AC–DC Transfer Difference of the PTB Multijunction Thermal Converter in the Frequency Range from 10 Hz to 100 kHz

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Abstract—The ac-dc transfer difference of the multijunction thermal converter (MJTC) developed at Physikalisch-Technische Bundesanstalt (PTB) has been evaluated for various heater resistances between 27 and 700 Ω. By optimizing the heater resistance for voltage transfer to 190 Ω and for current transfer to 27 Ω, transfer differences of a few parts in 10^7 are achieved up to 100 kHz. Transfer differences caused by thermoelectric effects could be shown to be smaller than 10^{-8}, due to the nearly ideal periodic structure of the converter and the special construction of the intermediate leads between heater and vacuum throughleads.

An increase of the transfer difference at low frequencies is caused partly by the change in the real part of the heater impedance, which can be compensated by adding series or parallel resistors to the heater. Another part is found to be a current transfer difference which can only be diminished to a few parts in 10^7 by increasing the number of thermocouples or decreasing the input power.

I. Introduction

During the last 20 years substantial work has been done on single-junction thermal converters (SJTC's) and multijunction thermal converters (MJTC's) to achieve an uncertainty in the ac–dc transfer below 10^{-6} in the optimum frequency range about 1 kHz, where the transfer difference is believed to be caused mainly by thermoelectric effects. In spite of this, differences of a few parts in 10^6 have been observed not only within a group of well-evaluated SJTC's [1], but also between several MJTC's of different design and manufacturers measured at Physikalisch-Technische Bundesanstalt (PTB). Moreover the mean value of a group of SJTC's showed disturbing deviations of 3×10^{-6} against a group of MJTC's [1]. In addition nonsystematic transfer differences of a few parts in 10^5 have been observed on MJTC's of different design at low frequencies [2] and at higher frequencies up to 100 kHz, which are in contradiction to the theoretical uncertainty estimated by Wilkins [3]. Looking for reasons to explain these discrepancies, systematic changes in the design of the PTB MJTC's [4] have been made to evaluate the ac–dc transfer difference theoretically, and by experiment, to the lowest uncertainty possible.

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II. Design of the MJTC

The basic design of the MJTC is shown in Fig. 1. Its functional element is a twisted bifilar heater of enameled quaternary alloy with a diameter of 10 μm up to 40 μm (depending on heater resistance), and 24 mm in length. The heater is thermally connected to and supported by the hot junctions of 56 thermocouples. The cold junctions are mechanically attached by an adhesive to the SiO-insulated copper heat sink. The series-connected thermocouples are produced by winding a helix (3.5 mm wide and 5.5 mm high) of 56 turns of CuNi44 wire (20 μm in diameter) and sputtering copper over some part of the helix. The thickness of the copper is carefully controlled during the sputtering process to obtain a low temperature coefficient of the output voltage.

The element is mounted in a heavy, hermetically sealed, OFHC copper housing, which is evacuated to a residual stable pressure of 10^{-6} mbar to reduce the convective loss of heat from the element and to improve the stability of the output voltage. A built-in getter absorbs some outgassing from the converter and can be reactivated several times.

The vacuum throughleads are made out of OFHC copper wire in an Al_2O_3 ceramic disc in order to obtain an almost frequency-independent heater circuit with very small thermoelectric voltages. Intermediate leads made out of the same alloy as the heater connect the heater to the vacuum throughleads. They are wound around a copper post to control the temperature of the spot-welded connections to the heater. An Al_2O_3 bead short circuits these connections thermally.

III. Transfer Difference Caused by Thermoelectric Effects

At mid-frequencies of about 400 Hz to 5 kHz the ac–dc transfer difference of thermal converters is believed to be caused mainly by thermoelectric effects. It has been shown by Zhang *et al.* [5] that an ideal MJTC with a periodic structure has no ac–dc transfer difference due to thermoelectric effects. Since any manufactured MJTC may depart from that ideal, the influence of temperature deviations along the heater has been investigated.

In the model (Fig. 2) the MJTC is represented by three thermocouples bearing a heater on top. The thermocou-

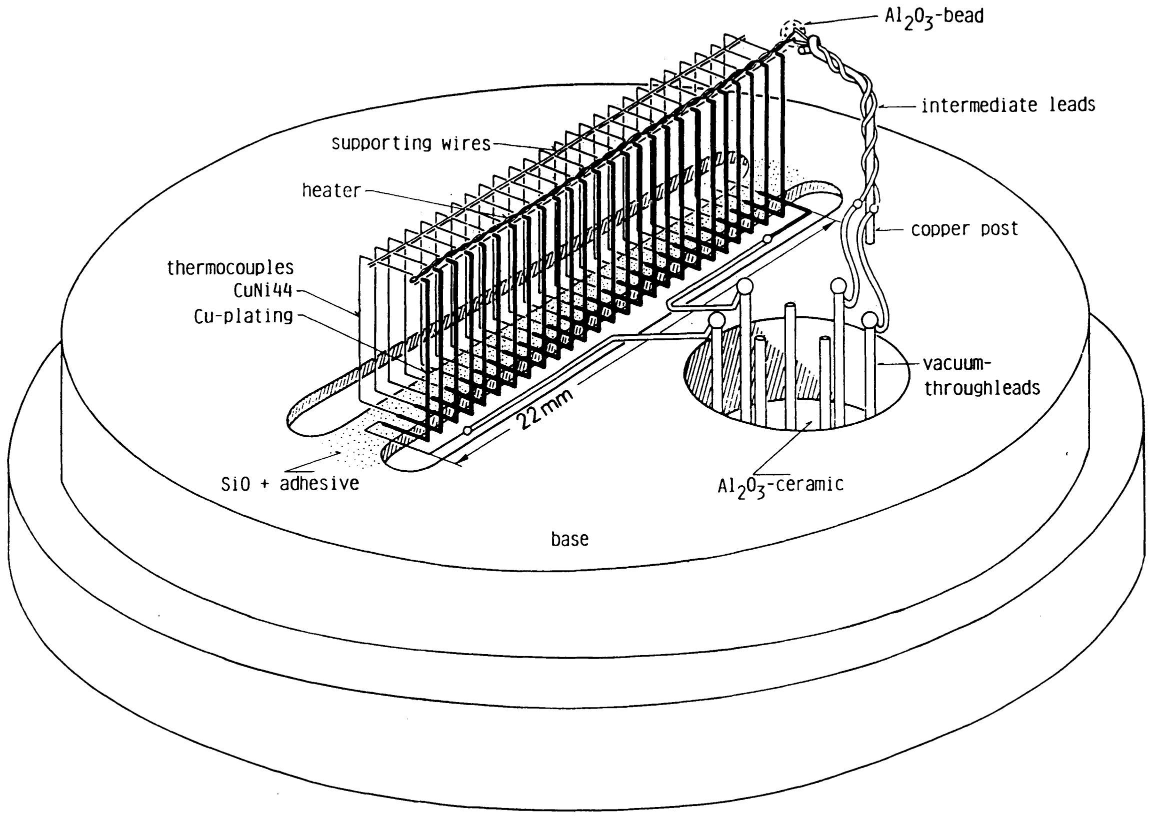

Fig. 1. MJTC with thermocouples in series made by partial copper plating of CuNi44 wire.

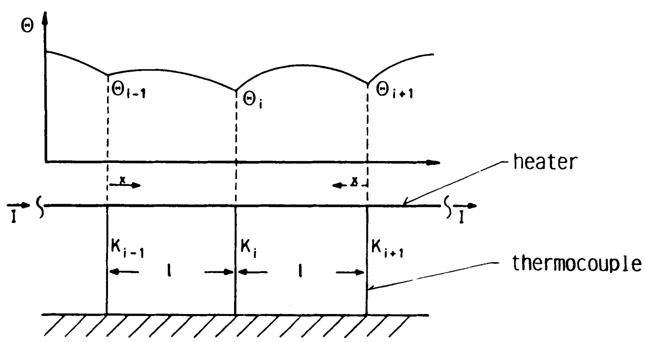

Fig. 2. Above: temperature distribution along the heater due to different heat conductance of the thermocouples. Below: section of three thermocouples with distances l and heat conductances K_{i-1}; K_i; K_{i+1}.

ples are equally spaced at a distance l, and the heater carries a current I. The temperatures Θ_{i-1}, Θ_i, and Θ_{i+1} are assumed not to be equal due to some asymmetry in the thermal conductance of the thermocouples.

Neglecting convective and radiative losses from the heater, all the Joule heat generated in the heater is conducted to the heat sink through the thermocouples.

To calculate the ac–dc transfer difference of the couple i due to the Thomson effect and nonperiodic temperature distribution, the differential equation for the heat flow along the heater may be solved using a similar approach to that used by Widdis [6] for the temperature distribution on a SJTC.

The temperature distribution along the heater is described by

$$\frac{d}{dx}\left(A\lambda\frac{d\Theta}{dx}\right) - \sigma I\frac{d\Theta}{dx} + \frac{I^2\rho}{A} = 0 \tag{1}$$

where

$d/dx(A\lambda(d\Theta/dx))\,dx$	heat flow due to thermal conductivity,
$-\sigma I(d\Theta/dx)\,dx$	Thomson heat, and
$(I^2\rho/A)\,dx$	Joule heat generated in the heater

where

- λ thermal conductivity,
- ρ resistivity,
- σ Thomson coefficient, and
- A cross section of the heater wire.

Writing

$$-\frac{\sigma I}{\lambda A} = C$$

and

$$\frac{I^2\rho}{\lambda A^2} = D$$

and using the boundary conditions given in Fig. 2 we get for Θ:

$$\Theta = -\frac{D}{C}x + \frac{\Theta_{i-1}e^{-Cl} - \Theta_i - \dfrac{Dl}{C}}{e^{-Cl} - 1} + \frac{\Theta_{i-1} - \Theta_i - \dfrac{Dl}{C}}{1 - e^{-Cl}} e^{-Cx}. \tag{2}$$

The temperature at the thermocouple i is given by equating the sum of the heat flow through the heater from both sides into point i and the heat flow through the thermocouple with its thermal conductance K_i.

$$A\lambda \frac{d\Theta}{dx}\bigg|_{x \to 1+0} + A\lambda \frac{d\Theta}{dx}\bigg|_{x \to 1-0} = -K_i\Theta_i. \tag{3}$$

By taking

$$1 + \frac{K_i l}{2\lambda A} = N$$

and $Cl << 1$ we get the approximate solution for one direction of the current:

$$\Theta_i = \frac{Dl^2}{2N}\left[1 - \frac{C^2l^2}{12N}\right] + \frac{\Theta_{i+1} + \Theta_{i-1}}{2N} \cdot \left[1 + \frac{C^2l^2}{12} - \frac{C^2l^2}{12N}\right] + \frac{Cl}{4N}(\Theta_{i+1} - \Theta_{i-1}). \tag{4}$$

This equation shows the linear influence of the Thomson effect, and some influences of second-order Thomson effect. Changing the direction of the direct current, the Thomson effect and hence also C change signs. By taking the mean of both directions (subscript o) the first-order Thomson effect is averaged out and only the second-order Thomson effect is left:

$$\Theta_{io} = \frac{Dl^2}{2N}\left[1 - \frac{C^2l^2}{12N}\right] + \frac{\Theta_{i+1} + \Theta_{i-1}}{2N} \cdot \left[1 + \frac{C^2l^2}{12} - \frac{C^2l^2}{12N}\right]. \tag{5}$$

At ac current (subscript f) there is no change of temperature distribution due to thermal inertia and C can be taken as zero:

$$\Theta_{if} = \frac{Dl^2}{2N} + \frac{\Theta_{i+1} + \Theta_{i-1}}{2N}. \tag{6}$$

The ac–dc current transfer difference may be defined [7] as

$$\delta = \frac{I_f - I_o}{I_o}\bigg|_{U_{af} = U_{ao}} \tag{7}$$

where

I_f rms ac current, and
I_o dc current which when reversed produces the same mean output voltage of the MJTC as I_f.

Substituting the heater temperatures Θ generated by the current I, we get for the transfer difference:

$$\delta = -\frac{1}{2}\frac{\Theta_f - \Theta_o}{\Theta_o}. \tag{8}$$

For small differences, Θ_o in the denominator can be substituted by Θ_f and we get with (5) and (6) for the transfer difference of the thermocouple at point i:

$$\delta_i = \left[\frac{\Theta_{i-1} + \Theta_{i+1}}{2\Theta_{if}} - 1\right]\frac{C^2l^2}{24N}. \tag{9}$$

The formula shows that for equal Θ_{i-1}, Θ_i, and Θ_{i+1} the transfer difference is zero. This indicates how to construct a thermal converter with low transfer difference and is the key to the MJTC. In (9) nothing is said about the number of thermocouples. Therefore an ideal MJTC can be made with three thermocouples if all three couples have the same temperature. However, this can only be achieved by using many couples to bear the heater and some special construction of the current-lead-in wires.

Equation (9) also shows that zero transfer difference can be achieved with a constant temperature gradient. Moreover for small deviations of the temperatures, the term in the brackets will always be smaller than 1 and will have positive and negative signs, which will cancel each other statistically in a well-made MJTC over a sufficiently large number of thermocouples.

In contrast to this in an SJTC, $\Theta_{i-1} + \Theta_{i+1}$ is always much smaller than $2\,\Theta_{if}$ and the transfer difference then is

$$\delta_i \approx -\frac{C^2l^2}{24N}. \tag{10}$$

An estimation for the PTB MJTC with an Isaohm heater having a Thomson coefficient of 1.7 $\mu V \cdot K^{-1}$, a length $l = 0.4$ mm, a temperature difference of 50 K between hot and cold junctions, and 1-K difference between Θ_{if} and the average of Θ_{i-1} and Θ_{i+1}, gives a transfer difference of $2 \times 10^{-9}\ K^{-1}$. To get an estimation of the temperature differences on the MJTC, a heater was made from CuNi44 with a Thomson coefficient of $-24\ \mu V \cdot K^{-1}$ and hence together with some different dimensions 250 times more transfer difference per kelvin than Isaohm. The transfer difference measured for this converter relative to a standard MJTC was only -1×10^{-6}. This leads to the conclusion that the transfer difference due to Thomson effect in the MJTC with an Isaohm heater is smaller than 4×10^{-9}. In fact at a frequency of 1 kHz no difference larger than 2×10^{-7} has been measured between more than 50 MJTC's, and about 80 percent of these had differences smaller than 1×10^{-7}.

Another transfer difference can be caused by the leads carrying the current to the heater, if the periodic temperature distribution is disturbed. This is qualitatively shown in Fig. 3. At the left side, the connection of a current lead with good thermal conductance is shown very near to the first thermocouple. Heat is drawn from the heater which will result in a temperature gradient along the elements.

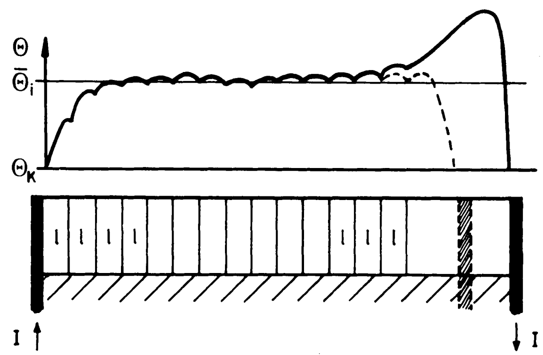

Fig. 3. Temperature distribution on the heater with current leads connected at different distances to the thermocouples. (Dotted lines show the ideal position.)

On the right side, the distance between the current connection and the first element is very large and the Joule heat leads to a rise of the temperature and again to a temperature gradient along the thermocouples. Such temperature gradients will lead to transfer differences in connection with the Thomson effect [5]. Therefore the length l_{HZ} of the heater between the welded connection to the intermediate leads and the first thermocouple is calculated so that the temperature of this region will rise in the same way as the regions between the elements and then come down to the temperature of the cold junctions at the welded points as is shown in Fig. 3 by the dotted lines. The distance l_{HZ} is calculated to 0.88 mm.

Another temperature change will be generated by the Peltier effect at the junction of the intermediate leads and the heater, if the materials are not the same. The first idea for properly designed intermediate leads was to make the leads and the heater out of the same material and to use for the leads only a larger diameter. To extract the Joule heat the intermediate leads are wound in good thermal connection around a copper post. The welded connections are thermally short-circuited by an Al_2O_3 bead to allow some heat exchange if there is any Peltier effect due to changes in the material by the welding process. The connection to the copper of the vacuum throughleads is made at the other end of these intermediate leads. Moreover a bifilar heater is used. Due to the opposite directions of the current in both parts of the heater, the Thomson effect theoretically should be averaged out for each cross section [3]. This construction, furthermore, reduces the transfer difference due to Thomson and Peltier effect as well as at the first thermocouple as at all the other thermocouples.

By using again CuNi44 material for both heater and intermediate leads to exaggerate the transfer difference of this construction, the transfer difference could be measured at the first element of the MJTC to be between -1×10^{-6} and -3×10^{-6}. Hence for Isaohm material the transfer difference is estimated to be smaller than 10^{-8}.

IV. High-Frequency Dependence of the Transfer Difference

At high frequencies dielectric losses in the heater insulation and bonding adhesive, which are constructional parts of the MJTC, are mainly responsible for transfer differences. Fig. 4 shows the schematic diagram of an infinitesimally small part of the bifilar heater with its distributed resistance R', its inductance L', and its distributed admittances due to capacitance C' and dielectric losses, represented by the conductance G', between the two halves of the heater with index H, and between heater and housing through the thermocouples with index G.

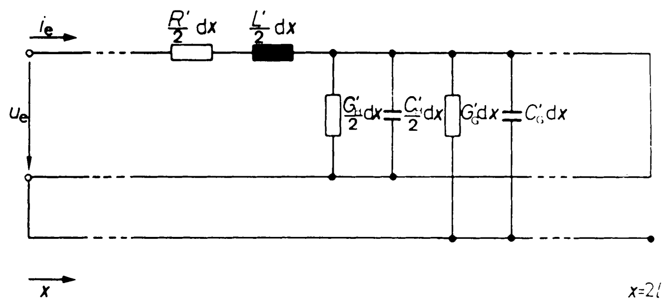

Fig. 4. Schematic diagram of the equivalent electrical circuit of the bifilar heater and the housing.

The input impedance of this circuit can be calculated using transmission-line theory. Adding the paralleled admittances and conductances, taking the lump sum of all distributed impedances and admittances, and neglecting all terms smaller than 10^{-10} up to 100 kHz, the approximate impedance Z is given by:

$$Z \approx R\left[1 - \frac{1}{3}RG + \frac{2}{3}\omega^2 LC - \frac{2}{15}\omega^2 R^2 C^2 + j\omega\left(\frac{L}{R} - \frac{1}{3}CR\right)\right]. \tag{11}$$

Equating the Joule heat generated in the heater at high frequency and at direct current and neglecting second-order terms we get for the voltage transfer difference δ_u

$$\delta_u \approx -\frac{1}{6}RG - \frac{1}{90}\omega^2 R^2 C^2 + \frac{1}{2}\frac{\omega^2 L^2}{R^2}. \tag{12}$$

For the current transfer difference δ_i we get

$$\delta_i \approx \frac{1}{6}RG - \frac{1}{3}\omega^2 LC + \frac{1}{15}\omega^2 R^2 C^2. \tag{13}$$

The measurement of the different parameters in these equations showed that the current transfer difference mainly depends on the dielectric losses represented by the conductance G, and is also proportional to the resistance of the heater. This leads to the requirement for a current converter to have a heater resistance as low as possible. For a heater of 27 Ω the current transfer difference at a frequency of 100 kHz is calculated to be less than 1×10^{-7}, and for a 700-Ω heater, to be 3.4×10^{-6}.

The voltage transfer difference is determined mainly by the first and the third term of (12) which cancel each other at a specific frequency and at an optimum value of heater resistance. For the heater alone they would cancel at 100 kHz for about 90 Ω, and δ_u would be -3×10^{-6} for 700 Ω, and $+2 \times 10^{-6}$ for 27 Ω. But there is also the effect

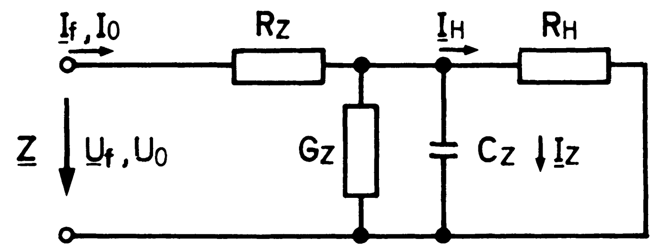

Fig. 5. Schematic diagram of the equivalent electrical circuit of the heater circuit and the intermediate leads.

of the intermediate leads to be considered, defining the temperature distribution on the heater. A schematic diagram of the intermediate leads is given in Fig. 5, with the series resistance R_Z, the dielectric losses represented by G_Z for a certain frequency and the capacitance C_Z between the two leads and also to the copper post. Neglecting second-order terms the transfer differences due to these leads are given by

$$\delta_u \approx 2G_ZR_Z + \omega^2 C_Z^2 R_H R_Z \tag{14}$$

$$\delta_i \approx R_H G_Z + \frac{1}{2}\omega^2 C_Z^2 R_H^2. \tag{15}$$

The current transfer difference again is proportional to the heater resistance and can be calculated at 100 kHz for a 27-Ω heater to be 3×10^{-7}, and for a 700-Ω heater, to be 8×10^{-6}. The voltage transfer difference due to the intermediate leads is always smaller than 2×10^{-7}.

Other sources of transfer difference are skin-effect and proximity-effect. Calculations show that the skin-effect is negligible in the thin Isaohm wires of the heater, but the vacuum throughleads made out of copper wire of 400-μm diameter and some parts of the intermediate leads made out of copper wire with 200-μm diameter show a significant skin-effect which is calculated by the formula for skin-effect at low frequencies:

$$\frac{\Delta R}{R_o} \approx \frac{1}{48} r^4 (\pi f \kappa \mu)^2 \tag{16}$$

where

- r radius of the conductor,
- f frequency,
- κ electrical conductivity, and
- μ magnetic permeability.

The skin-effect is enlarged by the proximity-effect in the vacuum throughleads by about 7 percent. For a heater of 27 Ω, the voltage transfer difference due to skin-effect and proximity-effect is calculated at 100 kHz to be 6×10^{-6}, and for a 700-Ω heater, to be 2×10^{-7}.

Transfer difference contributions for current are calculated from (13) and (15), and for voltage from (12), (14), and (16), and in each case they have to be added. They all depend on the value of the heater resistance and for current, therefore, the lowest heater resistance possible should be chosen. For a 27-Ω heater we calculated a transfer difference of 3×10^{-7} at 100 kHz.

For voltage at a heater resistance of 190 Ω, the transfer differences due to dielectric losses are compensated by the inductance and the skin-effect of the leads to $+3 \times 10^{-7}$ at a frequency of 100 kHz. Taking these converters as a reference the frequency dependence of the transfer difference for different heater resistances could be determined, and are given in Figs. 6 and 7. The measured transfer differences agreed to a few parts in 10^7 in the whole frequency range up to a 100 kHz with the calculated differences. This fact adds considerable weight to the correctness of the evaluation of the frequency dependence of the MJTC's with different heater resistances [8].

V. Low-Frequency Dependence of the Transfer Difference

With decreasing frequency from 1 kHz down to 10 Hz the current transfer differences grow increasingly more positive (Fig. 8). They depend on the resistance of the heater and grow larger with higher heater resistance. At half-rated input power the transfer difference at 10 Hz is nearly half, and at lower input power the transfer difference is even lower.

The voltage transfer differences grow increasingly more negative and also depend on the heater resistance and the input power (Fig. 9). The reference MJTC for these measurements will be explained at the end of this section.

If we increase the number of the thermocouples from 56 to 84 and 112, the temperature of the heater and the current transfer difference is decreased by the same factor for the same rated output voltage (Fig. 10). A similar decrease is measured for the voltage transfer difference, but here the change at half rated input power is very small (Fig. 11). Some part of this behavior at low frequencies is due to a change in the real part of the input impedance of the heater, which was measured in a low-frequency bridge against standard resistors. Fig. 12 shows the measured changes in the real part of the input impedance of a 190-Ω heater MJTC with 56 thermocouples for different input voltages (2.8 V extrapolated).

The different absolute values of current and voltage transfer differences show that there is also some part of a current transfer difference δ_i', the origin of which cannot be explained, but which is constant for different heater resistance values. Fig. 13 shows the voltage and current transfer differences and their components, the changes of the real part of the heater resistance and the current transfer difference δ_i'. For an increasing number of thermocouples this current transfer difference decreases. For a 112-thermocouple converter δ_i' at 10 Hz is 4×10^{-7}, and for half heater input power, is only 2×10^{-7}.

Hermach [2] showed, that transfer differences due to changes in the real part of the heater resistance can be compensated by series resistors for voltage and shunt resistors for current. With such a compensation only the current transfer difference δ_i' is left and will be the same for voltage and current. For a 56-thermocouple converter, at 10 Hz, and rated input power we get a transfer differ-

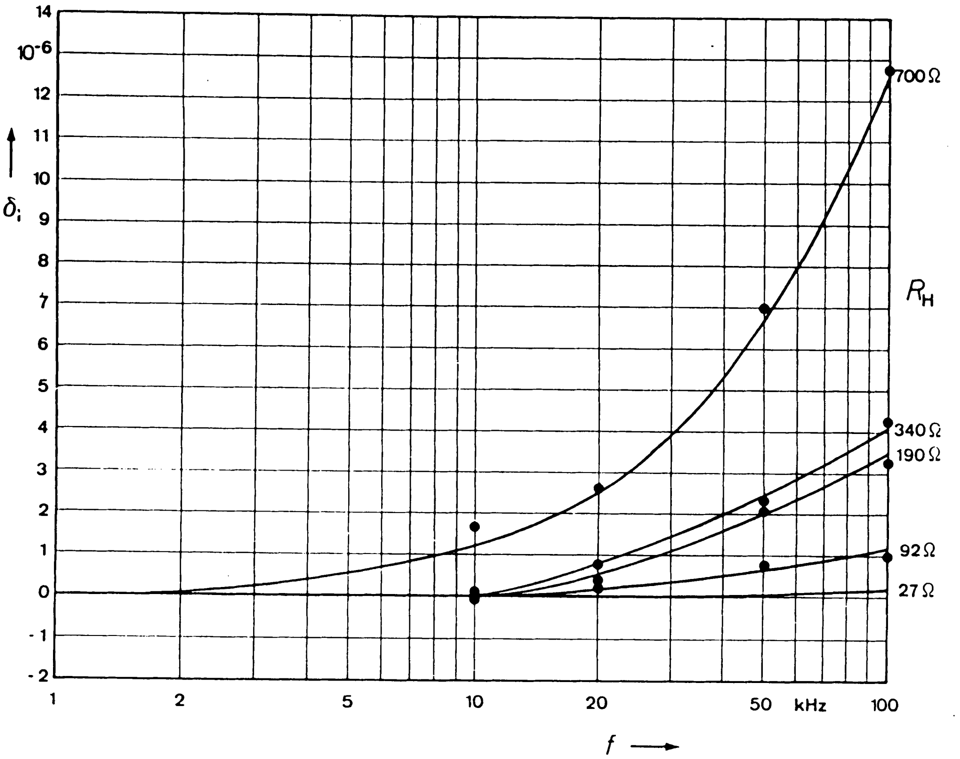

Fig. 6. High frequency dependence of the ac-dc current transfer difference δ_i for various heater resistances R_H.

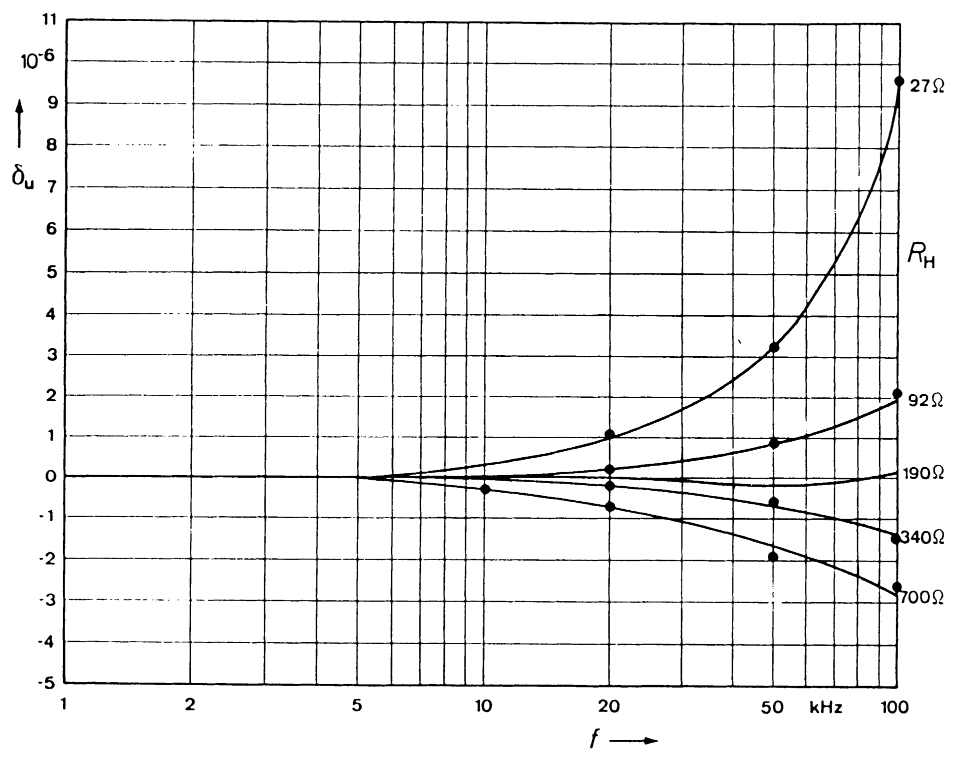

Fig. 7. High frequency dependence of the ac-dc voltage transfer difference δ_u for various heater resistances R_H.

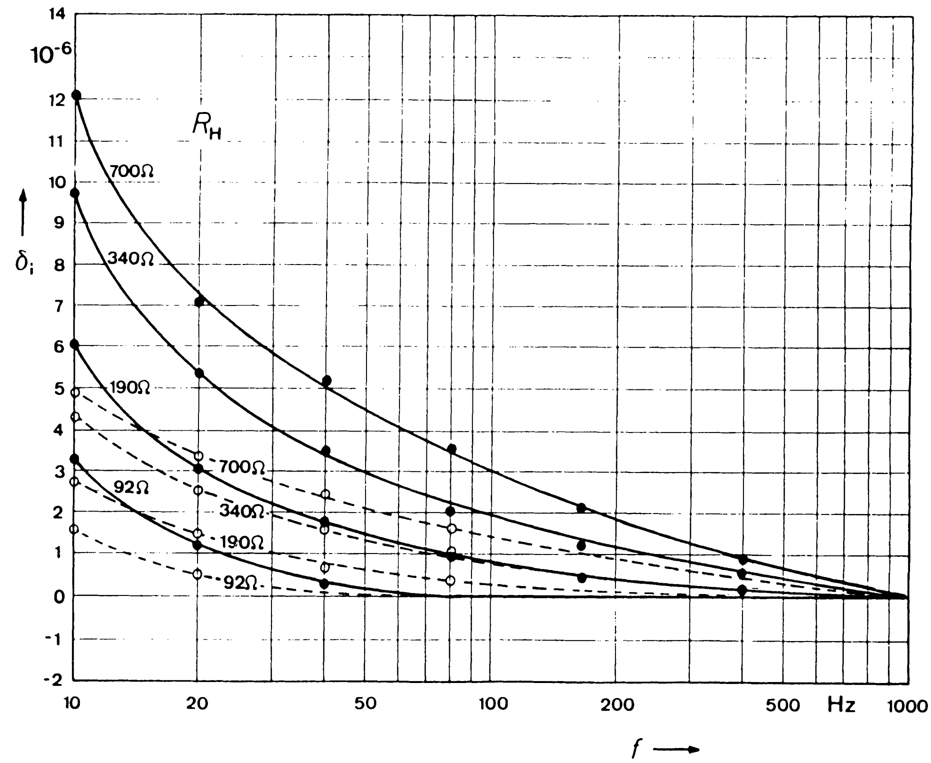

Fig. 8. Low-frequency ac–dc current transfer difference δ_i for different heater resistances R_H and input power adjusted for the same rated (——) and half rated (- - -) output voltage.

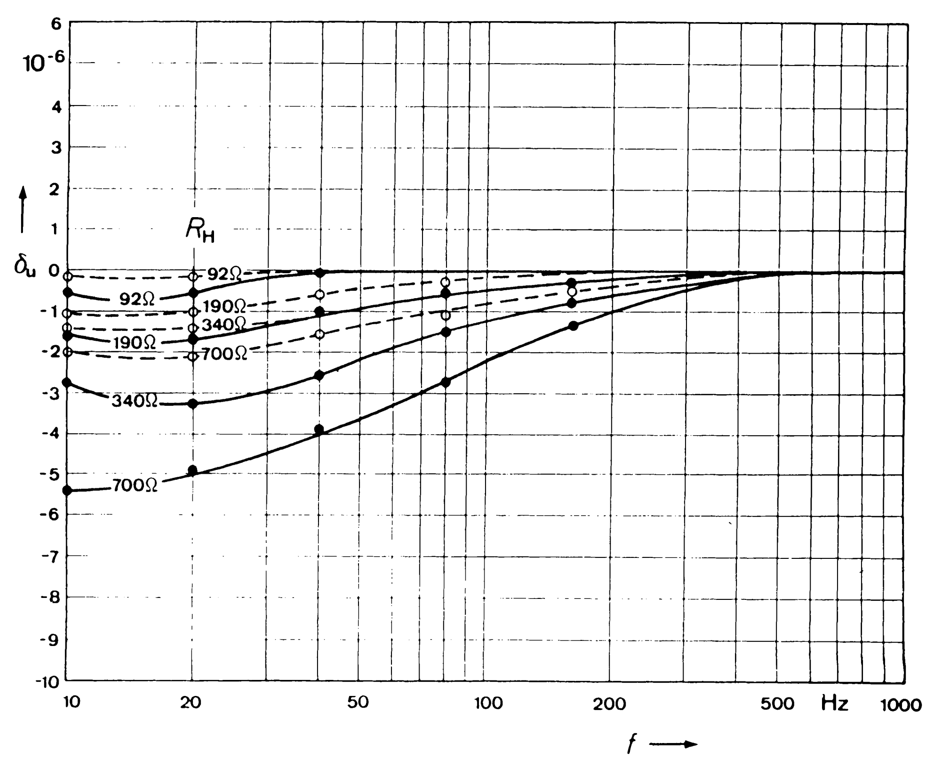

Fig. 9. Low-frequency ac–dc voltage transfer difference δ_u for different heater resistances R_H and input power adjusted for the same rated (——) and half rated (- - -) output voltage.

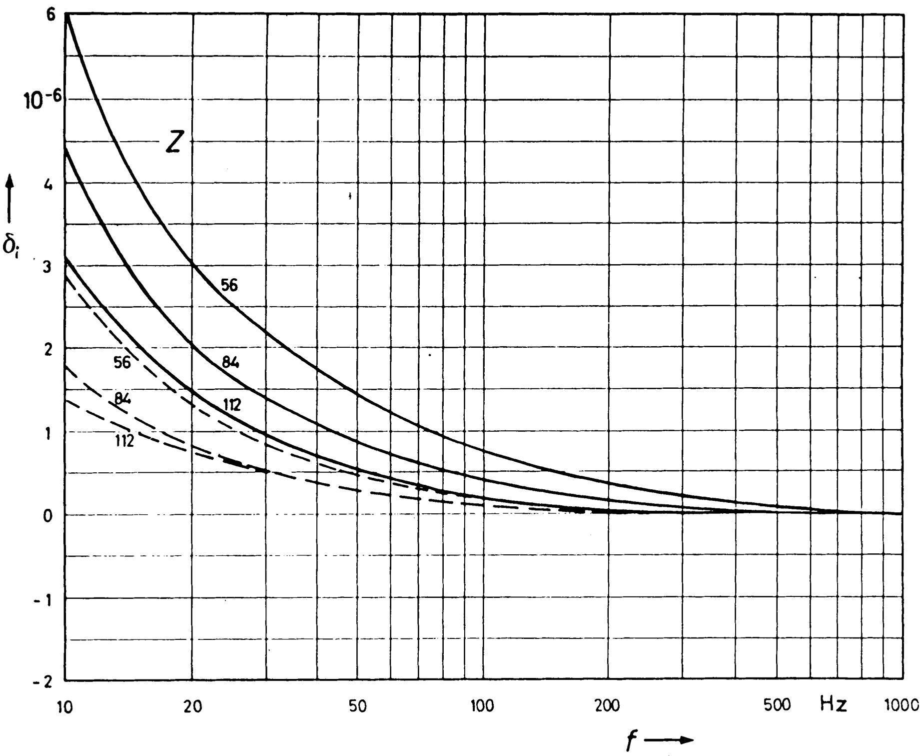

Fig. 10. Low-frequency ac-dc current transfer difference of a 190-Ω heater MJTC for different numbers Z of thermocouples with input power adjusted for the same rated (——) and half rated (- - -) output voltage.

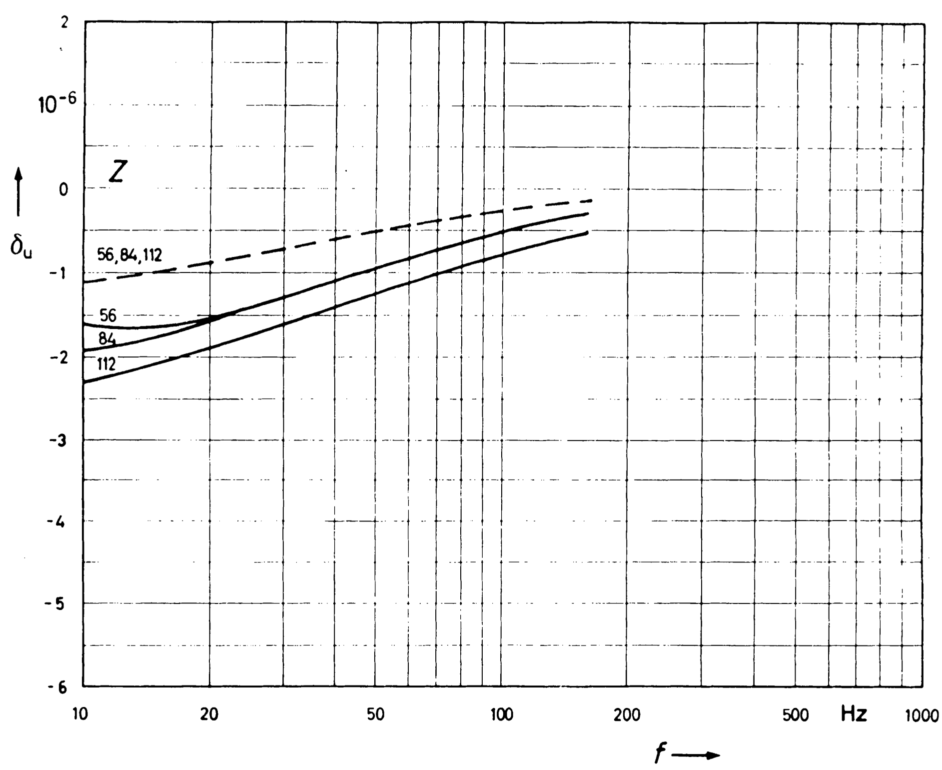

Fig. 11. Low-frequency ac-dc voltage transfer difference of a 190-Ω heater MJTC for different numbers Z of thermocouples with input power adjusted for the same rated (——) and half rated (- - -) output voltage.

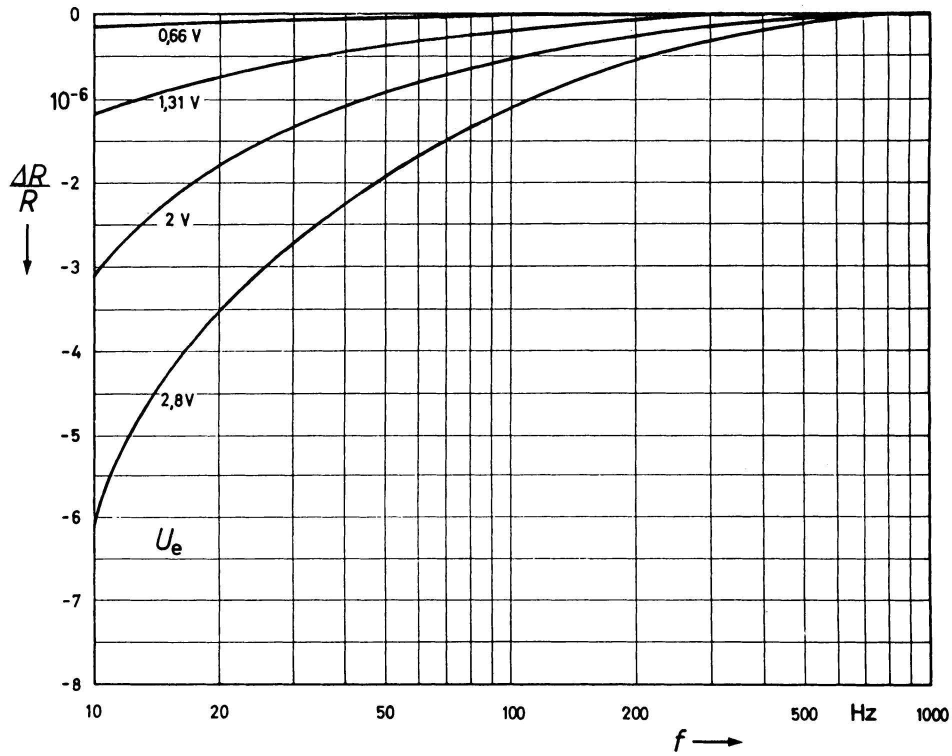

Fig. 12. Measured changes in the real part of the input impedance of a 190-Ω heater MJTC with 56 thermocouples at low frequencies for different input voltages (2.8 V extrapolated).

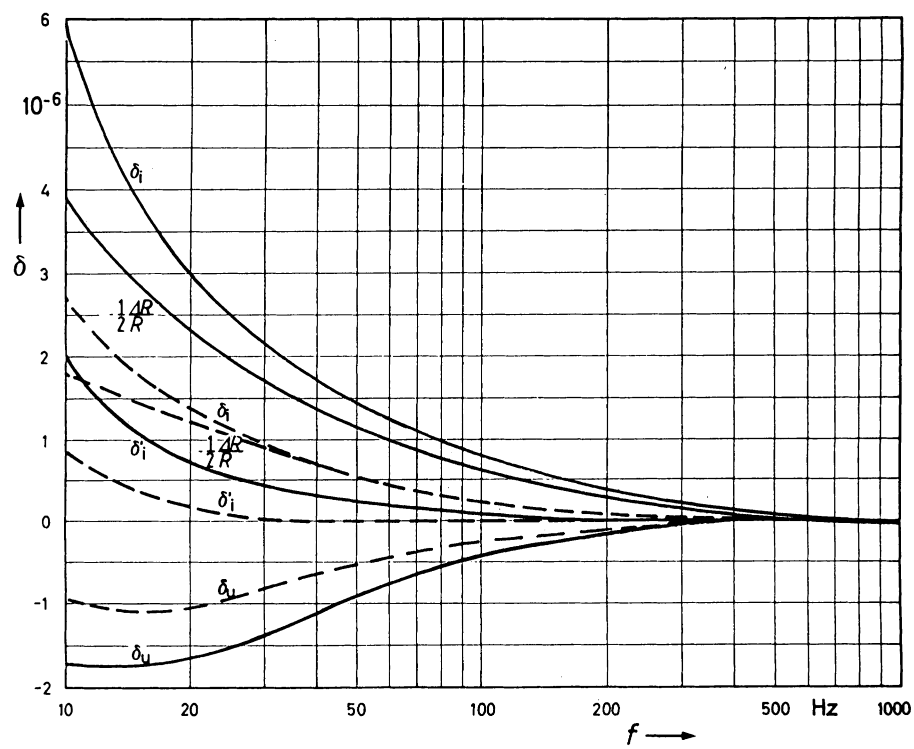

Fig. 13. Frequency dependence of the current and voltage transfer differences, of the change in the real part of a 190-Ω heater and of the current transfer difference δ_i' for a 56-thermocouple MJTC with input power adjusted for rated (———) and half rated (- - -) output voltage.

ence of 2×10^{-6}, and for 112-thermocouple converter, 4×10^{-7}. At half-rated input power this value decreases to 2×10^{-7}. Such a compensated 112-thermocouple converter served as the reference converter for all the measured transfer differences at low frequencies in Figs. 8–11, and 13.

VI. Conclusion

The proper design of the MJTC reduces the influence of thermoelectric effects to an insignificant level. But it shows frequency-dependent ac–dc transfer differences which are lowest in the mid-frequency region at about 1 kHz and increase at lower and higher frequencies. The equivalent circuit can be evaluated for a special design for different heater resistances with an uncertainty of a few parts in 10^7 for voltage and current up to 100 kHz. The measured transfer differences agreed to a few parts in 10^7 with the calculated differences, which gives considerable confidence to the correctness of the results. Also at low frequencies down to 10 Hz the transfer differences of compensated converters can be evaluated to a few parts in 10^7.

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