range at 103 Hz.

TABLE II
IN-PHASE DEVIATION FOR 100-μV OUTPUT VOLTAGE AT 103 Hz IN SEVERAL DIFFERENT CONFIGURATIONS

Input voltage (V)	Active Transformers ratio	Active Transformers configuration[a]	IVDref ratio	Deviation (μV/V)
0.1	0.001	T1+T2+T3	1.00	2421
0.1	0.010	T1+T2	0.10	2398
0.1	0.010	T2+T3	0.10	2425
0.1	0.100	T3	0.01	2434
0.1	0.100	T1	0.01	2404
0.1	0.100	T2	0.01	2421
0.1	0.001	T1+T2+T3	1.00	2420
0.2	0.001	T1+T2+T3	0.50	2421
0.5	0.001	T1+T2+T3	0.20	2418
1.0	0.001	T1+T2+T3	0.10	2410
2.0	0.001	T1+T2+T3	0.05	2402
5.0	0.001	T1+T2+T3	0.02	2384
0.1	0.001	T1+T2+T3	1.00	2399
			Average	**2412**
			Stdev	**14**

[a]The configuration indicates which of the three (T1, T2, T3) Active Transformers is used.

180° phase shifted. The measured quadrature signal was always negligible compared to the in-phase signal. The best uncertainty, at full range, is 70 parts in 10^6 with $k = 2$.

The same measurements series was repeated in the 1-μV range and gives a deviation of the in-phase signal around 2200 parts in 10^6 (Fig. 10). The duration of the measurement was 300 s per point.

C. Consistency Check

The ranges 100 μV and 1 mV were also measured but only at full scale. Because of the good short-term stability of the lock-in amplifier SR850, the measurement results of various combinations of the active transformers and voltages can be compared to check the consistency. Table II summarizes all results that generate a signal with the same voltage amplitude of 100 μV.

The average of deviations of all configurations is 2412 parts in 10^6, and the standard deviation is 14 parts in 10^6.

TABLE III
UNCERTAINTY BUDGET

Uncertainty components	Types	u (10^{-6}) 10 µV/100 Hz
IVDref	B	0.3
Active Transformers	B	0.9
Voltage source stability	B	15
Multimeter	B	10
Lock-in amplifier	A	26
Combined uncertainty *U* (*k* = 2)		**70**

- IVDref: relative uncertainty to the output with a ratio of 0.2
- Active dividers: relative uncertainty to the output with a ratio of 0.001
- Voltage source stability: short-term stability of the NI PXI 4461 output
- Multimeter: HP 3458A used in digitizing mode in the 100 mV range
- Lock-in amplifier: standard deviation of the 300 measurements with the SR850

This standard deviation is smaller than the standard deviation of a single measurement which is around 20 parts in 10^6.

V. UNCERTAINTY BUDGET

Table III lists the uncertainty budget of the measurement system for a voltage amplitude of 10 μV at 100 Hz.

VI. CONCLUSION

A new system based on the cascaded active transformers capable to generate ac voltages with amplitudes down to 100 nV, from 20 Hz to 1 kHz, was presented. This system can calibrate the absolute linearity of lock-in amplifiers in ranges from 1 μV to 1 mV. As an example, a typical uncertainty, of 70 parts in 10^6 ($k = 2$) can be reached at 10 μV and 100 Hz. A lock-in amplifier like the SR850 can be used as a high-precision ac voltmeter due to its very good linearity, stability, and low noise.

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REFERENCES

[1] E. Theocharous, "Absolute linearity characterization of lock-in amplifiers," *Appl. Opt.*, vol. 47, no. 8, pp. 1090–1096, 2008.

[2] F. Raso, A. Hortelano, and M. M. Izquierdo, "A calibration method of the linearity of lock in amplifiers," in *Proc. Conf. Precis. Electromagn. Meas. (CPEM)*, Ottawa, ON, Canada, Jul. 2016, pp. 1–2.

[3] L. Callegaro, V. D'Elia, M. Pisani, and A. Pollarolo, "A Johnson noise thermometer with traceability to electrical standards," *Metrologia*, vol. 46, no. 5, p. 409, 2009.

[4] D. Corminboeuf and F. Overney, "Inductive voltage divider calibration with sampling method," in *Proc. EPJ Web Conf.*, vol. 77, 2014, p. 00014, doi: 10.1051/epjconf/20147700014.

[5] D. Corminboeuf, "Calibration of the absolute linearity of lock-in amplifiers," in *Proc. Conf. Precis. Electromagn. Meas. (CPEM)*, Paris, France, Jul. 2018, pp. 1–2.

[6] D. Corminboeuf, "Calibration of bridge standard for strain gauge bridge amplifier," in *Proc. 17th Int. Congr. Metrol.*, 2015, p. 04004, doi: 10.1051/metrology/201504004.

[7] S. Awan, B. Kibble, and J. Schurr, *Coaxial Electrical Circuits for Interference-Free Measurements*. Edison, NJ, USA: IET, 2011.

[8] *3458A Multimeter User's Guide*, 7th ed, Keysight, Santa Rosa, CA, USA, Aug. 2014.

[9] R. L. Swerlein, "A 10 ppm accurate digital AC measurement algorithm," in *Proc. NCSL*, Albuquerque, NM, USA, Aug. 1991, pp. 17–36.

[10] G. Betta, C. Liguori, and A. Pietrosanto, "Propagation of uncertainty in a discrete Fourier transform algorithm," *Measurement*, vol. 27, pp. 231–239, Jun. 2000.

[11] *MODEL SR850 DSP Lock-In Amplifier, Revision 2.0*, Stanford Res. Syst., Sunnyvale, CA, USA, Jan. 2009.

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