ORIGINAL COMMUNICATION.

NON-INDUCTIVE, WATER-COOLED STANDARD RESISTANCES FOR PRECISION ALTERNATING-CURRENT MEASUREMENTS.

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CONTENTS.

INTRODUCTION.

In the measurement of power in alternating-current networks when the highest accuracy is desired, it is difficult to obtain standard resistances in which the time constant (the inductance divided by the resistance) is sufficiently low to render the effect of inductance negligible, and at the same time the resistance of which does not vary with the temperature or the frequency.

This is especially the case when large currents are to be dealt with, for in this instance the comparatively large amount of energy to be dissipated renders it almost impossible to get rid of the heat by natural radiation sufficiently rapidly to prevent a rise in temperature beyond the limit at which even the best material will maintain its resistance sufficiently constant.

In the case of resistances for the measurement of large currents, although much care may be taken to reduce the self-inductance, the comparatively low value of the resistance makes it very difficult to reduce the time constant sufficiently to be negligible at low power factors.

In dealing with the question of alternating-current measurement at the National Physical Laboratory, we have had to aim at an accuracy considerably higher than 1 part in 1,000, and not the least difficult part of this problem has been to construct a series of resistances ranging from 0·04 to 0·001 ohm, and capable of operating at from 50 to upwards of 2,000 amperes.

It is not proposed in this paper to explain in detail the electrostatic instruments and methods employed for measuring respectively the volts, amperes, and watts, but rather to limit the discussion to a description of the standard resistances used with the electrometer, and the methods employed for surmounting the difficulties indicated.

WATER-COOLED RESISTANCES.

It is generally recognised that for laboratory purposes it is not desirable to try to limit the amount of power used in the measuring apparatus (provided the heat can be effectually dissipated) if extra torque and increased accuracy and reliability can be secured in the standard instruments.

In the resistances in question we have been able, without difficulty, to use a drop of 2 volts and more, across the potential points, for standard resistances up to 2,000 amperes, and we have adopted this figure as a basis for alternating current and power measurements.

It will be readily appreciated that the only practicable method of dealing with this amount of power (4 k.w. in the 2,000-ampere resistance) is by some artificial method of cooling. The device adopted by Messrs. Crompton many years ago of tubular resistances cooled by water constantly flowing through them suggests itself at once as the most convenient form for the purpose.* Attempts were first made to draw Eureka tubes, and Messrs. The London Electric Wire Company went to considerable trouble in an endeavour to make some seamless tubes of this material; but the attempt had eventually to be abandoned in favour of manganin. This alloy, although more troublesome on account of the necessity of hard soldering all joints, appears to lend itself better to the drawing of seamless and uniform tubes. The tubes used have been obtained from Messrs. Goliasch, of Berlin, and have proved most satisfactory in every way. (For detailed dimensions see Table II.)

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* See "The Potentiometer and its Adjuncts," by Clark Fisher, p. 77; also *Electrician*, vol. 38, p. 22, 1896.

CURRENT DENSITY.

In order to be able to obtain high wattmeter readings at low power factors it is desirable to be able to use each resistance at a current considerably in excess of that for which it is normally rated, thus obtaining a greater voltage drop and proportionally larger deflection. In the following table of values, therefore, column 2 gives the normal current for each resistance, viz., that causing a 2-volt drop across its terminals; whilst column 4 gives the value to which the current in each resistance can be raised before a greater change than 2 parts in 10,000 takes place in its value, due to rise of temperature.

It is possible, of course, by applying a correction to use the tubes for higher currents than these. For instance, the 0·001 ohm standard has a total change in resistance of 1 part in 1,000 from 0 to 3,500 amperes, and a similar increase in their ranges may be obtained from the other tubes without the necessity of a correction, if changes in resistance up to 0·1 per cent. may be neglected.

TABLE I.

Resistance.	"Normal" Current.	Volt Drop with Normal Current.	"Maximum" Current.	Volt Drop with Maximum Current.
Ohms.	Amperes.			
0·040	50	2	115	4·6
0·020	100	2	260	5·2
0·010	200	2	450	4·5
0·002	1,000	2	1,300	2·6
0·001	2,000	2	2,500	2·5

The figures in Table I. were obtained experimentally from the tubes actually constructed. They will necessarily vary somewhat according to the temperature-resistance characteristic of the manganin used, but they serve to indicate the range and the order of accuracy obtainable from this class of apparatus.

The watts (per square centimetre) which are dissipated in these tubes under the "maximum" load conditions vary from 7½ to 12. An upper limit of 10 watts per square centimetre may be taken as an average value for the design of manganin tubes up to 1·5 mm. thickness of wall. This corresponds to a current density of about 25 amperes per square millimetre (16,000 amperes per square inch) in tubes 0·3 mm. thick. The authors' experience up to the present goes to show that within certain limits, from the point of view merely of efficient cooling, the thickness of the walls of the tubes is not of great consequence. It will be apparent, however, from consideration of the magnitude of the self-induction, that a thin-walled tube of large diameter, working at a high current density, has great advantages. Hence it is a real gain to be able to work with current densities up to 25 amperes per square millimetre.

The experimental results given above were obtained with a flow of cooling water from the ordinary main supply of about 15 litres per minute, but the flow may be varied considerably without greatly affecting the result.

It is probable that if more perfect arrangements were made for churning the water in the tubes as it passes through them, a much smaller volume of water would suffice to give as favourable results as those obtained with the larger amount of flow. The difficulty is that the water tends to pass up the tubes without mixing sufficiently, the inside of the column remaining cold with a skin of relatively hot water on the outside next to the metal. In the larger sized resistances a length of glass tube fills up most of the space inside (see Fig. 3), and helps to diminish the flow whilst increasing the velocity at the surface. The difference in the temperature of the inflow and outlet water in the case of the 0·002-ohm tube when dissipating 6½ k.w. was found to be only 4·7° C. The actual difference of temperature between the bottom and top of the tube, however, was much greater than this amount, showing that the mixing was not perfect. The water must be led in to the bottom of the tube, which is for this reason used in a vertical position, so that it always remains full. Should, then, the current be switched on by mistake without turning on the water no damage to the standard will ensue.

CONDUCTANCE OF THE WATER.

In order to test whether the conductance of ordinary London tap water has any effect on the apparent resistance of the tubes the following test was made: A glass tube of length and diameter, similar to that of the tube of the 0·002-ohm resistance through which a flow of water was maintained, was connected up in series with a high variable resistance. When the latter was 40,000 ohms, and 110 volts connected across the two in series, an electro-static voltmeter gave equal deflections on the two portions of the circuit. This showed that the resistance of the column of water was of the order of 40,000 ohms, an entirely negligible quantity when shunting these low resistances.

DURABILITY.

It is of some interest to notice that a water tube manganin standard similar to the 0·002-ohm size described here was recently taken to pieces. It had been in use for about four years, and showed no signs of deterioration due to the constant flow of water through it. The thin internal enamelled surface appeared to be as clean as when new, and the resistance value of the tube had also not altered.

CHANGE IN RESISTANCE DUE TO VARIATION OF LOAD.

The diagram in Fig. 2 illustrates the change in the resistance of the 0·001-ohm manganin tube as the current increases from 0 to 4,000 amperes. The shape of this curve may be taken as typical of the curves obtained by experiment on all the standards, which exhibit the well-known characteristic of manganin alloy for variation of the temperature coefficient. In good manganin there is a total rise in the resistance of about 2/10,000 from 8° C. to 35 or 40° C., after which, as the temperature increases, the resistance tends to fall with greater rapidity than the original rate of increase.

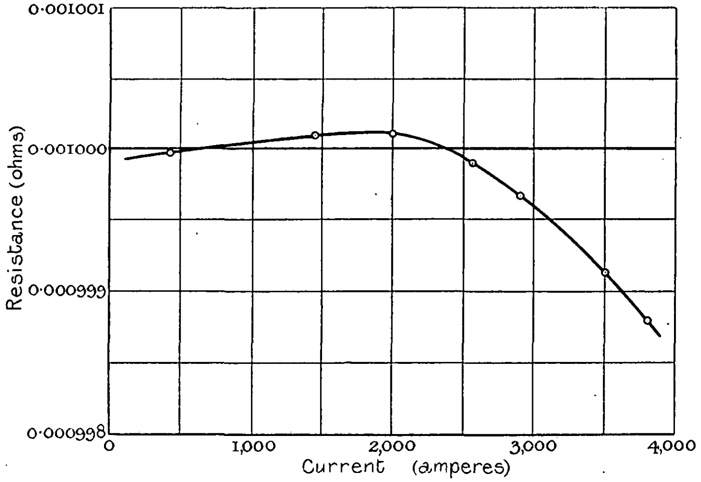

FIG. 2.—0·001 Ohm Standard Resistance. Curve of Change of Resistance with Current.

CONSTRUCTION.

The cross-section of each tube is proportioned so that a length of about 40 cms. (15½ in.), has the desired resistance. Five sizes have been made up to the present, and three resistances of each size except of the largest. These enable 3-phase watt measurements to be made, in which case one resistance is connected in each phase, and the standard wattmeter changed over from one to the other.

The approximate dimensions of the tubes are given below in Table II.

These tubes are silver soldered into heavy copper ends which form the potential points, and which serve to lead the current into the tube. Each copper end has a hole through it equal to the bore of the tube, so that a stream of cooling water can pass continuously through the tube carrying away the heat, and maintaining the resistance alloy at an even temperature.

The working drawing of one of the complete resistances (0·002 ohm for 1,000 amperes) is shown in Fig. 3. It will be seen from this that the current is led in from the two terminal posts in a way which maintains a well-closed circuit and thus avoids giving rise to fields which may affect other apparatus in the neighbourhood. The narrow ring soldered to the centre of the tube forms the common point for the volt and current circuits in wattmeter measurements with the quadrant electrometer.

TABLE II.

Approximate Dimensions of the Manganin Tubes.

Resistance.	Outer Diameter.	Thickness of Wall.	Length.
	Millimetres.	Millimetres.	Centimetres.
0·040	6	0·25	35½
0·020	10	0·30	40
0·010	15	0·40	39
0·002	30	1·00	48
0·001	40	1·50	42½

The manganin tubes are seamless, and the first treatment is to thoroughly anneal them at a red heat, in order, as far as possible, to secure subsequent constancy in their resistance. The heavy copper cylindrical ends and centre rings are then sweated on with silver solder, care being taken to ensure that the resistances are a few per cent. lower than the values which are finally required.

It is then necessary, especially with the very thin walled tubes, to wash over all the inner surfaces with acid so as to remove the scale. If this is not done the tubes will have a comparatively large temperature coefficient. After removing all traces of acid the inner surface is coated with enamel which is hardened in the usual way by stoving. This prevents subsequent oxidation of the surface. The tubes are then ready for closer resistance adjustment, which is best

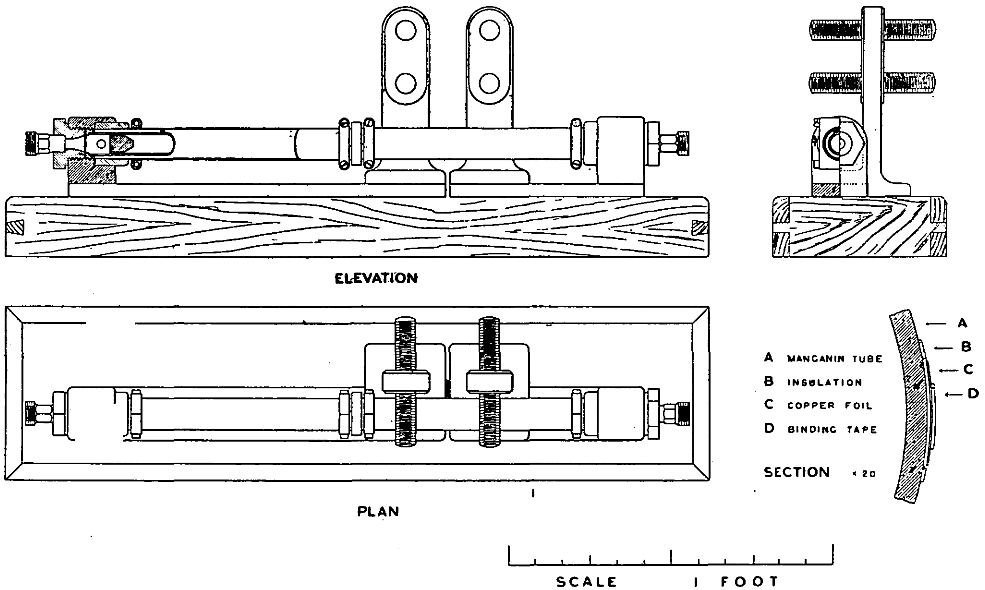

FIG. 3.—Working Drawing of 0·002 Ohm (1,000 Amperes) Resistance.

The small portion of the tube and sheath in section, shown on the right-hand side, is enlarged twenty times. The inner tube (shown in the elevation) is of glass, and leads the water along the inner surface of the metal.

done by sand-papering their outer surfaces in a lathe. When the resistance is within ½ per cent. of the final value the tube is soft soldered into the brass or copper end castings and mounted up. The final adjustment is then made and the outer surface lacquered to prevent oxidation.

A photograph of the largest and smallest of the thirteen tubes at present made is reproduced in Fig. 1.

SELF-INDUCTION.

The method usually adopted for rendering a low sheet resistance as non-inductive as possible is to double the sheet back upon itself. Dr. Orlich (of the Reichsanstalt) has recently constructed a set of oil-cooled standards on this principle, with very active mechanical stirring of the oil; but as he has not up to the present published details of the results attained, the authors are not in a position to discuss them.

In order to bring the volt drop across the terminals of the water-tube standards into phase with the currents passing through them, a device has been adopted which was suggested by Mr. Albert Campbell,* in connection with non-inductive resistances.

It consists in leading the potential wires back along the resistances in such a way that the magnetic flux, which causes the back E.M.F. of self-induction in the resistance, will also give rise to a practically equal back E.M.F. in the lead of the potential circuit and so neutralise the induction effect. In order to make this elimination more complete and also to render the calculation of the inductance possible, it is necessary that the potential lead shall take the form of a closely fitting cylindrical sheath running the whole length of the tube and insulated from it at all points except at one end. In the tubes in question the thickness of this insulation is 0·2 m͏m.

The cross-section of one of the tubes with its sheath clamp is shown in the top right-hand portion of the working drawing in Fig. 3. Underneath this is shown a section (to a very much magnified scale) of a small part of the tube, insulation, and sheath in order to illustrate their relative proportions. On account of its closeness to the tube it has not been possible to draw the sheath in the elevation, only its clamp being shown.

The magnified section, as reproduced, is twenty times the scale of the remainder of the drawing. An inspection of these drawings will show that the only flux which can cause a self-inductive effect which is not eliminated by the sheath is that which cuts the manganin tube while it does not cut the sheath. Of these linkages those outside the sheath are completely eliminated by it and those inside it can

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* *Electrician*, vol. 61, pp. 1000–1001, 1908. Dr. Orlich has kindly brought to the authors' notice an article by Leo Lichtenstein in the *Dinglers Polytechnisches Journal*, vol. 321, pp. 38, 109, and 118, 1906, in which this method is foreshadowed of neutralising the self-induction of an alternating-current circuit.

be divided into two classes: namely, the linkages of the flux in the insulating material with the whole current, and the linkages of the flux in the tube itself with part of the current. It will be seen how the concentration of all the current in a very thin tube of large radius assists the non-inductive quality of the resistance as compared with the case in which it is distributed over a solid conductor.

The most perfect condition is, of course, approached as both the thickness of the wall of the tube and the distance between it and the sheath are reduced, and also as the diameter of the tube is increased.

If we use the formula due to Lord Rayleigh for calculating the self-induction of the tubes, the inductance, which is introduced by the potential sheath not being infinitely close to the tube, can be separated from the inductance, due to the walls of the tube being of finite thickness. This formula for the self-inductance of concentric cylinders is given in a convenient form by Russell,* and has recently been more fully developed by the same author in a paper before the Physical Society of London.†

The decrease of the self-induction of these standards due to skin effect, calculated from the formulæ given in that paper, are not appreciable at the ordinary frequencies met with in practice, and it is sufficient to take the simple formula reproduced from Dr. Russell's book below.

If b_2, b_1, a_2, and a_1 (in centimetres) are the outer and the inner radii of the outer and inner cylinders respectively, then—

$$\frac{L}{l} = \left[2 \log_\epsilon \frac{b_1}{a_2} \right] + \left[\frac{2 a_1^4}{(a_2^2 - a_1^2)^2} \log_\epsilon \frac{a_2}{a_1} + \frac{1}{2} \frac{a_2^2 - 3 a_1^2}{a_2^2 - a_1^2} \right]$$

$$+ \left[\frac{2 b_2^4}{(b_2^2 - b_1^2)^2} \log_\epsilon \frac{b_2}{a_1} - \frac{1}{2} \frac{3 b_2^2 - b_1^2}{b_2^2 - b_1^2} \right],$$

where L is the inductance in centimetres and l is the length of the tubes. (1 cm. inductance = 1 henry × 10^{-9}.)

Now it will be seen from this that the first term deals with the flux which pulsates in the insulating medium between the two cylinders. The second term takes account of the flux which is restricted to the walls of the inner cylinder and which does not emerge from its outer surface; whilst the last term, which we do not require in our problem, deals in a similar way with the flux confined to the outer return conductor of a concentric main. We are clearly not concerned with the linkages due to the current in the return conductor since its inductive effects on the sheath and the inner tube are practically equal and opposite. In our case, therefore, we are only concerned with the two first portions

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* "Alternating Currents," vol. i., p. 53.

† *Proceedings of the Physical Society of London*, vol. 21 (1909).

of the formula. The insulation between the tubes and their sheaths consists of a single layer of varnished cloth, 0·2 mm. thick, the sheath itself being made of copper foil, 0·04 mm. thick, bound tightly round the varnished cloth. It is hardly practicable from the point of view of the security of the insulation to attempt to make the latter thinner than 0·2 mm. Much may be gained, however, by making the tubes of large diameter with thin walls.

Table III. gives the values of the effective inductances and time constants of the various resistances, and shows the large amount of power which it is possible to dissipate with very low time constants in this type of standard.

The diagram in Fig. 4 has been drawn from the calculated values in order to indicate graphically the relative effect of the two linkages. The resistances of the tubes are plotted as abscissæ, and the inductances in centimetres for the tubes as ordinates. The upper curve gives the total inductance, and the lower one shows what portion of the inductance is due to the thickness of the insulation between a tube and its sheath. The curves are merely given to indicate the order of inductance obtained in these particular tubes, and they only, of course, give the approximate values. These values and also those in Table III. must necessarily be modified to some extent as the dimensions of any given tube are changed. For instance, the inductance of the 0·002-ohm tube could have been reduced from 3·7 cms. to 1·2 cm., for the same cooling surface, if it had been made 4 instead of 3 cms. in diameter, with a wall of 0·5 instead of 1 mm. thickness. As already pointed out, however, mechanical considerations prevent the use of tubes the walls of which are too thin, although tubes with 0·5 mm. walls are probably quite practicable.

TABLE III.

Resistance.	Current Range.	Kilowatts Dissipated at Full Load.	$\frac{\text{Inductance}}{\text{Resistance}}$ (Time Constant).	Effective Inductance.
	Amperes.			Centimetres.
0·040	115	0·53	$1{\cdot}6 \times 10^{-7}$	6·5
0·020	260	1·35	$2{\cdot}7 \times 10^{-7}$	5·4
0·010	450	2·00	$3{\cdot}4 \times 10^{-7}$	3·4
0·002	1,300	3·40	$18{\cdot}5 \times 10^{-7}$	3·7
0·001	2,500	6·25	$30{\cdot}0 \times 10^{-7}$	3·0

With the data and experience gained with the tubes already constructed, the authors are now designing standards to work with 5,000 and 10,000 amperes in which the inductances will be more favourable than those calculated in Table III.

The necessity of reducing the value of the inductances of low resistances to the lowest possible limit is illustrated in the diagram in Fig. 5. This shows how the angle of phase displacement between the current and the volts on the potential terminals of the resistances varies at a frequency of 50 ∿. The scale on the left-hand side shows the error introduced into wattmeter readings at 0·1 power factor in which these resistances are used. The curve illustrates the fact already alluded to, that for standards up to 2,000 or 3,000 amperes, although the inductances may be as low as 3 cms., viz., three thousandths of a microhenry, the low value of the resistance and consequent high time constant makes it necessary to use a correction, if a closer accuracy than 1 per cent. is desired at power factors of 0·1. With resistances for 200 and 300 amperes the correction is only 0·1 per cent. at 0·1 power factor.

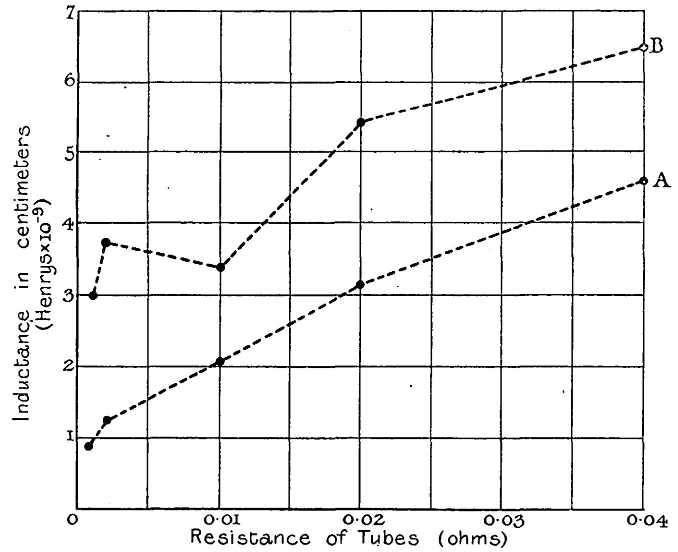

FIG. 4.—Diagram showing the Relative Proportion of the Self-induction of Tube Resistances caused by the Thickness of Insulation between Tube and Sheath.

Curve A shows the proportion of the inductance which is due to a thickness of insulation of 0·2 mm.
Curve B gives the total inductance.

THE EFFECT OF FREQUENCY ON THE RESISTANCE OF TUBES.

Owing to their large diameter it was necessary to calculate the effect of a variation of frequency on the resistance of the tubes.

For this purpose the authors used Dr. Heaviside's formula, an independent proof of which has recently been given by Dr. Russell (*l.c. ante*). It was found that owing to the thinness of the tubes and the high volume resistivity of manganin the variation of resistance with frequencies of 100 or less only affected the hundreth-thousandth place and was absolutely negligible. Hence in using these tubes to an accuracy of one or two parts in 10,000 it is only necessary to make a slight correction for their very minute inductance in cases when low power-factor circuits are being dealt with.

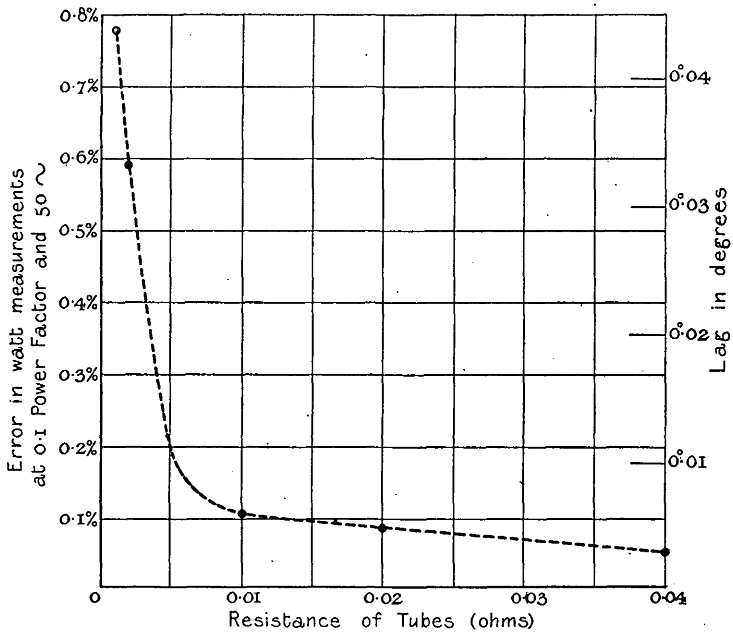

FIG. 5.—Diagram giving Phase Displacements between Current and Volts for the different Resistances, and the Error caused in Wattmeter Readings thereby at 0·1 Power Factor and 50 ∿ per second.

EXPERIMENTAL TESTS TO DEMONSTRATE THE SCREENING EFFECT OF THE SHEATHS.

We are not aware of any method of measuring the low values of effective inductance of such resistances as these to anything approaching the accuracy with which they can be calculated. The following experiment is of interest, however, and embodies a method which has been found of great value in measuring small differences of phase in current transformers and similar apparatus. The method is similar in principle to that which the authors find has been utilised and described by E. and W. H. Wilson in connection with some interesting tests on commercial shunts.*

R and R_1, Fig. 6, are two similar water-tube resistances supplied with current by one of two alternators coupled together. The quadrants of a sensitive electrometer are connected across one of the resistances, while the needle is connected to the other alternator whose voltage is displaced 90° with relation to the phase of the machine which supplies the current. The phase difference is adjusted till the deflection of the electrometer is zero, the quadrants being connected to one of the resistances. If these are now changed over to the other one, any deflection will be proportional to the sine of the phase angle between the E.M.F.'s on the two resistances.

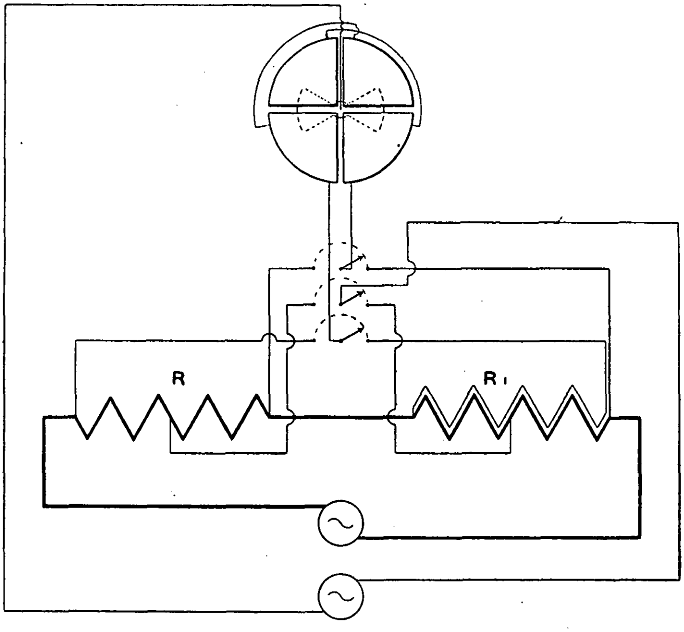

FIG. 6.—Diagram of Connections for Test to Demonstrate the Screening Action of the Potential Sheaths.

In the tests two similar standards were used, one with its sheath

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* *Electrician*, vol. 56, p. 464, 1906.

and the other without, and it is possible thus to test the difference of effective inductances of the two tubes and to illustrate the screening action of the sheaths.

Measured in this way the change of phase angle of the 0·002-ohm resistance with and without its sheath was 0·67° at 50 ∿ per second. It will thus be seen (by comparison with Fig. 6) that the sheath corrects for over 90 per cent. of the inductance of this tube. If the tube be surrounded by one or two sheets of iron the deflection without the sheath can be increased 10 times—but no effect whatever is produced by the iron on the deflection when the screen is used.

COMPARISON WITH OTHER APPARATUS.

It is interesting to compare the relative advantages of the authors' tube type of non-inductive resistance with the more usual strip type. A comparison cannot be made on the basis of inductance alone since the capacity of the tube resistances to dissipate energy may be said to be roughly 300 times that of air-cooled standards. This gives them a great advantage from the points of view of overload capacity and constancy of resistance at all loads. It is clear that a strip doubled back on itself could not be satisfactorily cooled with water on account of the nearness of the ends and the consequent leakage which would take place through the water, so that the only alternative is to use the oil cooling mentioned earlier in the paper.

The authors are of opinion, however, that the water-tube type of resistance must compare very favourably with any other similar standards from the points of view of cooling efficiency, compactness, and general convenience. Quite apart from these characteristics, it would appear that advantage also lies with the tube resistances from the point of view of low self-inductance.

Applying the formula given by E. B. Rosa[*] to calculate the order of inductance to be obtained from flat sheet resistances the following case has been taken. A sheet of manganin 86 cms. long, 20 cms. broad, and 2 mm. thick, thus giving a resistance of approximately 0·001 ohm, is assumed doubled back on itself. The formula assumes the sheet to be of negligible thickness so that the case is quite hypothetical. Suppose, however, we assume these two negligibly thin plates to have an average distance between their centres of 2 mm., then the inductance works out at 5·3 cms. as compared with 3·0 cms. for the water-cooled type of the same value of resistance. If this resistance is air cooled its maximum capacity will be about 250 amperes instead of 2,500 in the case of the water-cooled type in spite of its having more than three times the cooling surface. The case taken assumes that these two large plates can be satisfactorily mounted so that the distance between them is of the order of 0·5 mm. It is,

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[*] Bulletin of the Bureau of Standards, vol. 4, No. 2, p. 324.

however, questionable whether this can be achieved, and it is more probable that a much greater distance would be necessary.

The following data are given for the purpose of indicating the magnitude of the inductances found in commercial apparatus. Messrs. E. and H. Wilson (*ref. cit.*) give the value of a commercial corrugated straight manganin shunt for use with a hot wire amperemeter as 160 cms. Its value was 0·0045 ohm, being constructed for 200 amperes. The phase displacement of this shunt at 50 ∿ is of the order of 0·6°.

Messrs. Siemens and Halske publish the results of phase displacement tests between the primary and secondary of one of their high-grade current transformers, ratio 1,200/5, 50 ∿. These tests show phase displacements varying from 0·5° at $\frac{1}{10}$th load to 0·2° at full load. Our own measurements on the phase differences in current transformers also give values of this order.*

CONCLUSION.

The following are the principal points dealt with in this paper :—

1. The authors demonstrate that by applying the novel device of a thin potential sheath in close proximity to the outside of a tube of resistance alloy such as manganin, standard resistances of low value and up to any current capacity can be constructed whose effective self-induction is as low as 2 cms. or 3 cms. (two or three thousandths of a microhenry).

2. By cooling these standards with a stream of water flowing through the tube (as in the well-known type of "water-tube" resistance) a current density of 16,000 amperes per square inch may be attained, and upwards of 10 k.w. with 3,000 amperes be dissipated in a 1½ in. tube 18 in. long, without undue heating. The capability of the resistances to dissipate energy is thus some 300 times greater than that of standards which merely depend on natural air cooling.

3. The standards actually constructed have values 0·04, 0·02, 0·01, 0·002, 0·001 ohm, with capacities of 115, 260, 450, 1,300 and 2,550 amperes respectively. These current values are attained with a total change of resistance due to heating of not more than 2 parts in 10,000 from no load to full load.

4. The values of the resistances are not affected by so much as 1 part in 10,000 by "skin effect" at ordinary frequencies. The conductivity of the ordinary tap water used for cooling does not change their values.

5. Comparisons with other apparatus of a similar nature show that these tubes have such very great advantages for laboratory purposes that in the authors' opinion the inconvenience of having to connect

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* See also "The Use of Shunts and Transformers with Alternate-current Measuring Instruments," by C. V. Drysdale, *Philosophical Magazine*, vol. 16, p. 136, 1909.

up the standards to a water supply for cooling purposes is of very small importance.

In connection with the work on these standards, we desire to acknowledge the obligations we are under to Dr. R. T. Glazebrook, Director of the National Physical Laboratory. We wish to express our indebtedness to Mr. A. Campbell and Dr. A. Russell for their ready counsel and advice. Our thanks are also due to Mr. S. W. Melsom for help with the resistance measurements and to Mr. A. Kinnes for assistance in many of the calculations.