Calibration of a Kelvin-Varley Voltage Divider

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Summary—**Earlier descriptions of methods for providing corrections to be applied to a Kelvin-Varley voltage divider can be misconstrued, particularly as the reasons for applying the corrections in the manner outlined are not obvious. The new method removes any ambiguity and presents the theoretical reasons leading to the procedure which may be carried as far as is justified by the instrument. The magnitude of the terms being neglected may be estimated at any stage, to serve as a guide to the reliability of the corrected data. Techniques of calibrating a divider, which do not require internal connections to the instrument, are also presented.**

INTRODUCTION

A MULTIPLE-DECADE Kelvin-Varley voltage divider[1] is frequently used to measure or to establish ratios of voltages both at dc and at ac up to several kilocycles. It is this complete coverage of the frequency spectrum which often dictates use of a Kelvin-Varley resistive voltage divider rather than alternative methods, notably the inductive voltage divider. With the advent of precision resistance components which have excellent characteristics at audio frequencies, high-precision Kelvin-Varley dividers with as many as six or seven decades of voltage ratio variation have become possible and are available for use from dc to several kilocycles.

Resistance elements are wound from wire of one of the metallic alloys in general use and have always been subject to variation of resistance in the time following manufacture. Various methods of artificial aging of the finished resistor are most successful in reducing the time variations to a reasonably small level, but it is seldom that a resistance coil is obtained that does not change with time, at least by a few parts in a million (ppm) per year. If six and seven-decade Kelvin-Varley dividers are to be made and used, then it is necessary to be able to calibrate them periodically so as to obtain corrected ratios with equivalent high precision and reliability.

In the earlier methods[2,3] of providing corrections to be applied to the dial setting of a Kelvin-Varley divider, there is an ambiguity in the method of applying the calibration data, and if the wrong approach is used, an error will be caused. For a good divider, the discrepancy is slight and in earlier years it was of less importance, but as the units age and the demand for accuracy and reliability in electrical measurements increases, improved methods and techniques are required, as well as better instruments.

The new method herein described removes any ambiguity and leads to easily applied correction terms which agree with the errors observed to a high degree of consistency. The magitude of the neglected correction terms may be estimated from the calibration data, thus permitting an assessment of the reliability of the corrected data obtained from the instrument. In addition, a method of autocalibration has been developed which does not require any internal connections to the divider and, in effect, calibrates the unit in the manner in which it is used.

THE KELVIN-VARLEY DIVIDER

A Kelvin-Varley voltage divider consists of several decades of resistive elements interconnected as shown in Fig. 1. The first decade is made up of eleven equal resistors of value R_1, and at any given setting of the first decade switch, two of these resistors are shunted by a second decade element of resistors of value R_2 with an input resistance equal to the value of the two resistors shunted. Thus the input to the first decade looks like ten equal resistors in series, with dial steps of one-tenth the total applied voltage, up to a maximum of nine-tenths of the input voltage.

The second, third, and higher decades are also Kelvin-Varley elements, covering the range of one step of the preceding decade in tenth steps up to nine-tenths. The last decade in the chain consists of ten equal resistors and may be set to values of zero, one, two, *etc.* through ten. It is only by use of this eleventh setting of the last dial that the divider can be set to full scale at a reading of 9, 9, $\cdots$, X, where X is used to symbolize the setting TEN.

To obtain the decade relationship desired for the successive dials, it is necessary that $2R_1=10R_2$, $2R_2=10R_3$, *etc.* and the values of the resistors in each succeeding decade become smaller by a factor of five. Manufacturing or adjustment difficulties can arise from this cause, but may be overcome by the technique illustrated in the last two decades of Fig. 1, where the whole dial is parallelled by an additional resistor so as to match the resistance of the double step of the preceding dial. The form of Kelvin-Varley divider considered here will be the unshunted type, but the conclusions drawn will be of general application.

A detailed example of a four-decade Kelvin-Varley divider is shown in Fig. 2. Each resistance value is identified in the form $R(1+\Delta)$, where R is the nominal value of resistance and Δ is the proportional deviation from nominal. In the interests of brevity only four decades are considered, although the analysis has been

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[1] C. F. Varley, "On a new Method of Testing Electrical Resistance," Math. and Phys. Sec., Brit. Assoc. Adv. Sci., Nottingham, England, Rept., pp. 14–15; 1866.

[2] M. L. Morgan and J. C. Riley, "Calibration of a Kelvin-Varley standard divider," IRE TRANS. ON INSTRUMENTATION, vol. I-9, pp. 237–243; September, 1960.

[3] L. C. Fryer, "A voltage divider standard," *AIEE Trans.*, (*Communications and Electronics*) vol. 81, pp. 128–135; May, 1962.

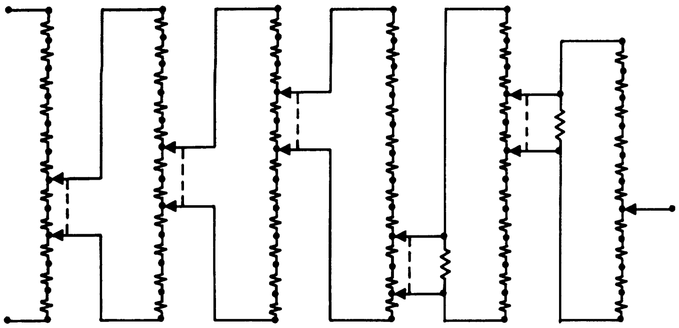

Fig. 1—Schematic diagram of six-decade Kelvin-Varley voltage divider.

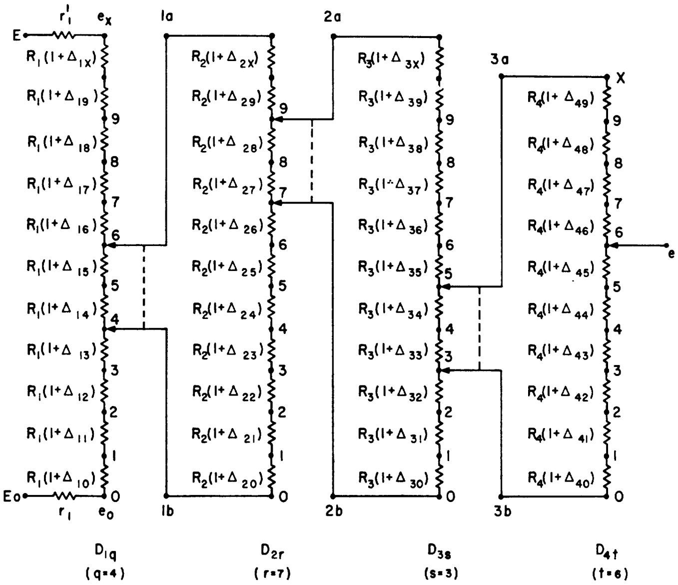

Fig. 2—Detailed wiring diagram of four-decade Kelvin-Varley voltage divider.

extended. It is shown in Appendix I that the ratio of potential between points e and e_o to the potential between points e_x and e_o in Fig. 2 is

$$\frac{e-e_o}{e_x-e_o} = D_{1q}+D_{2r}+D_{3s}+D_{4t}+c_{qrst}$$
$$= \frac{q}{10}+\frac{r}{100}+\frac{s}{1000}+\frac{t}{10000}+c_{qrst}, \qquad (25)$$

where D_{1_q}, D_{2r}, D_{3s}, D_{4t} refer to the nominal settings of the four dials set to positions q, r, s, t, respectively, and c_{qrst} is a correction to the ratio due to the departure from nominal value of the resistors making up the divider. The explicit expression for c_{qrst} is given in (26).

For greatest convenience, the correction data for the settings of each dial should be separable and be capable of being listed in a simple tabular form. Assume, therefore, that the correction to a setting of the divider may be expressed in a form such as

$$c_{qrst} = c_{q000}+c_{0r00}+c_{00s0}+c_{000t}+\epsilon, \qquad (1)$$

where ϵ is a term which should be negligible for most convenient use and the other terms represent the measured corrections for each dial setting with all other dials set to zero. By expanding the terms in (1) in the form c_{qrst} given by (26), the following relation is established:

$$\begin{aligned}
c_{qrst} &= c_{q000}+c_{0r00}+c_{00s0}+c_{000t} \\
&+\frac{13}{40}\left(\frac{r+s/10+t/100}{100}\right)[\Delta_{1q}+\Delta_{1(q+1)}-\Delta_{10}-\Delta_{11}] \\
&+\frac{233}{800}\left(\frac{s+t/10}{1000}+\frac{3q}{2330}\right)[\Delta_{2r}+\Delta_{2(r+1)}-\Delta_{20}-\Delta_{21}] \\
&+\frac{4633}{16000}\left(\frac{t}{10000}+\frac{30q+33r}{463300}\right) \\
&\cdot[\Delta_{3s}+\Delta_{3(s+1)}-\Delta_{30}-\Delta_{31}] \\
&= c_{q000}+c_{0r00}+c_{00s0}+c_{000t} \\
&+\frac{13}{40}(D_{2r}+D_{3s}+D_{4t})[\Delta_{1q}+\Delta_{1(q+1)}-\Delta_{10}-\Delta_{11}] \\
&+\frac{233}{800}\left(D_{3s}+D_{4t}+\frac{3}{233}D_{1q}\right)[\Delta_{2r}+\Delta_{2(r+1)}-\Delta_{20}-\Delta_{21}] \\
&+\frac{4633}{16000}\left(D_{4t}+\frac{3}{4633}(D_{1q}+11D_{2r})\right) \\
&\cdot[\Delta_{3s}+\Delta_{3(s+1)}-\Delta_{30}-\Delta_{31}]. \qquad (2)
\end{aligned}$$

It will be seen that as long as the successive pairs of resistors making up each dial are matched in value, the terms are separable and the simplified form of (1) is applicable. However, this simple equation is not adequate to express the correction data for many Kelvin-Varley voltage dividers in use today, and the reason is to be found in the matching of the resistors making up the divider.

An expression for the mismatch term for the first dial is obtained by expanding (2) in the form $c_{q99X}-c_{q000}$, from which

$$\begin{aligned}
&\frac{13}{40}\cdot\frac{1}{10}[\Delta_{1q}+\Delta_{1(q+1)}+\Delta_{10}-\Delta_{11}] \\
&= c_{q99X}-c_{q000}-c_{099X}-\frac{3}{800}D_{1q}\Big[\Delta_{29}+\Delta_{2X}-\Delta_{20} \\
&\qquad -\Delta_{21}+\frac{1}{20}(\Delta_{39}+\Delta_{3X}-\Delta_{30}-\Delta_{31})\Big] \qquad (3)
\end{aligned}$$

and the expression for c_{qrst} may be rewritten as

$$\begin{aligned}
c_{qrst} &= c_{q000}+c_{0r00}+c_{00s0}+c_{000t} \\
&+10(D_{2r}+D_{3s}+D_{4t})(c_{q99X}-c_{q000}-c_{099X}) \\
&+\frac{233}{800}\left(D_{3s}+D_{4t}+\frac{3}{233}D_{1q}\right)[\Delta_{2r}+\Delta_{2(r+1)}-\Delta_{20}-\Delta_{21}] \\
&+\frac{4633}{16000}\left[D_{4t}+\frac{3}{4633}(D_{1q}+11D_{2r})\right] \\
&\cdot[\Delta_{3s}+\Delta_{3(s+1)}-\Delta_{30}-\Delta_{31}] \\
&-\frac{3}{80}D_{1q}(D_{2r}+D_{3s}+D_{4t})\Big[\Delta_{29}+\Delta_{2X}-\Delta_{20}-\Delta_{21} \\
&\qquad +\frac{1}{20}(\Delta_{39}+\Delta_{3X}-\Delta_{30}-\Delta_{31})\Big]. \qquad (4)
\end{aligned}$$

The term c_{q99X} introduced in this expression is the measured correction for the ten positions of Dial 1 when all the following dials are set to their maximum values, and

TABLE I

CALIBRATION OF A SIX-DECADE KELVIN-VARLEY VOLTAGE DIVIDER (PPM OF UNITY RATIO)

Dial Setting	c_{q00000}	c_{q999X}	c_{0r0000}	c_{0r999X}	c_{00s000}	c_{00s99X}
0	0.00	+1.88	0.00	+0.13	0.03	−0.08
1	−0.80	+1.13	+0.56	+0.62	−0.03	−0.08
2	+3.38	+3.81	+0.91	+0.91	−0.05	−0.08
3	+1.93	+2.27	+1.16	+1.03	−0.06	−0.08
4	+1.69	+2.00	+1.20	+0.78	−0.08	−0.08
5	+0.32	+0.89	+0.97	+0.91	−0.08	−0.08
6	+0.41	+1.27	+0.81	+1.04	−0.05	−0.07
7	−0.02	+0.95	+1.34	+1.39	+0.01	−0.02
8	+0.32	+1.00	+1.66	+1.73	+0.03	+0.05
9	−0.40	0.00	+1.93	+1.88	+0.11	+0.13

Dial Settieg	c_{000t00}	c_{000t9X}	c_{0000v0}	c_{0000vX}	c_{00000w}
0	0.00	−0.03	0.00	−0.02	0.00
1	−0.02	−0.03	0.00	−0.01	−0.03
2	−0.01	−0.02	−0.01	−0.01	−0.04
3	−0.01	0.00	−0.01	−0.01	−0.04
4	0.00	0.00	0.00	−0.01	−0.04
5	+0.02	+0.02	+0.01	−0.01	−0.03
6	0.00	+0.02	+0.02	−0.01	−0.03
7	+0.02	+0.02	+0.02	0.00	0.03
8	+0.02	+0.05	+0.02	0.00	−0.02
9	0.00	−0.08	+0.03	−0.03	−0.02
X					−0.02

it represents the additional measurements which must be made to account for the mismatch of the resistors in Dial 1.

Although the effect of the term c_{q99X} may be calculated for any dial setting of the divider according to (4), probably the easiest way to take account of the additional terms is by graphical interpolation.

An expression for the mismatch term for the second dial may be obtained by expanding (2) in the form $c_{0r9X} - c_{0r00}$ to give

$$\frac{233}{800}\cdot\frac{1}{100}\left[\Delta_{2r} + \Delta_{2(r+1)} - \Delta_{20} - \Delta_{21}\right] = c_{0r9X} - c_{0r00} - c_{009X} - \frac{33}{16000} D_{2r}(\Delta_{39} + \Delta_{3X} - \Delta_{30} - \Delta_{31}). \quad (5)$$

By making the additional measurements represented by the term c_{0r9X}, an assessment of the matching of the resistors of Dial 2 may be made.

In the same way the matching of resistors in Dial 3 may be checked by making the measurements c_{00sX}, according to the expression

$$\frac{4633}{16000}\cdot\frac{1}{1000}\left[\Delta_{3s} + \Delta_{3(s+1)} - \Delta_{30} - \Delta_{31}\right] = c_{00sX} - c_{00s0} - c_{000X}. \quad (6)$$

If the effects of matching of resistors in Dials 2 and 3 are not negligible, they may be accounted for by combining (4), (5), and (6) which leads to

$$\begin{aligned} c_{qrst} = {} & c_{q000} + c_{0r00} + c_{00s0} + c_{000t} \\ & + 10(D_{2r} + D_{3s} + D_{4t})(c_{q99X} - c_{q000} - c_{099X}) \\ & + 100(D_{3s} + D_{4t})(c_{0r9X} - c_{0r00} - c_{009X}) \\ & + 1000 D_{4t}(c_{00sX} - c_{00s0} - c_{000X}) \\ & + \frac{300}{233} D_{1q}(c_{0r9X} - c_{0r00} - c_{009X}) \\ & - \frac{3000}{233} D_{1q}(D_{2r} + D_{3s} + D_{4t})(c_{009X} - c_{0900} - c_{009X}) \\ & + \frac{3000}{4633}(D_{1q} + 11 D_{2r})(c_{00sX} - c_{00s0} - c_{000X}) \\ & - 7.4\left[D_{1q}D_{2r} + 96.4\left(D_{2r} - \frac{D_{1q}}{400}\right)(D_{3s} + D_{4t})\right] \\ & \cdot\left[c_{009X} - c_{0090} - c_{000X}\right]. \end{aligned} \quad (7)$$

The first terms in (7) represent the initial correction terms applicable to a divider, and these may be combined from the basic calibration data in a number of ways. By rewriting the first half of (7) in the following form:

$$\begin{aligned} c_{qrst} = {} & c_{q000} + 10(D_{2r} + D_{3s} + D_{4t})(c_{q99X} - c_{q000}) \\ & + c_{0r00} - 10 D_{2r} c_{099X} \\ & + 100(D_{3s} + D_{4t})\left(c_{0r9X} - c_{0r00} - \frac{1}{10} c_{099X}\right) \\ & + c_{00s0} - 100 D_{3s} c_{009X} \\ & + 1000 D_{4t}\left(c_{00sX} - c_{00s0} - \frac{1}{10} c_{009X}\right) \\ & + c_{000t} - 1000 D_{4t} c_{000X}, \end{aligned} \quad (7a)$$

it will be found to be exactly the analytical expression which represents the method of interpolation of linearity deviations proposed earlier[2,3] when derived from the calibration data presented in Table I.

TABLE II
CALIBRATION OF A SIX-DECADE KELVIN-VARLEY VOLTAGE DIVIDER
(PPM OF UNITY RATIO)

Dial Setting	c_1	c_2	c_3	c_4	c_5	c_6	$\frac{300}{233}(c_{2x}-c_{20}-c_{3x})$
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	−0.80	+0.56	−0.03	−0.02	0.00	−0.03	−0.09
2	+3.38	+0.91	−0.05	−0.01	−0.01	−0.04	−0.17
3	+1.93	+1.16	−0.06	−0.01	−0.01	−0.04	−0.34
4	+1.69	+1.20	−0.08	0.00	0.00	−0.04	−0.71
5	+0.32	+0.97	−0.08	+0.02	+0.01	−0.03	−0.24
6	+0.41	+0.81	−0.05	0.00	+0.02	−0.03	+0.13
7	−0.02	+1.34	+0.01	+0.02	+0.02	−0.03	−0.10
8	+0.32	+1.66	+0.03	+0.02	+0.02	−0.02	−0.08
9	−0.40	+1.93	+0.11	0.00	+0.03	−0.02	−0.23
X						−0.02	

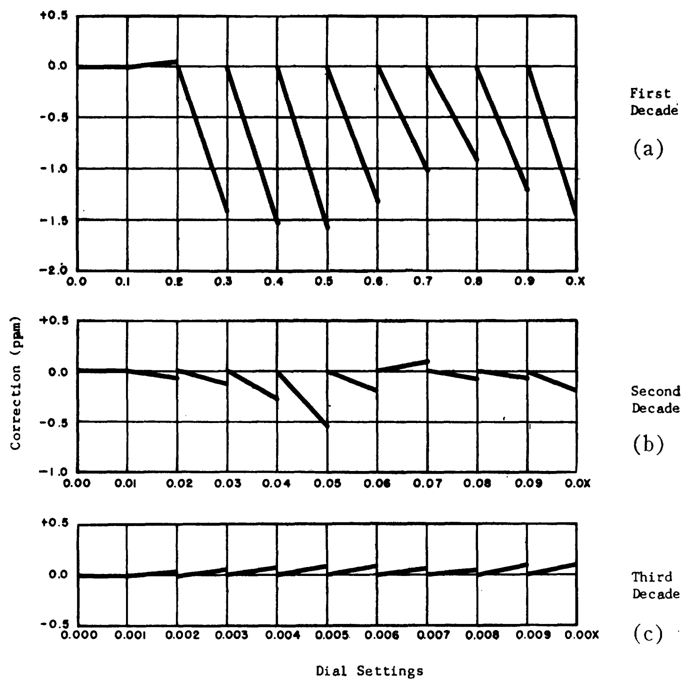

Fig. 3—The abscissae for these graphs are as follows:
(a) $D_{1q}+D_{2r}+D_{3s}+D_{4t}+D_{5v}+D_{6w}$.
(b) $D_{2r}+D_{3s}+D_{4t}+D_{5v}+D_{6w}$.
(c) $D_{3s}+D_{4t}+D_{5v}+D_{6w}$.

The second half of (7) represents contributions to the total correction term coming from the mismatch of dial resistors which are proportional to the settings of the preceding dials rather than the following dials. If the matching of the second and following dials requires correction, the residual terms proportional to the settings of the preceding dials must be recognized. No simple form of doing this suggests itself, and the magnitude of this effect remains as a limit of the accuracy of convenient use of the divider. The effect of neglecting the residual terms may be assessed from the expressions in (3), (5), and (6).

As an example of the use and usefulness of this method, the results of a calibration of a six-figure voltage divider are presented. In Table I is shown the complete calibration data obtained for the divider, while Table II together with Fig. 3 indicate a combined graphical-tabular form of presentation, based on (7), which has been found convenient for use. The columns headed $c_1, c_2, \cdots, c_6$ are the basic correction terms represented by the first four terms of the equation. The column heading

$$\frac{300}{233}(c_{2X}-c_{20}-c_{3X})$$

is a shorthand form for

$$\frac{300}{233}(c_{0r9X}-c_{0r00}-c_{009X})$$

for which the values are computed and listed for the ten positions of Dial 2; these entries must be multiplied by the appropriate value of D_{1q} to account for the eighth term in (7). The interpolation graphs are a means of obtaining values for the fifth, sixth, and seventh terms of the equation. In each term the coefficient of the cor-

TABLE III

CORRECTIONS TO KELVIN-VARLEY VOLTAGE DIVIDER (PPM OF UNITY RATIO)

1 Dial Setting	2 Observed Correction (c)	3 Correction From (8) (c')	4 $c'-c$	5 Correction From (9) (c'')	6 $c''-c$	7 Correction From (10) c'''	8 $c'''-c$	9 Correction From (11) c''''	10 $c''''-c$
.090909	+2.01	+1.91	−0.10	+1.91	−0.10	+1.89	−0.12	+1.89	−0.12
.181818	+0.80	+0.83	+0.03	+0.87	+0.07	+0.86	+0.06	+0.85	+0.05
.272727	+3.41	+4.65	+1.24	+3.59	+0.18	+3.56	+0.15	+3.54	+0.13
.363636	+1.55	+2.64	+1.09	+1.66	+0.11	+1.70	+0.15	+1.74	+0.19
.454545	+1.39	+2.57	+1.18	+1.71	+0.32	+1.62	+0.23	+1.52	+0.13
.545454	+0.28	+1.41	+1.13	+0.81	+0.53	+0.51	+0.23	+0.15	−0.13
.636364	+0.70	+1.49	+0.79	+1.12	+0.42	+0.95	+0.25	+0.75	+0.05
.727273	+0.36	+0.87	+0.51	+0.62	+0.26	+0.53	+0.17	+0.41	+0.05
.818182	+0.61	+0.87	+0.26	+0.65	+0.04	+0.59	−0.02	+0.52	−0.09
.909091	−0.38	−0.29	+0.09	−0.39	−0.01	−0.39	−0.01	−0.39	−0.01
.888889	+0.75	+2.03	+1.28	+0.96	+0.19	+0.19	+0.16	+0.85	+0.10
.777778	+0.34	+1.35	+1.01	+0.64	+0.30	+0.58	+0.24	+0.51	+0.17
.666667	+0.40	+1.16	+0.76	+0.48	+0.08	+0.55	+0.15	+0.63	+0.23
.555556	+0.14	+1.21	+1.07	+0.48	+0.34	+0.38	+0.24	+0.26	+0.14
.444444	+1.69	+2.77	+1.08	+2.07	+0.38	+1.83	+0.14	+1.55	−0.14
.333333	+2.28	+2.97	+0.69	+2.46	+0.18	+2.37	+0.09	+2.27	−0.01
.222222	+3.81	+4.18	+0.37	+3.86	+0.05	+3.83	+0.02	+3.80	−0.01
.111111	−0.31	−0.32	−0.01	−0.31	0.00	−0.32	−0.01	−0.33	−0.02
.09999*X*	+1.88	+2.05	+0.17	+2.05	+0.17	+1.87	−0.01	+1.87	−0.01
.19999*X*	+1.13	+1.25	+0.12	+1.30	+0.17	+1.12	−0.01	+1.10	−0.03
.29999*X*	+3.81	+5.43	+1.62	+3.98	+0.17	+3.80	−0.01	+3.75	−0.06
.39999*X*	+2.27	+3.98	+1.71	+2.44	+0.17	+2.26	−0.01	+2.19	−0.08
.49999*X*	+2.00	+3.74	+1.74	+2.17	+0.17	+1.99	−0.01	+1.90	−0.10
.59999*X*	+0.89	+2.37	+1.48	+1.06	+0.17	+0.88	−0.01	+0.76	−0.13
.69999*X*	+1.27	+2.46	+1.19	+1.44	+0.17	+1.26	−0.01	+1.12	−0.15
.79999*X*	+0.95	+2.03	+1.08	+1.12	+0.17	+0.94	−0.01	+0.78	−0.17
.89999*X*	+1.00	+2.37	+1.37	+1.17	+0.17	+0.99	−0.01	+0.80	−0.20
.99999*X*	0.00	+1.65	+1.65	+0.17	+0.17	−0.01	−0.01	−0.24	−0.24
			+0.9±0.9		+0.2±0.25		+0.1±0.2		0.0±0.2

rection increases linearly from zero to unity for the full range of the following dials; the graphs are plotted with a value of zero at an integral dial setting (c_{q000}) increasing to the full value of the correction term ($c_{q99X}-c_{q000}-c_{099X}$) at the next equivalent integral dial setting (c_{q99X}). In this way the approximate magnitudes of the additional correction terms are obvious by inspection. From the graphs in Table II it is immediately obvious that interpolation for Dials 2 and 3 need only be done for positions 3, 4, 5 of Dial 2 if corrections to within 0.5 ppm are all that are required.

Table III presents the results of successive approximations to the correction terms measured for a number of multiple dial settings. Column four in this table is the difference between the observed corrections and those derived from the basic equation

$$c'_{qrstvw} = c_{q00000} + c_{0r0000} + c_{00s000} + c_{000t00} + c_{0000v0} + c_{00000w}. \quad (8)$$

For this divider, the discrepancy rises to almost 2 ppm of unity ratio. Column six lists the discrepancy found when the first dial approximation is added, that is

$$c''_{qrstvw} = c'_{qrstvw} + 10(D_{2r} + D_{3s} \cdots)(c_{q9999X} - c_{q00000} - c_{09999X}) \quad (9)$$

and the error is found to have been reduced to less than 0.5 ppm. Column eight is derived from the first and second dial approximations, where

$$c'''_{qrstvw} = c''_{qrstvw} + 100(D_{3s} + D_{4t} \cdots)(c_{0r999X} - c_{0r0000} - c_{00999X}) \quad (10)$$

and the error is now less than 0.3 ppm. Column ten is derived from the equation

$$c''''_{qrstvw} = c'''_{qrstvw} + \frac{300}{233} D_{1q}(c_{0r999X} - c_{0r0000} - c_{00999X}). \quad (11)$$

It is seen that the inclusion of the term representing the effect of the setting of Dial 1 on the correction for Dial 2 reduces the error to about 0.2 ppm. Inclusion of the correction term for the variation of Dial 3 does not improve the results, due both to the magnitude of the neglected terms and to the limit of precision of the calibration data. A convenient limit to the reliability of the calibration procedure on this divider is about 0.3 ppm, represented by (10), column eight.

The measurements recorded here were made with a precision of approximately ±0.02 ppm, and it is believed that the relative values are reliable to about ±0.1 ppm, although the accuracy of the ratio measurements is probably no better than ±0.2 ppm in an absolute sense. Twenty-eight multiple dial settings have been examined in detail in Table III, and it is believed that these do represent the operation of a Kelvin-Varley divider.

There are many ways of actually calibrating a Kelvin-Varley voltage divider, but the simplest method is to compare it with a standard divider, as was done here. This may be done at either dc or ac against another Kelvin-Varley divider, or at ac against an inductive voltage divider. In case the frequency characteristics of the Kelvin-Varley divider are not adequate or an inductive voltage divider of sufficient accuracy is not available, a new form of complete autocalibration of a standard Kelvin-Varley divider at dc is given in detail in Appendix II.

For a great many uses of a Kelvin-Varley divider, the calibration method proposed provides all that is necessary for use of the divider. However, for other uses, *i.e.*, where the divider is not artificially zeroed at both the maximum and minimum settings but is used simply as a voltage divider across a known voltage source, the end wiring connections r_1' and r_1 between the points e_x and E and e_o and E_o in Fig. 2 must be taken into account.

With a voltage E''' applied between the terminals E and E_o, the voltages appearing across r_1' and r_1 will be e_r' and e_r, respectively. The division ratio of the divider will now become

$$\begin{aligned}\frac{e - E_o}{E - E_o} &= \frac{(e - e_o) + e_r}{e_r' + (e_x - e_o) + e_r} = \\ &= \frac{e - e_o}{e_x - e_o}\left[1 + \frac{e_r}{e - e_o} - \frac{e_r + e_r'}{e_x - e_o}\right] \\ &= \frac{e - e_o}{e_x - e_o} + \frac{e_r}{e_x - e_o} - \frac{e - e_o}{e_x - e_o}\cdot\frac{e_r + e_r'}{e_x - e_o} \\ &= D_{qrst} + c_{qrst} + (c_0)_0 - D_{qrst}[(c_0)_0 + (c_0)_X],\end{aligned}$$

if

$$\begin{aligned}(c_0)_0 &= e_r/(e_x - e_o) \approx e_r/E''' \\ (c_0)_X &= e_r'/(e_x - e_0) \approx e_r'/E'''.\end{aligned}$$

The corrections $(c_0)_0$ and $(c_0)_X$ may be readily determined by measuring the appropriate voltage drops when some convenient voltage E''' is applied to the divider. The correction $(c_0)_0$ may refer to either the input zero terminal E_o or an output zero terminal E_o', if such exists at a different potential from E_o. The simplest manner of handling these corrections in subsequent use of the divider is by linear interpolation from a graph.

Conclusions

The error in ratio of a Kelvin-Varley voltage divider has been shown to be due to the departure from nominal value of the resistors making up the divider chain, either from initial adjustment of resistance values or from subsequent changes in the resistors due to aging or other causes.

Although the effects of individual dials cannot be delineated in the error expression, a method of treating the corrections obtained for the various dial settings has been established which enables the calibration data to be used with knowledge of the residual terms which are being neglected. The complete correction data is best presented as a combination of tabular data and interpolation graphs.

The autocalibration technique referred to above and treated in detail in Appendix II determines the corrections to be applied, without the necessity of referring the instrument to a national standardizing laboratory.

Appendix I

Derivation of Correction Terms for a Kelvin-Varley Divider

A detailed example of a four-decade Kelvin-Varley divider is shown in Fig. 2. Each resistance value is indicated in the form $R(1+\Delta)$, where R is the nominal value and Δ is the proportional deviation from nominal. It is assumed initially that these deviations from nominal are all small enough that second and higher-order terms are negligible.

The total resistance of Dial 4 is given by

$$\begin{aligned}R_{T4} &= R_4(1 + \Delta_{40}) + R_4(1 + \Delta_{41}) + \cdots + R_4(1 + \Delta_{49}) \\ &= 10R_4\left[1 + \frac{1}{10}(\Delta_{40} + \Delta_{41} + \cdots + \Delta_{49})\right] \\ &= 10R_4(1 + \bar{\Delta}_4), \end{aligned} \tag{12}$$

where

$$\begin{aligned}\bar{\Delta}_4 &= \frac{1}{10}(\Delta_{40} + \Delta_{41} + \cdots + \Delta_{49}) \\ &= \text{mean deviation of } R_4 \text{ resistors.}\end{aligned}$$

The total resistance of Dial 3, when set to dial position s, is given by

$$\begin{aligned}R_{T3} = {} & R_3(1 + \Delta_{30}) + R_3(1 + \Delta_{31}) + \cdots \\ & + R_3(1 + \Delta_{3(s-1)}) \\ & + \frac{[R_3(1 + \Delta_{3s}) + R_3(1 + \Delta_{3(s+1)})]\cdot 10R_4(1 + \bar{\Delta}_4)}{R_3(1 + \Delta_{3s}) + R_3(1 + \Delta_{3(s+1)}) + 10R_4(1 + \bar{\Delta}_4)} \\ & + R_3(1 + \Delta_{3(s+2)}) + \cdots + R_3(1 + \Delta_{3X}).\end{aligned}$$

As $2R_3 = 10R_4$ in the design and construction of the divider, the parallel element term becomes

$$\begin{aligned}& \frac{R_3[1 + \Delta_{3s} + 1 + \Delta_{3(s+1)}]\cdot 2R_3(1 + \bar{\Delta}_4)}{R_3[1 + \Delta_{3s} + 1 + \Delta_{3(s+1)} + 2 + 2\bar{\Delta}_4]} \\ &\quad = \frac{4R_3^2[1 + \frac{1}{2}(\Delta_{3s} + \Delta_{3(s+1)})]\cdot[1 + \bar{\Delta}_4]}{4R_3[1 + \frac{1}{4}(\Delta_{3s} + \Delta_{3(s+1)}) + \frac{1}{2}\bar{\Delta}_4]} \\ &\quad = R_3[1 + \tfrac{1}{4}(\Delta_{3s} + \Delta_{3(s+1)}) + \tfrac{1}{2}\bar{\Delta}_4]\end{aligned} \tag{13}$$

and

$$\begin{aligned}R_{T3} = 10R_3\Bigg[1 + \frac{1}{10}\Bigg(&\Delta_{30} + \Delta_{31} \cdots \Delta_{3(s-1)} \\ &+ \frac{\Delta_{3s} + \Delta_{3(s+1)}}{4} + \Delta_{3(s+2)} \cdots \Delta_{3X} + \frac{1}{2}\bar{\Delta}_4\Bigg)\Bigg]\end{aligned}$$

$$= 10R_3\left[1 + \frac{1}{10}\left(\Delta_{30} + \Delta_{31} \cdots \Delta_{3X} - \frac{3}{4}(\Delta_{3s} + \Delta_{3(s+1)}) + \frac{1}{2}\Delta_4\right)\right]$$

$$= 10R_3\left[1 + \frac{11}{10}\bar{\Delta}_3 + \frac{1}{20}\bar{\Delta}_4 - \frac{3}{40}(\Delta_{3s} + \Delta_{3(s+1)})\right]$$

$$\equiv 10R_3[1 + \bar{\bar{\Delta}}_3], \tag{14}$$

where

$$\bar{\Delta}_3 = \frac{\Delta_{30} + \Delta_{31} \cdots \Delta_{3X}}{11} = \text{mean deviation of } R_3 \text{ resistors}$$

and $\bar{\bar{\Delta}}_3$ is an equivalent deviation of the total resistance of the dial.

Similarly, the total resistance of Dial 2 when set to dial position r is given by

$$R_{T2} = 10R_2\left[1 + \frac{1}{10}(\Delta_{20} + \Delta_{21} \cdots \Delta_{2X} - \frac{3}{4}(\Delta_{2r} + \Delta_{2(r+1)}) + \frac{1}{2}\bar{\bar{\Delta}}_3)\right]$$

$$= 10R_2\left[1 + \frac{11}{10}\bar{\Delta}_2 + \frac{1}{20}\bar{\bar{\Delta}}_3 - \frac{3}{40}(\Delta_{2r} + \Delta_{2(r+1)})\right]$$

$$\equiv 10R_2[1 + \bar{\bar{\Delta}}_2], \tag{15}$$

where

$$\bar{\Delta}_2 = \frac{\Delta_{20} + \Delta_{21} \cdots \Delta_{2X}}{11} = \text{mean deviation of } R_2 \text{ resistors.}$$

In the same way, the total resistance of Dial 1 when set to dial position q is given by

$$R_{T1} = 10R_1\left[1 + \frac{11}{10}\Delta_1 + \frac{1}{20}\bar{\bar{\Delta}}_2 - \frac{3}{40}(\Delta_{1q} + \Delta_{1(q+1)})\right]$$

$$\equiv 10R_1[1 + \bar{\bar{\Delta}}_1], \tag{16}$$

where

$$\bar{\Delta}_1 = \frac{\Delta_{10} + \Delta_{11} \cdots \Delta_{1X}}{11} = \text{mean deviation of } R_1 \text{ resistors.}$$

It is to be noted that $\bar{\Delta}_1$ represents the deviation of the R_1 resistors from the absolute design value. It is quite satisfactory to take the mean value of the R_1 resistors as the fundamental unit of comparison, thus making $\bar{\Delta}_1 = 0$. However, $\bar{\Delta}_2$, $\bar{\Delta}_3$, and $\bar{\Delta}_4$ must now refer to this new fundamental unit. In order to eliminate possible misunderstanding, everything will be based on the usual absolute resistance unit, and $\bar{\Delta}_1$ retained in the equations.

If a voltage E is applied between the points e_x and e_o in Fig. 2, then the ratios of potentials appearing across the various dials will be as follows, starting from (13) and (14) for Dial 4:

$$\frac{E_4}{E_3} = \frac{e_{3a} - e_{3b}}{e_{2a} - e_{2b}} = R_3\left[1 + \frac{1}{4}(\Delta_{3s} + \Delta_{3(s+1)}) + \frac{1}{2}\bar{\Delta}_4\right] \Big/ R_{T3}$$

$$= \frac{1}{10}\left[1 + \frac{1}{4}(\Delta_{3s} + \Delta_{3(s+1)}) - \bar{\bar{\Delta}}_3 + \frac{1}{2}\bar{\Delta}_4\right] \tag{17}$$

$$\frac{E_3}{E_2} = \frac{e_{2a} - e_{2b}}{e_{1a} - e_{1b}} = R_2\left[1 + \frac{1}{4}(\Delta_{2r} + \Delta_{2(r+1)}) + \frac{1}{2}\bar{\bar{\Delta}}_3\right] \Big/ R_{T2}$$

$$= \frac{1}{10}\left[1 + \frac{1}{4}(\Delta_{2r} + \Delta_{2(r+1)}) - \bar{\bar{\Delta}}_2 + \frac{1}{2}\bar{\bar{\Delta}}_3\right] \tag{18}$$

$$\frac{E_2}{E_1} = \frac{e_{1a} - e_{1b}}{e_x - e_o} = \frac{1}{10}\left[1 + \frac{1}{4}(\Delta_{1q} + \Delta_{1(q+1)}) - \bar{\bar{\Delta}}_1 + \frac{1}{2}\bar{\bar{\Delta}}_2\right]. \tag{19}$$

Under these same conditions, the ratio of potential selected by Dial 1 will be

$$\frac{e_{1b} - e_o}{e_x - e_o} = [R_1(1 + \Delta_{10}) + R_1(1 + \Delta_{11}) + \cdots + R_1(1 + \Delta_{1(q-1)})]/R_{T1}$$

$$= qR_1\left[1 + \frac{1}{q}(\Delta_{10} + \Delta_{11} + \cdots + \Delta_{1(q-1)})\right] \Big/ R_{T1}$$

$$= \frac{q}{10}\left[1 + \frac{1}{q}(\Delta_{10} + \Delta_{11} + \cdots + \Delta_{1(q-1)}) - \bar{\bar{\Delta}}_1\right]. \tag{20}$$

Similarly,

$$\frac{e_{2b} - e_{1b}}{e_{1a} - e_{1b}} = rR_2\left[1 + \frac{1}{r}(\Delta_{20} + \Delta_{21} + \cdots + \Delta_{2(r-1)})\right] \Big/ R_{T2}$$

$$= \frac{r}{10}\left[1 + \frac{1}{r}(\Delta_{20} + \Delta_{21} + \cdots + \Delta_{2(r-1)}) - \bar{\bar{\Delta}}_2\right] \tag{21}$$

$$\frac{e_{3b} - e_{2b}}{e_{2a} - e_{2b}} = \frac{s}{10}\left[1 + \frac{1}{s}(\Delta_{30} + \Delta_{31} + \cdots + \Delta_{3(s-1)}) - \bar{\bar{\Delta}}_3\right] \tag{22}$$

$$\frac{e - e_{3b}}{e_{3a} - e_{3b}} = \frac{t}{10}\left[1 + \frac{1}{t}(\Delta_{40} + \Delta_{41} + \cdots + \Delta_{4(t-1)}) - \bar{\Delta}_4\right]. \tag{23}$$

The ratio of potential between points e and e_o to the potential between points e_x and e_o will be given by

$$\frac{e-e_o}{e_x-e_o}=\frac{e_{1b}-e_o}{e_x-e_o}+\frac{e_{2b}-e_{1b}}{e_x-e_o}+\frac{e_{3b}-e_{2b}}{e_x-e_o}+\frac{e-e_{3b}}{e_x-e_o}, \quad (24)$$

and by combining the expressions in (17) to (23) this becomes

$$\frac{e-e_o}{e_x-e_o}=\frac{q}{10}+\frac{r}{100}+\frac{s}{1000}+\frac{t}{10000}+c_{qrst}$$
$$=D_{1q}+D_{2r}+D_{3s}+D_{4t}+c_{qrst}, \quad (25)$$

where D_{1q}, D_{2r}, D_{3s}, D_{4t} refer to the nominal settings of the respective dials and c_{qrst} is a correction to the ratio due to the departure from nominal value of the resistors making up the divider. The over-all expression for c_{qrst} is found to be

$$\begin{aligned}c_{qrst}=&\frac{1}{10}\Big[(\Delta_{10}+\Delta_{11}+\cdots+\Delta_{1(q-1)})-\frac{11q}{10}\bar\Delta_1-\frac{11q}{200}\bar\Delta_2-\frac{11q}{4000}\bar\Delta_3-\frac{q}{8000}\bar\Delta_4\\&+\frac{3q}{40}(\Delta_{1q}+\Delta_{1(q+1)})+\frac{3q}{800}(\Delta_{2r}+\Delta_{2(r+1)})+\frac{3q}{16000}(\Delta_{3s}+\Delta_{3(s+1)})\Big]\\&+\frac{1}{100}\Big[(\Delta_{20}+\Delta_{21}+\cdots+\Delta_{2(r-1)})-\frac{11r}{10}\bar\Delta_1-\frac{121r}{200}\bar\Delta_2-\frac{121r}{4000}\bar\Delta_3-\frac{11r}{8000}\bar\Delta_4\\&+\frac{13r}{40}(\Delta_{1q}+\Delta_{1(q+1)})+\frac{33r}{800}(\Delta_{2r}+\Delta_{2(r+1)})+\frac{33r}{16000}(\Delta_{3s}+\Delta_{3(s+1)})\Big]\\&+\frac{1}{1000}\Big[(\Delta_{30}+\Delta_{31}+\cdots+\Delta_{3(s-1)})-\frac{11s}{10}\bar\Delta_1-\frac{121s}{200}\bar\Delta_2-\frac{2321s}{4000}\bar\Delta_3-\frac{211s}{8000}\bar\Delta_4\\&+\frac{13s}{40}(\Delta_{1q}+\Delta_{1(q+1)})+\frac{233s}{800}(\Delta_{2r}+\Delta_{2(r+1)})+\frac{633s}{16000}(\Delta_{3s}+\Delta_{3(s+1)})\Big]\\&+\frac{1}{10000}\Big[(\Delta_{40}+\Delta_{41}+\cdots+\Delta_{4(t-1)})-\frac{11t}{10}\bar\Delta_1-\frac{121t}{200}\bar\Delta_2-\frac{2321t}{4000}\bar\Delta_3-\frac{4211t}{8000}\bar\Delta_4\\&+\frac{13t}{50}(\Delta_{1q}+\Delta_{1(q+1)})+\frac{233t}{800}(\Delta_{2r}+\Delta_{2(r+1)})+\frac{4633t}{16000}(\Delta_{3s}+\Delta_{3(s+1)})\Big].\end{aligned} \quad (26)$$

From the expression for c_{qrst} it will be seen that

$$c_{0000}=c_{999X}=0 \quad (27)$$

as

$$(\Delta_{i0}+\Delta_{i1}+\cdots+\Delta_{i(0-1)})\equiv 0$$

which is required by the tacit assumption that the switch contacts and interconnecting wiring are of negligible resistance. If additional elements ($r_2, r_2', r_3, r_3', r_4, r_4'$) are included in Fig. 2 to represent these resistances, the expression for c_{qrst} must be modified by the addition of

$$\begin{aligned}\overset{*}{c}_{qrst}=&\frac{1}{10}\left[-q\left(\frac{1}{20}\delta_2+\frac{1}{400}\delta_3+\frac{1}{8000}\delta_4\right)\right]\\&+\frac{1}{100}\left[\frac{r_2}{R_2}-r\left(\frac{11}{20}\delta_2+\frac{11}{400}\delta_3+\frac{11}{8000}\delta_4\right)\right]\\&+\frac{1}{1000}\left[\frac{r_3}{R_3}-s\left(\frac{11}{20}\delta_2+\frac{211}{400}\delta_3+\frac{211}{8000}\delta_4\right)\right]\\&+\frac{1}{10000}\left[\frac{r_4}{R_4}-t\left(\frac{11}{20}\delta_2+\frac{211}{400}\delta_3+\frac{4211}{8000}\delta_4\right)\right],\end{aligned} \quad (28)$$

where

$$\delta_2=\frac{r_2+r_2'}{10R_2},\quad \delta_3=\frac{r_3+r_3'}{10R_3},\quad \delta_4=\frac{r_4+r_4'}{10R_4}$$

and

$$\overset{*}{c}_{0000}=\frac{1}{10R_1}\left(\frac{r_2}{2}+\frac{r_3}{4}+\frac{r_4}{8}\right) \quad (29)$$

$$\overset{*}{c}_{999X}=-\frac{1}{10R_1}\left(\frac{r_2'}{2}+\frac{r_3'}{4}+\frac{r_4'}{8}\right). \quad (30)$$

It is found that the effect of these additional terms is identically zero in the method of manipulating the observed calibration data proposed, and hence they need not be included, provided they remain constant during use. However, the unavoidable variations of contact resistance in a set of switches must set a limit to the precision with which a Kelvin-Varley voltage divider may be used. The magnitude of the effect at maximum or minimum dial setting is given by

$$\delta c=\frac{1}{10R_1}\left(\frac{\delta r_2}{2}+\frac{\delta r_3}{4}+\frac{\delta r_4}{8}\right), \quad (31)$$

where δr_i represents the variation of the switch contact resistance for each dial.

From the way in which the correction term c_{qrst} has been derived from the summation of contribution from four sources, it is tempting to ascribe an appropriate correction to an individual dial and assume that it is possible to calibrate the effect of each dial separately. Unfortunately, the effects of the settings of each dial are completely intermixed in the expression in (26), and it is impossible to separate the different dials in this expression in any useful manner.

Appendix II

Autocalibration of Six-Dial Kelvin-Varley Divider

Although there are many networks in which a Kelvin-Varley divider may be connected to perform an autocalibration of its dial positions, the following arrangement and procedure has proven to be most suitable and reliable, and is the procedure normally followed in this laboratory.

Let the divider to be calibrated be connected in parallel with another source of subdivided voltage (another divider is satisfactory, its corrections need not be known), as shown in Fig. 4, along with a suitable potentiometer, microvolt potentiometer (variable microvolt source), and sensitive galvanometer in series between the two variable taps on the dividers. The battery E' may be any stable source of low current, but 12 v is probably most suitable, as in this way the potential drop across the dividers may be adjusted to approximately 10 v which provides a convenient operating value.

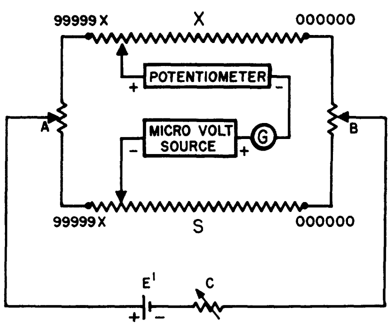

Fig. 4—Autocalibration network for Kelvin-Varley voltage dividers.

Initial adjustment is by the rheostats A and B to ensure that the potential difference between variable tap points is zero when dials of both dividers are set to maximum and minimum settings. Repeated setting of the dials of each or both dividers to the checking position will indicate the reliability and repeatability of the switch contact resistances, and hence the limit of precision and accuracy to be expected from the calibration.

When the two dividers have been set up and found to repeat EMF's at their maximum and minimum settings, set divider S to 0.000000 and X to 0.100000, potentiometers to measure 1.00005 v or some suitable value, and adjust rheostat C for balance (*i.e.*, approximately 10 v across the divider pair). This setting becomes the check value to establish the stability of the current flowing in the X divider during the calibration.

The end resistance corrections of divider X may now be determined by measuring the EMF appearing between the tap point and the appropriate connection terminal with the divider dials set to either maximum or minimum setting. The two EMF's measured are termed e_{9X} and e_{00}, which lead to the terminal resistance corrections by the relations

$$(c_0)_0 = \frac{e_{00}}{E} \qquad (c_0)_X = \frac{e_{9X}}{E}, \tag{32}$$

where E is the total voltage appearing across the divider (approximately 10 v).

With divider S set to 0.900000, potentiometer to 1.000000, and divider X to 0.99999X (maximum), zero current flow indicated by the galvanometer is obtained by variation of the microvolt source, with a reading δ_{1X}' at balance. The balance equation under these conditions is

$$(D_{9X} + c_{9X})E_1 = D_{90}'E_1 + e_1 + \delta_{1X}'. \tag{33}$$

Leaving divider S untouched, reset potentiometer to 0.000000 and divider X to 0.900000 and obtain second balance (δ_{1X}'') by means of microvolt source. The balance equation is now

$$(D_{90} + c_{90})E_1 = D_{90}'E_1 + e_1' + \delta_{1X}''. \tag{34}$$

The difference between the two equations is thus

$$(D_{9X} - D_{90})E_1 + (c_{9X} - c_{90})E_1 = (e_1 - e_1') + \delta_{1X}' - \delta_{1X}''$$

$$c_{9X} - c_{90} = \frac{e_1 - e_1' - 0.1E_1}{E_1} + \frac{\delta_{1X}' - \delta_{1X}''}{E_1}$$

$$= \epsilon_1 + \Delta_{1X}. \tag{35}$$

Divider S is now reset to 0.800000, potentiometer to 1.000000, and X left unchanged, and reading (δ_{19}) of the microvolt source obtained at balance, from which

$$(D_{90} + c_{90})E_1 = D_{80}'E_1 + e_1 + \delta_{19}. \tag{36}$$

Divider X is reset to 0.89999X and reading (δ_{19}') obtained at balance, leading to

$$(D_{8X} + c_{8X})E_1 = D_{80}'E_1 + e_1 + \delta_{19}'. \tag{37}$$

Divider X is reset to 0.800000, potentiometer to 0.000000, and reading (δ_{19}'') obtained at balance, when

$$(D_{80} + c_{80})E_1 = D_{80}'E_1 + e_1' + \delta_{19}''. \tag{38}$$

The difference between (36) and (38) leads to

$$c_{90} - c_{80} = \frac{e_1 - e_1' - 0.1E_1}{E_1} + \frac{\delta_{19} - \delta_{19}''}{E_1} = \epsilon_1 + \Delta_{19}, \tag{39}$$

and the difference between (36) and (37) provides

$$c_{8X} - c_{90} = \frac{\delta_{19}' - \delta_{19}}{E_1} = \theta_{19}. \tag{40}$$

By repeating this procedure for each successively lower group of dial settings of Dial 1, the following set of equations is obtained:

$$\begin{array}{ll}
c_{9X} - c_{90} = \epsilon_1 + \Delta_{1X} & \\
c_{90} - c_{80} = \epsilon_1 + \Delta_{19} & c_{8X} - c_{90} = \theta_{19} \\
c_{80} - c_{70} = \epsilon_1 + \Delta_{18} & c_{7X} - c_{80} = \theta_{18} \\
\cdots\cdots\cdots & \cdots\cdots \\
\cdots\cdots\cdots & \cdots\cdots \\
\underline{c_{10} - c_{00} = \epsilon_1 + \Delta_{11}} & c_{0X} - c_{10} = \theta_{11},
\end{array} \tag{41}$$

$$c_{9X} - c_{00} = 10\epsilon_1 + \sum_{i=1}^{10} \Delta_{1i}, \tag{42}$$

and, generally,

$$c_{q0} - c_{00} = q\epsilon_1 + \sum_{i=1}^{q} \Delta_{1i}. \tag{43}$$

The basic assumptions require that

$$c_{99999X} = c_{000000} = 0,$$

and thus (42) leads to

$$\epsilon_1 = -\frac{1}{10}\sum_{1}^{10} \Delta_{1i} \tag{44}$$

and

$$c_{q00000} = \sum_{1}^{q} \Delta_{1i} - \frac{q}{10}\sum_{1}^{10} \Delta_{1i} \tag{45}$$

which provides a set of self-consistent values for the ten primary corrections to Dial 1. The secondary corrections to Dial 1 needed to account for the lack of matching of the resistors in Dial 1 are obtained from the secondary equations derived above, that is

$$c_{(q-1)9999X} = c_{q00000} + \theta_{1q}. \tag{46}$$

A similar process is repeated for each group of dial settings for Dial 2, leading to equations of the form

$$\begin{aligned} c_{09X} - c_{090} &= \frac{e_2 - e_2' - 0.01E_2}{E_2} + \frac{\delta_{2X}' - \delta_{2X}''}{E_2} \\ &= \epsilon_2 + \Delta_{2X} \end{aligned}$$

$$\begin{array}{ll} c_{090} - c_{080} = \epsilon_2 + \Delta_{29} & c_{08X} - c_{090} = \theta_{29} \\ c_{080} - c_{070} = \epsilon_2 + \Delta_{28} & c_{07X} - c_{080} = \theta_{28} \\ \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ \underline{c_{010} - c_{000} = \epsilon_2 + \Delta_{21}} & c_{00X} - c_{010} = \theta_{21}, \end{array} \tag{47}$$

$$c_{09X} - c_{000} = 10\epsilon_2 + \sum_{i=1}^{10} \Delta_{2i} \tag{48}$$

from which

$$\epsilon_2 = -\frac{1}{10}\sum_{1}^{10} \Delta_{2i} + \frac{1}{10} c_{09999X}, \tag{49}$$

and

$$c_{0r0000} = \sum_{1}^{r} \Delta_{2i} - \frac{r}{10}\sum_{1}^{10} \Delta_{2i} + \frac{r}{10} c_{09999X}, \tag{50}$$

and also

$$c_{0(r-1)999X} = c_{0r0000} + \theta_{2r}. \tag{51}$$

By continuing the same process for each of the dials, further sets of equations are obtained, as follows:

$$c_{00s000} = \sum_{1}^{s} \Delta_{3i} - \frac{s}{10}\sum_{1}^{10} \Delta_{3i} + \frac{s}{10} c_{00999X} \tag{52}$$

$$c_{00(s-1)000} = c_{00s000} + \theta_{3s} \tag{53}$$

$$c_{000t00} = \sum_{1}^{t} \Delta_{4i} - \frac{t}{10}\sum_{1}^{10} \Delta_{4i} + \frac{t}{10} c_{00099X} \tag{54}$$

$$c_{0000v0} = \sum_{1}^{v} \Delta_{5i} - \frac{v}{10}\sum_{1}^{10} \Delta_{5i} + \frac{v}{10} c_{00009X} \tag{55}$$

$$c_{00000w} = \sum_{1}^{w} \Delta_{6i} - \frac{w}{10}\sum_{1}^{10} \Delta_{6i} + \frac{w}{10} c_{00000X}. \tag{56}$$

The procedure outlined will provide a fully self-consistent set of corrections to a Kelvin-Varley voltage divider, similar to those which are obtained when the divider is directly calibrated in terms of a known standard divider. If, however, Dial 2 is not directly correlated with Dia. 1 as outlined, but instead the assumption is made that

$$c_{09999X} = c_{000000} = 0,$$

then a calibration is obtained which duplicates the calibrations proposed earlier.[2,3]

In listing the general equations for the calibration, it has been implied that the secondary corrections need not be obtained beyond Dial 3. This was done because when matching of the resistors in Dial 3 must be corrected, then the residual terms proportional to the preceding dials must also be taken into account. This latter is very difficult to achieve in any convenient manner, and it is better to establish a limit to the working precision of the divider. Thus graphical interpolation beyond the second or third dial position is no longer useful.

It must be remembered that in order to correlate each dial with the following one, the relation between the lowest step of the dial and the maximum settings of the following dials must be measured. Alternatively, the sequence for the lower dials may start directly from the lowest step of the preceding dial. These modifications to the sequence of operations are minor, and depend to a large degree on the method to be used for subsequent handling of the data.

The validity of this method of calibration rests on the assumption that during the measurements on any dial the potential E across the two dividers and the potential e selected by the main potentiometer remain constant, or at least change by the same proportional amount. To assess this effect, rewrite (36), (37), and (38) to include terms representing variations of these two voltages; that is

$$\begin{aligned} &(D_{90} + c_{90})E(1 + \alpha_9) \\ &\qquad = D_{80}'E(1 + \alpha_9) + e(1 + \beta_9) + \delta_{19} \end{aligned} \tag{36a}$$

$$\begin{aligned} &(D_{8X} + c_{8X})E(1 + \alpha_9') \\ &\qquad = D_{80}'E(1 + \alpha_9') + e(1 + \beta_9') + \delta_{19}' \end{aligned} \tag{37a}$$

$$\begin{aligned} &(D_{80} + c_{80})E(1 + \alpha_9'') \\ &\qquad = D_{80}''E(1 + \alpha_9'') + e'(1 + \beta_9'') + \delta_{19}'' \end{aligned} \tag{38a}$$

from which

$$c_{90} - c_{80} = \frac{e - e' - 0.1E}{E} + \frac{\delta_{19} - \delta_{19}''}{E} + \frac{e - e'}{E}(\beta_9 - \alpha_9)$$
$$= \epsilon_1 + \Delta_{19} + 0.1(\beta_9 - \alpha_9) \quad (57)$$

$$c_{8X} - c_{90} = \frac{\delta_{19}' - \delta_{19}}{E} + \frac{e}{E}(\beta_9' - \beta_9) - 0.1(\alpha_9' - \alpha_9)$$
$$= \theta_{19} + 0.1[(\beta_9' - \alpha_9') - (\beta_9 - \alpha_9)]. \quad (58)$$

Thus it is seen that the simplified expressions are valid providing $0.1(\beta-\alpha)$ remains negligible. For each successively lower dial the stringency of this requirement decreases by a factor of ten. It is to be noted that the requirements are only on the stability of the potentials selected and on the reproducibility of the dial settings. The accuracy of the potentiometer need not be known, as each voltage increment selected by the potentiometer is separately adjusted to the conditions imposed by the assumption $c_{9X}-c_{00}=0$. The possible indeterminacies in the first dial measurements indicated here are further reduced by a form of compensation of the measurements, coming from the assumption which requires there be no indeterminacy at c_{99999X} or c_{000000}.

ACKNOWLEDGMENT

The primary purpose of this investigation was to establish the most convenient method of calibration of a Kelvin-Varley voltage divider which could be performed in any laboratory. As a better understanding of the detailed operation of a divider not only leads to improved calibration data for existing dividers but also to better instruments becoming available, the active interest and suggestions made by personnel in three instrument manufacturing companies is gratefully acknowledged.

Design and Error Analysis of High-Accuracy DC Voltage-Measuring Systems

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Summary—**A systematic analysis of the errors inherent in the application of a resistance voltage divider to the measurement of high voltages is presented. The effects of potentiometer errors, leakage errors, and rod resistance are considered in turn. A newly developed high accuracy volt box providing output voltages of 0.5 v, 1.0 v, and 1.5 v for all input voltages ranging from 1.5 to 1600 v in steps of 0.5 v is then described.**

INTRODUCTION

MODERN SCIENTIFIC and engineering projects have confronted the instrument industry with increasingly stringent requirements for high precision, high accuracy instruments. Among the electrical variables which must be measured to high accuracy, dc and low-frequency ac voltages are among the most important. DC voltage measurements are generally based upon comparisons with a standard cell or a standard 1.0-v or 1.5-v potentiometer. A voltage divider, commonly termed a volt box, must ordinarily be used in conjunction with these devices in order to permit the calibration of higher voltages.[1]

Although simple in concept, consisting merely of a series circuit of electrical resistors, the attainment of the desired combination of accuracy and flexibility in volt-box design constitutes a most difficult and challenging problem, one which has been a subject of considerable interest in recent years.[2,3] The present paper includes a systematic analysis of the errors attending the utilization of resistance voltage dividers and describes a newly designed instrument in which a heretofore unattainable combination of accuracy, voltage range, and simplicity of operation is realized.

MAJOR SOURCES OF ERROR

The basic measurement principle underlying the operation of a volt box is illustrated in Fig. 1. Since continuously variable voltage dividers with a sufficiently high accuracy and precision are unavailable, it is necessary to provide a working circuit consisting of the series connection of a large number of fixed precision resistors.

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