

# Characterizing Artefact Standards for Use with Coaxial Vector Network Analyzers at Millimeter-wave Frequencies

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**Abstract** — Traceability for vector network analyzer (VNA) measurements in coaxial lines smaller than 2.4 mm is problematic. The common method for traceability is to use precision coaxial air lines as primary standards. Since slotless lines are not commercially available in lines of this size, the air lines used are difficult to connect, extremely delicate, and expensive to replace. This paper describes a method of characterizing artefacts to use as standards, including specially designed and manufactured air-dielectric shielded open-circuits of various offset lengths as well as offset short-circuits and well-matched loads available in commercial VNA calibration kits. This will provide traceability using more robust and easier to use standards for calibration of the VNA. The proposed method is illustrated by measurement results obtained in 1.85 mm coaxial line (to 65 GHz.)

**Index Terms** — Measurement standards, metrology, microwave measurements, millimeter wave measurements, transmission line measurements.

## I. INTRODUCTION

Providing traceable vector network analyzer (VNA) measurements in coaxial lines smaller than 2.4 mm has significant challenges. Precision coaxial air lines [1] – [3] have become an important tool for providing traceability through TRL [4] or LRL [5] calibrations. In lines smaller than 2.4 mm, however, slotless air lines are not commercially available. Unsupported slotted air lines are difficult to connect, extremely delicate, and expensive to replace. More robust and easy to use standards are therefore required.

Recent work [6] has introduced air-dielectric shielded open-circuits (or, “air opens” for short) as coaxial primary standards for millimeter-wave frequencies. These one-port devices can be used with the ‘three-known-loads’ calibration technique. Three air opens with different offset lengths provide three known, but different, values of voltage reflection coefficient (VRC) to achieve calibration at each required frequency. It is relatively straight-forward to realize air opens with three different offset lengths and this provides reliable calibrations at very high frequencies (where the phase separation between the standards is sufficiently large). However, as the required frequency decreases (i.e. as the wavelength increases), the phase separation produced by these standards decreases and this ultimately causes the calibrations to become less reliable.

New standards are needed to substitute for the air opens at these lower frequencies and hence improve calibration reliability. This paper describes how these new standards are realized (i.e. the process that is used to characterize their

performance as standards). The standards are offset short-circuits and near-matched loads. Both types of standard are found in commercially available VNA calibration kits.

The paper also describes how these new standards can be used, in conjunction with air opens, to provide traceable measurements over the full frequency range of a given coaxial connector. This is demonstrated in this paper for the 1.85 mm connector, where the frequency range is DC to 65 GHz.

## II. CHARACTERIZING THE VNA CALIBRATION STANDARDS

Three different types of artefacts are characterized in this study for use as one-port VNA calibration standards: (i) air dielectric shielded open-circuits (air opens) [6], (ii) offset short-circuits such as those found in commercial VNA calibration kits, and (iii) near-matched loads such as those found in commercial VNA calibration kits.

The characterization of an artefact involves a two step process: (i) Construction of a mathematical model for the VRC of the artefact as a function of frequency. This model depends upon certain parameters whose values need to be estimated; (ii) Estimation of the model parameters for the particular artefact.

Methods to estimate the model parameters include: (i) Estimation of a parameter based on Electromagnetic Theory; (ii) Estimation of a parameter from measurements made on the artefact (either VRC measurements, DC resistance measurements, or length measurements); (iii) Estimation of a parameter by using a value from a standard specification of the artefact i.e. using a “nominal” value for the parameter; (iv) Estimation of a parameter by using a value from a table of physical constants i.e. using an “accepted” value for the parameter.

The models used for air opens and offset short-circuits are **physical models** based on the physics of the operation of the artefact. The parameters in these models correspond to physical properties of the artefact such as the radius of the inner conductor of the coaxial line or the resistivity of the coaxial line conductors. On the other hand, the model used for near-matched loads is an **empirical model** based on a least squares polynomial fit to the measured VRC of the load as a function of frequency and the measured DC resistance of the load. In this case, the model parameters are the coefficients in the polynomial representation of the frequency dependence of

the VRC and do not correspond directly to physical properties of the artefact.

The models and methods of parameter estimation for the three types of artefact will now be described. Since both air opens and offset short-circuits incorporate a length of lossy coaxial transmission line, a model for such a section of line will first be presented. This lossy line model is incorporated into the models for both air opens and offset short-circuits.

#### A. Model for a section of lossy coaxial line

A model for a lossy coaxial transmission line is given in [7]. The propagation constant  $\gamma$  and characteristic impedance  $Z_0$  of the transmission line are given by

$$\gamma = g(a, b, \rho, \epsilon_r, \mu_r, f) \quad (1)$$

and

$$Z_0 = h(a, b, \rho, \epsilon_r, \mu_r, f), \quad (2)$$

where the complex-valued functions  $g$  and  $h$  can be determined from [7] and where  $a$  is the radius of the inner conductor of the coaxial line,  $b$  is the inner radius of the outer conductor,  $\rho$  is the resistivity of the conductors,  $\epsilon_r$  is the relative permittivity of the dielectric between the conductors,  $\mu_r$  is the corresponding relative permeability and  $f$  is the frequency. Note that because the line is lossy both the characteristic impedance and the propagation constant are complex-valued. It follows from general transmission line theory (see for example [8]) that if one end of a length  $l$  of lossy line is terminated by an impedance  $Z$  with VRC  $\Gamma_0$  then the VRC seen looking into the other end of the transmission line,  $\Gamma_{in0}$ , is given by

$$\Gamma_{in0} = \Gamma_0 \exp(-2\gamma l), \quad (3)$$

where

$$\Gamma_0 = \frac{Z - Z_0}{Z + Z_0}. \quad (4)$$

The VRC  $\Gamma_{in0}$  is normalized to the complex-valued characteristic impedance of the lossy line namely  $Z_0$ . The corresponding VRC renormalized to  $50 \Omega$ ,  $\Gamma_{in}$ , is obtained using the following equation

$$\Gamma_{in} = \frac{Z_0 \left( \frac{1 + \Gamma_{in0}}{1 - \Gamma_{in0}} \right) - 50}{Z_0 \left( \frac{1 + \Gamma_{in0}}{1 - \Gamma_{in0}} \right) + 50}. \quad (5)$$

(1) to (5) constitute a mathematical model for the input VRC at frequency  $f$  of a length  $l$  of lossy transmission line with parameters  $a$ ,  $b$ ,  $\rho$ ,  $\epsilon_r$ ,  $\mu_r$  terminated by an impedance  $Z$ . In what follows, this model is applied to both air opens and offset short-circuits. For an air open, the terminating

impedance is assumed to be a frequency dependent capacitance whilst for an offset short-circuit, the terminating impedance is assumed to be an ideal short-circuit.

#### B. Characterizing air opens

**Physical model for an air open:** An air open consists of a length  $l$  of coaxial line after which the inner conductor is truncated and the outer conductor continues thereby forming a section of circular waveguide below cut-off. It is modelled as a length  $l$  of lossy coaxial line terminated in a frequency dependent capacitance  $C(f)$  as described in [6], [9] and [10]. In the model, the terminating impedance is given by

$$Z = \frac{-j}{\omega C(f)}, \quad (6)$$

where  $j = \sqrt{-1}$ ,  $\omega = 2\pi f$  and

$$C(f) = C_0 + C_1 f + C_2 f^2 + C_3 f^3. \quad (7)$$

(1) to (5) and (6) to (7) together constitute a mathematical model for the input VRC at frequency  $f$  of an air open.

#### Estimating the model parameters for an air open:

Table I summarises the model parameters for an air open and the methods used to estimate those parameters for a particular air open.

TABLE I  
PARAMETER ESTIMATION FOR AN AIR OPEN

Component of artefact	Parameter	Method of parameter estimation
-	Frequency ( $f$ )	Accurately set and indicated by the VNA (especially if an external frequency reference is used)
Lossy coaxial line	Coaxial line conductor radii ( $a$ and $b$ )	Nominal values from IEEE std 287 (2007) [11]
	Coaxial line conductors' resistivity ( $\rho$ )	From VRC measurements
	Coaxial line air dielectric relative permittivity and permeability ( $\epsilon_r$ , $\mu_r$ )	Accepted values from a table of physical constants [10]
	Coaxial line length ( $l$ ) (i.e. offset length of air open)	Disassemble air open and measure length of inner conductor
Terminating impedance (frequency dependent capacitance)	Capacitance coefficients ( $C_0$ , $C_1$ , $C_2$ , $C_3$ )	From Electromagnetic Theory [12]

As an example, the resistivity ( $\rho$ ) of an air open with a 6 mm offset and fitted with a 1.85 mm connector was estimated. The VRC of the offset air open was measured on a VNA up to 65 GHz with reference to a TRL calibration. A single air line was used to calibrate the VNA (as well as a zero length thru and male and female short-circuits with identical

VRCs). The calibration only gives good measurements at those frequencies where the difference between the transmission phase of the line standard and the zero length thru standard is more than  $30^\circ$  away from the known calibration failure points at  $n \times 180^\circ$  (where  $n = 0, 1, 2, \dots$ ). With all the other model parameters already estimated, the resistivity value was chosen to give the best agreement in a least squares sense between the measured VRC values and the corresponding VRC values predicted by the model. An iterative process was used to find the best fit resistivity. A frequency range of 30 GHz to 65 GHz was chosen for this fitting procedure for two reasons: (i) this is the frequency range over which three air opens can provide a robust calibration of the VNA and (ii) at lower frequencies, the loss due to signal propagation in the air open is extremely small making it difficult to accurately estimate resistivity from VRC at those lower frequencies (because the VRC values predicted by the model at these frequencies are fairly insensitive to the resistivity). Fig. 1 shows the measured and modelled values for the linear magnitude of VRC of the 6 mm air open for the chosen value of resistivity.

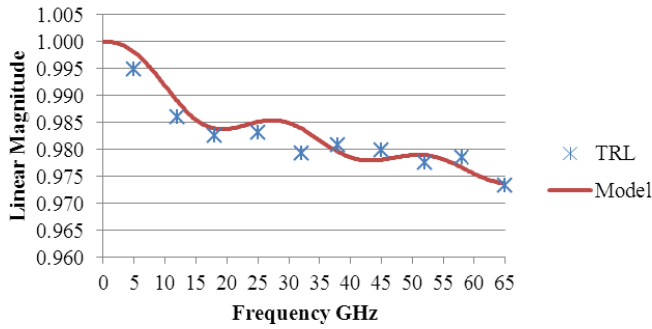


Fig. 1. Linear magnitude of VRC for the air open with an offset length of 6 mm. Shown are the VRC predicted by the model and the VRC measured on a VNA calibrated using a TRL calibration.

### C. Characterizing offset short-circuits

**Physical model for an offset short-circuit:** An offset short-circuit consists of a length  $l$  of coaxial line closed at one end by a conducting disk short-circuiting the inner and outer conductors. It is modelled as a length  $l$  of lossy coaxial line terminated in a perfect (lossless) short-circuit<sup>1</sup>. In the model, the terminating impedance,  $Z$ , is given by

$$Z = 0. \quad (8)$$

(1) to (5) and (8) together constitute a mathematical model for the input VRC at frequency  $f$  of an offset short-circuit.

**Estimating the model parameters for an offset short-circuit:** Table II summarises the model parameters for an

<sup>1</sup> It is fairly straightforward to incorporate a lossy terminating short-circuit into the model, but that is not done here where a lossless terminating short-circuit is assumed.

offset short-circuit and the methods used to estimate those parameters for a particular offset short-circuit. As an example, the resistivity ( $\rho$ ) and offset length ( $l$ ) of an offset short-circuit fitted with a 1.85 mm connector were estimated. A three-known-loads calibration [13] – [14] of the VNA was performed using 5 mm, 6 mm, and 7 mm air open standards, characterized using the techniques described in sub-section II.B, over the frequency range 30 – 65 GHz (below 30 GHz the three air opens do not provide a robust calibration of the VNA due to reduced phase separation between the three air opens). The offset short-circuit was then measured to obtain corrected VRC data which was used to estimate the length and resistivity of the offset line section of the short-circuit.

From (3) and (5) it follows that

$$l = \frac{-1}{2\gamma} \ln \left( \frac{1}{\Gamma_0} \frac{50 \left( \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \right) - Z_0}{50 \left( \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \right) + Z_0} \right). \quad (9)$$

TABLE II  
PARAMETER ESTIMATION FOR AN OFFSET SHORT-CIRCUIT

Component of artefact	Parameter	Method of parameter estimation
-	Frequency ( $f$ )	Accurately set and indicated by the VNA (especially if an external frequency reference is used)
Lossy coaxial line	Coaxial line conductor radii ( $a$ and $b$ )	Nominal values from IEEE std 287 (2007) [11]
	Coaxial line conductor resistivity ( $\rho$ )	From VRC measurements
	Coaxial line air dielectric relative permittivity and permeability ( $\epsilon_r, \mu_r$ )	Accepted values from a table of physical constants [10]
	Coaxial line length ( $l$ ) [offset length of short-circuit]	From VRC measurements (Offset short-circuits cannot be disassembled in order to measure the length of the inner conductor)
Terminating impedance (ideal short-circuit)	Impedance of ideal short-circuit ( $Z$ )	Nominal value ( $Z = 0$ so that $\Gamma_0 = -1$ )

(9) allows the length  $l$  to be estimated given an initial estimate for the resistivity  $\rho$  and estimates of all the other parameters. Measured values are used for  $\Gamma_{in}$ ,  $\Gamma_0 = -1$  (from (4) and (8)),  $\gamma$  and  $Z_0$  are determined from (1) and (2) which depend on  $\rho$  as well as on other parameters. In fact, (9) provides multiple estimates for the length (one for each frequency at which the VRC was measured) which are then

summarised into a single length value. Once the length has been estimated, the resistivity can be estimated using the same iterative process as described in sub-section II.B for air opens. If necessary, the estimation of length and resistivity can be repeated starting with the new estimate of resistivity until convergence is obtained. Fig. 2 shows the measured and modelled values for the linear magnitude of VRC of the offset short-circuit for the chosen values of length and resistivity.

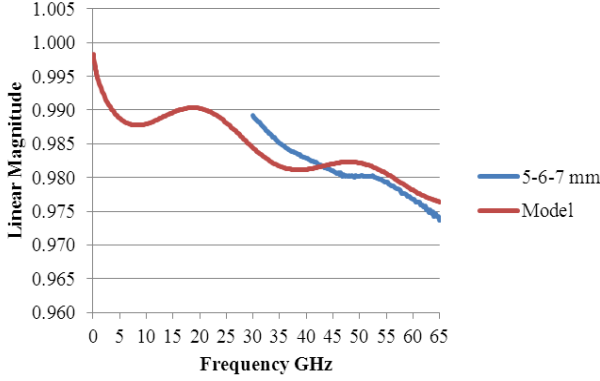


Fig. 2. Linear magnitude of VRC for the offset short-circuit. ‘Model’ is the calculated VRC predicted by the model and ‘5-6-7 mm’ is the corrected measured VRC using a calibration with air open standards of nominal lengths 5 mm, 6 mm and 7 mm.

#### D. Characterizing near-matched loads

**Empirical model for a near-matched load:** The impedance of a near-matched load at frequency  $f$ ,  $Z_{in}(f)$ , is represented, over a specified frequency range, by means of two real polynomials in  $f$ , one representing the real component of impedance (resistance) and the other representing the imaginary component of impedance (reactance) as follows:

$$\text{Re}(Z_{in}(f)) = d_0 + d_1 f + d_2 f^2 + d_3 f^3 + d_4 f^4, \quad (10)$$

$$\text{Im}(Z_{in}(f)) = e_0 + e_1 f + e_2 f^2 + e_3 f^3 + e_4 f^4. \quad (11)$$

In terms of the impedance, the VRC of the near-matched load at frequency  $f$ ,  $\Gamma_{in}(f)$ , is given by

$$\Gamma_{in}(f) = \frac{Z_{in}(f) - 50}{Z_{in}(f) + 50}. \quad (12)$$

(10) to (12) together constitute a mathematical model for the input VRC at frequency  $f$  of a near-matched load over a specified frequency range.

**Estimating the model parameters for a near-matched load:** Table III summarises the model parameters for a near-matched load and the methods used to estimate those parameters for a particular load. The VRC measurements,  $\Gamma_{in}(f)$ , are transformed into impedances,  $Z_{in}(f)$ , prior to estimating the impedance coefficients according to the equation

$$Z_{in}(f) = 50 \left( \frac{1 + \Gamma_{in}(f)}{1 - \Gamma_{in}(f)} \right). \quad (13)$$

TABLE III  
PARAMETER ESTIMATION FOR A NEAR-MATCHED LOAD

Parameter	Method of parameter estimation
Frequency ( $f$ )	Accurately set and indicated by the VNA (especially if an external frequency reference is used)
Impedance coefficients ( $d_0, d_1, d_2, d_3, d_4, e_0, e_1, e_2, e_3, e_4$ )	From DC resistance and VRC measurements

As an example, the impedance coefficients of a near-matched load fitted with a 1.85 mm connector were estimated over the frequency range from 0 to 10 GHz. The load resistance was measured at DC and the VRC of the load was measured at frequencies from 0.3 to 10 GHz with a VNA calibrated using a 5 mm air open, a 7 mm air open (both characterized using the techniques in sub-section II.B), and an offset short-circuit (characterized using the techniques in sub-section II.C). The impedance of the near-matched load was calculated from the VRC measurements. A 4<sup>th</sup> order polynomial least-squares fit was obtained for the real part of the impedance including the resistance measurement at DC – see Fig. 3. Another 4<sup>th</sup> order polynomial fit was also obtained for the imaginary part of the impedance using the defined value of zero for the reactance at DC – see Fig. 4. The resulting curve fits allow the impedance and hence the VRC for the near-matched load to be predicted at any frequency from DC to 10 GHz.

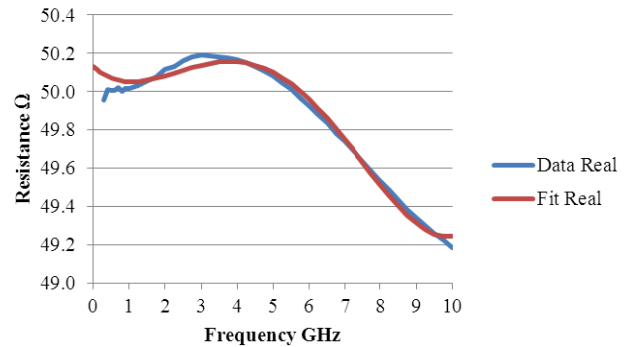


Fig. 3. ‘Data Real’ is the real part of the impedance value calculated from the measured VRC. The measurement was made with a calibration utilizing 5 mm and 7 mm air opens and an offset short-circuit. ‘Fit Real’ is the 4<sup>th</sup> order least squares fit to ‘Data Real’.

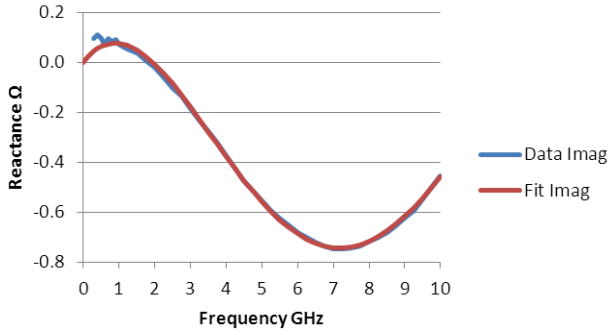


Fig. 4. ‘Data Imag’ is the imaginary part of the impedance value calculated from the measured VRC. The measurement was made with a calibration utilizing 5 mm and 7 mm air opens and an offset short-circuit. ‘Fit Imag’ is the 4<sup>th</sup> order least squares fit to ‘Data Imag’.

### III. BROADBAND VNA CALIBRATION USING THE CHARACTERIZED STANDARDS

A broadband one-port VNA calibration scheme for the frequency range DC to 65 GHz is presented in Table IV. The calibration scheme employs three air opens (with offset lengths 5 mm, 6 mm and 7 mm), an offset short-circuit and a near matched load. Above 30 GHz, the phase separation of the three air opens in the VRC plane is sufficient to provide a good calibration of the VNA. However, as the frequency is decreased below 30 GHz, the phase separation between the air opens reduces so that a calibration based on three air opens is no longer reliable. To overcome this difficulty a calibration involving two air-opens and one offset short-circuit is used at intermediate (microwave) frequencies and a calibration involving an air open, an offset short-circuit and a near-matched is used at low frequencies. Weighted combinations of two calibrations are used over certain frequency ranges to ensure a reliable transition between one calibration scheme and the other.

### IV. MEASUREMENT RESULTS FROM THE CALIBRATED VNA

In this section some VRC measurements made on two devices are presented. The devices are: (i) an air open with a 9 mm offset and (ii) a 3 dB attenuator with one port terminated in a short-circuit. The VRCs of both devices were measured using both a TRL calibration (for reference purposes) and also the broadband calibration scheme described in this paper. Figs. 5 to 7 compare the linear magnitude and phase of the VRCs measured using the two calibration techniques and good agreement is observed between the two sets of VRC measurements.

TABLE IV  
A PROPOSED BROADBAND VNA CALIBRATION SCHEME FOR THE FREQUENCY RANGE DC TO 65 GHz

Frequency Range	Calibration Type	Standards
0 – 8 GHz	Three-known-loads	5 mm air open, offset short-circuit, near-matched load
8 – 10 GHz	Weighted combination of two calibrations using three-known-loads	1 <sup>st</sup> set: 5 mm air open, offset short-circuit, near-matched load 2 <sup>nd</sup> set: 5 mm air open, 7 mm air open, offset short-circuit
10 – 20 GHz	Three-known-loads	5 mm air open, 7 mm air open, offset short-circuit
20 – 30 GHz	Weighted combination of two calibrations using three-known-loads	1 <sup>st</sup> set: 5 mm air open, 7 mm air open, offset short-circuit 2 <sup>nd</sup> set: 5 mm air open, 6 mm air open, 7 mm air open
30 – 65 GHz	Three-known-loads	5 mm air open, 6 mm air open, 7 mm air open

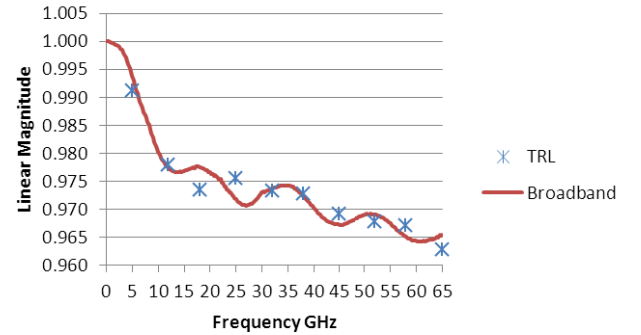


Fig. 5. Linear magnitude of VRC for a 9 mm air open from DC to 65 GHz. Shown are the VRC measured on a VNA calibrated using the broadband calibration and also calibrated using the TRL calibration.

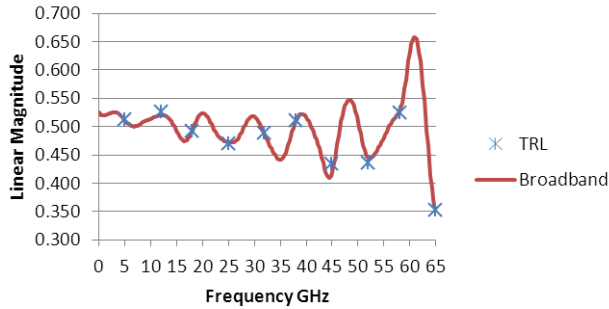


Fig. 6. Linear magnitude of VRC for a shorted 3 dB attenuator from DC to 65 GHz. Shown are the VRC measured on a VNA calibrated using the broadband calibration and also calibrated using the TRL calibration.

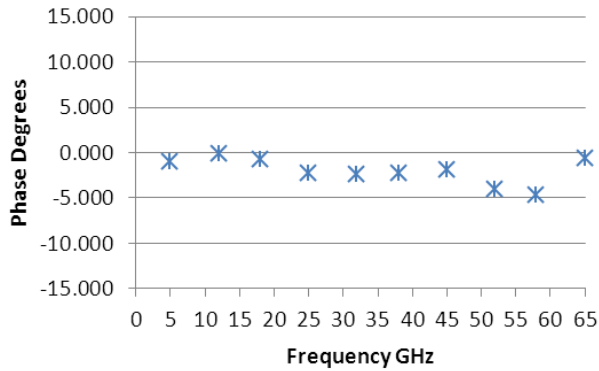


Fig. 7. Phase difference between the VRCs for a 9 mm offset air open measured on a VNA calibrated using the broadband calibration and also calibrated using the TRL calibration

## V. CONCLUSIONS

This paper has shown different methodologies for characterizing VNA calibration standards. The combination of these different devices provides a means to calibrate a coaxial VNA fitted with 1.85 mm connectors from DC to 65 GHz. This provides a more robust methodology for calibration than line based methods such as TRL and LRL.

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## REFERENCES

- [1] R. E. Spinney and I. A. Harris, "The realization of high-frequency impedance standards using air spaced coaxial lines," *IEEE Trans. Instrumentation & Measurement*, vol. 13 no. 4, pp. 265-272, Dec. 1964.
- [2] B. O. Weinschel, "Air-filled coaxial lines as absolute impedance standards," *Microwave Journal*, vol. 7, pp. 47-50, Apr. 1964.
- [3] K. H. Wong, "Using precision coaxial air dielectric transmission lines as calibration and verification standards," *Microwave Journal*, vol. 31, pp. 83-92, Dec. 1998.
- [4] G. F. Engen and C. A. Hoer, "'Thru-Reflect-Line': An improved technique for calibrating the dual six-port automatic network analyzer," *IEEE Trans. Microwave Theory & Techniques*, vol. 27, no. 12, pp 987-993, Dec. 1979.
- [5] C. A. Hoer and G. F. Engen, "On-line accuracy assessment for the dual six-port ANA: extension to nonmating connectors," *IEEE Trans. Instrumentation & Measurement*, vol. 36 no. 2, pp 524-529, Jun. 1987.
- [6] N. M. Ridler and A. G. Morgan, "New primary reference standard for vector network analyser calibration at millimetre wavelengths in coaxial line," *Measurement Science and Technology*, vol. 19, 065103, 2008.
- [7] W C Daywitt, "First-order symmetric modes for a slightly lossy coaxial transmission line", *IEEE Trans MTT-38*, No 11, pp 1644-1651, Nov. 1990
- [8] S Ramo, J R Whinnery & T Van Duzer, "Fields and waves in communications electronics", 3<sup>rd</sup> Edition, Wiley, 1994
- [9] M. Horibe, N. Ridler, M. Salter and C. Eio, "Characterization and Verification of Coaxial Open-circuit Primary Standards for Millimeter-wave Vector Network Analyzer Calibration," in *76th ARFTG Microwave Measurement Conference digest*, pp 89-100, Marriott Suites on Sand Key, Clearwater Beach, FL, USA, 2010.
- [10] N. M. Ridler and J. C. Medley, "An uncertainty budget for VHF and UHF reflectometers", *NPL Report DES 120*, Appendix B, 1992.
- [11] "IEEE Standard for Precision Coaxial Connectors (DC to 110 GHz)", IEEE std 287 (2007)
- [12] T E Hodgetts, "The calculation of the equivalent circuits of coaxial-line step discontinuities", RSRE Memorandum No 3422, Royal Signals and Radar Establishment, Great Malvern, 1981
- [13] D. Rytting, "An Analysis of Vector Measurement Accuracy Enhancement Techniques," in *RF & Microwave Symposium and Exhibition*, 1980.
- [14] D. Rytting, "Network Analyzer Error Models and Calibration Methods," in *RF & Microwave Measurements for Wireless Applications (ARFTG/NIST Short Course Notes)*, 1996.