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AN ULTRA-HIGH RESOLUTION FREQUENCY METER

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Summary

A novel instrument for measuring the frequency of a periodic signal contaminated by phase noise is presently being developed at the National Bureau of Standards. This frequency meter averages overlapping time intervals using a simple algorithm implemented with standard logic circuits. Because of the signal-to-noise improvement inherent in the averaging process, the standard deviation of a single measurement contaminated by white phase noise is proportional to $\tau^{-1.5}$, where τ is the time for the measurement. In contrast, the measurement uncertainty using a standard frequency counter is proportional to τ^{-1} in the white phase noise regime[1]. For many potential applications of the frequency meter, the measurement uncertainty due to contaminating noise may thereby be reduced by several orders of magnitude in comparison with a measurement over the same time interval using presently available instruments.

Introduction

For this analysis, we assume the test signal to be given by

$$V(t) = V_0 \sin[2\pi\nu t + \phi(t)] \quad (1)$$

where ν is the nominal frequency of the test signal, and where we explicitly neglect amplitude noise and assume the phase noise $\phi(t)$ ($|\phi| << 1$) to be white. The reference or clock signal at ν_0 is assumed to be noise free, and is taken to be the higher of the two frequencies. The reference frequency drives a clock counter whose value at any instant defines a time scale (or equivalently, a phase scale) t. The phase of the test frequency is used to derive a set of time values $\{t_i\}$ from the clock counter, where t_i is, e.g., the time of the i^{th} positive-going zero crossing of the test signal. In the absence of noise, the frequency ratio of the two signals is the slope of t_i vs. i. The effect of white phase noise in the test signal is to introduce uncorrelated errors in $\{t_i\}$ as shown in Figure 1.

Background

A conventional high resolution counter[2] counts the clock for a time interval τ, defined as a predetermined number (n-1) of cycles of the test signal. If the clock counter was initially cleared, then at the end of the time interval τ, it contains the value N, and the frequency of the test signal is given by

$$\nu = \frac{n-1}{N} \nu_0 \quad (2)$$

This method is equivalent to estimating the slope of the data in Figure 1 by drawing a straight line through the first and last points. If the clock quantization error of ±1 count is the dominant source of white phase noise, then the normalized frequency uncertainty equals the numerical resolution

$$\frac{\delta\nu_Q}{\nu} = \frac{1}{N} \simeq \frac{1}{\nu_0 \tau} \quad (3)$$

Alternatively, if the measurement is limited by white phase noise in the test signal, the normalized uncertainty is

$$\frac{\delta\nu_N}{\nu} = \frac{\delta\phi}{2\pi N} \simeq \frac{\delta\phi}{2\pi\nu_0\tau} \quad (4)$$

where the phase fluctuation amplitude is $\delta\phi$. For both situations, the measurement uncertainty is proportional to τ^{-1}.

Frequency Meter Algorithm

The frequency meter incorporates a simple hardware digital filter based on an algorithm previously used to find zero-slope points of interferometer fringes[3], and similar to an algorithm independently proposed by Gamlen[4]. As shown schematically in Figure 2, the data $\{t_i\}$, (i=1 to 2n) are divided into n overlapping time intervals $\{\Delta t_i\}$, each of length n cycles of the test signal where n is a predetermined value. Thus each of the n intervals is approximately half the length of the total interval. For white phase noise, the n intervals may be considered independent and their average length is given as

$$\langle N \rangle = \frac{1}{n} \sum_{i=1}^{n} \Delta t_i \quad (5)$$

The uncertainty in the computed average is reduced by the averaging process as $\sqrt{n}$ in comparison with the uncertainty in each interval. The frequency of the test signal is computed similarly to Eq. 2:

$$\nu = \frac{n}{\langle N \rangle} \nu_0 \quad (6)$$

Implementation

The algorithm may be implemented with commercially available TTL logic circuits. As shown in Figure 3, a clock at frequency ν_o ($\nu_o \gtrsim 20$ MHz) drives a counter which defines accumulated time. Strobe pulses derived from the test signal ($\nu \lesssim 1$ MHz) are used to sequentially latch the instantaneous values of the clock counter without interrupting its operation. During the total measurement time $\tau \equiv (2n-1)/\nu$, the strobed values of the clock counter are first subtracted from an initially cleared digital accumulator for the first n cycles of the test signal and then added to the accumulator for the last n cycles. At the end of the measurement interval τ, the accumulator contains the value

$$A = -\sum_{i=1}^{n} t_i + \sum_{i=n+1}^{2n} t_i = \sum_{i=1}^{n} \Delta t_i = n\langle N\rangle \qquad (7)$$

where t_i is the instantaneous value of the clock counter at the time of the i^{th} strobe pulse, and Δt_i is the length of the i^{th} interval in units of $1/\nu_o$. The test frequency is found from Eqs. (6) and (7) to be

$$\nu = \frac{n^2}{A}\nu_o \qquad (8)$$

The numerical resolution of the frequency meter is $1/A = 1/n\langle N\rangle$, where $\langle N\rangle$ is the average value of the n overlapping intervals. The uncertainty due to clock quantization noise is

$$\frac{\delta\nu_Q}{\nu} = \frac{1}{\langle N\rangle\sqrt{n}} = \frac{2}{\nu_o\tau}\sqrt{\frac{2}{\nu\tau}} \qquad (9)$$

and the uncertainty due to white phase noise in the signal is

$$\frac{\delta\nu_N}{\nu} = \frac{\delta\phi}{2\pi\langle N\rangle\sqrt{n}} = \frac{2\delta\phi}{2\pi\nu_o\tau}\sqrt{\frac{2}{\nu\tau}} \qquad (10)$$

From Eqs. (3) and (4), we see that for the same measurement interval $\tau \equiv (2n-1)/\nu$, the frequency meter reduces the uncertainty due to clock quantization or other white phase noise by $\sim\sqrt{n}/2$ in comparison with conventional frequency counting techniques. In addition, unlike the situation with conventional counters the numerical resolution is a factor of $\sqrt{n}$ finer than the clock quantization uncertainty. The benefit of this very high numerical resolution is that quantization noise in a series of measurements will appear to be continuous rather than discrete.

Digital Filter Analysis

A conventional frequency counter that measures only the phase difference over the measurement interval τ may be described as a digital filter consisting of a negative-going delta function at t=0 and a positive-going delta at $t=\tau$. In the frequency domain, this filter has a simple $\sin(\omega)$ response; for small ω the filter differentiates the accumulated phase to give the frequency whereas for large ω the response oscillates between ± some constant. The time-domain character of the digital filter used in the frequency meter is a single cycle of a square wave of amplitude one and period τ. As shown in Figure 4, this filter has a frequency response of $\sin(\omega)^2/\omega$, and therefore also acts as a simple differentiator for small ν, but in addition is a low-pass filter with a cut-off at $\omega \sim 1/\tau$. It is the reduction in the high-frequency phase noise that is responsible for the improved resolution of the frequency meter.

There are a number of interesting extensions of these very simple digital techniques that involve convolutions of $\sin(\omega)$ (phase difference) filters with $\sin(\omega)/\omega$ (phase accumulation) filters that give additional stages of high-frequency phase noise reduction and/or different degrees of differentiation at the origin. For example, the $\sin(\omega)^3/\omega$ filter of Figure 5 gives directly the first difference of the frequency. This filter has been shown by Allan and Barnes[5] to resolve the ambiguity in the standard definition of the Allan variance between white phase and flicker phase noise in an oscillator.

Experimental Results

A preliminary version of the frequency meter has been tested by measuring the frequency of a 21 kHz signal generated by a low-phase-noise synthesizer[6] in comparison with its 10 MHz internal reference. Both the Allan variance for a conventional frequency counter and a similar variance for the frequency meter were computed using measurement times τ ranging from $\sim 10^{-3}$ to $\sim 10^{3}$ seconds. In Figure 6 sigma, the square root of the conventional Allan variance, has a slope of τ^{-1} due to the white phase noise of the clock quantization error. Sigma as measured by the frequency meter has a slope of $\tau^{-1.5}$ out to about 1 second where it changes to τ^{-1}. As shown by Allan and Barnes[5], the slope of τ^{-1} identifies a regime dominated by flicker phase noise. The abrupt change of the slope to τ^{+1} at about 100 seconds is due to a frequency noise peak with a period of about one hour and amplitude of $\sim$3 microhertz. This frequency error is shown in Figure 7. Its source is unknown but is believed due to temperature cycling within the synthesizer.

A hardware implementation of the frequency meter is under construction using standard TTL logic. This instrument will use clock frequencies above 20 MHz to measure frequencies as high as 1 MHz. An immediate application[7] of this instrument will be to the development of interferometric laser wavelength comparators with resolution exceeding a part in 10^{11}.

References:

[1]J.A. Barnes et al., "Characterization of Frequency Stability," NBS Technical Note 394, October 1970.

[2]For example, the Hewlett Packard Model 5345A. The use of company or brand names is for illustrative purposes only and does not imply any product endorsement by the National Bureau of Standards.

[3] J.J. Snyder, "Algorithm for Fast Digital Analysis of Interference Fringes," Appl. Opt. 19, 1223 (1980).

[4] M.W. Gamlen, to be published.

[5] D.W. Allan and J.A. Barnes, "A Modified 'Allan Variance' with Increased Oscillator Characterization Ability," 35th Proceedings of the Annual Symposium on Frequency Control, U.S. Army Electronics Command, Ft. Monmouth, NJ. Copies available from Electronic Industries Association, 2001 "I" Street, NW, Washington, D.C. 20006.

[6] Fluke Model 644A Frequency Synthesizer. The use of company or brand names is for illustrative purposes only and does not imply any product endorsement by the National Bureau of Standards.

[7] J.J. Snyder, T. Baer, L. Hollberg and J.L. Hall, presented at the Conference on Lasers and Electro-Optics, Washington, D.C., June 1981.

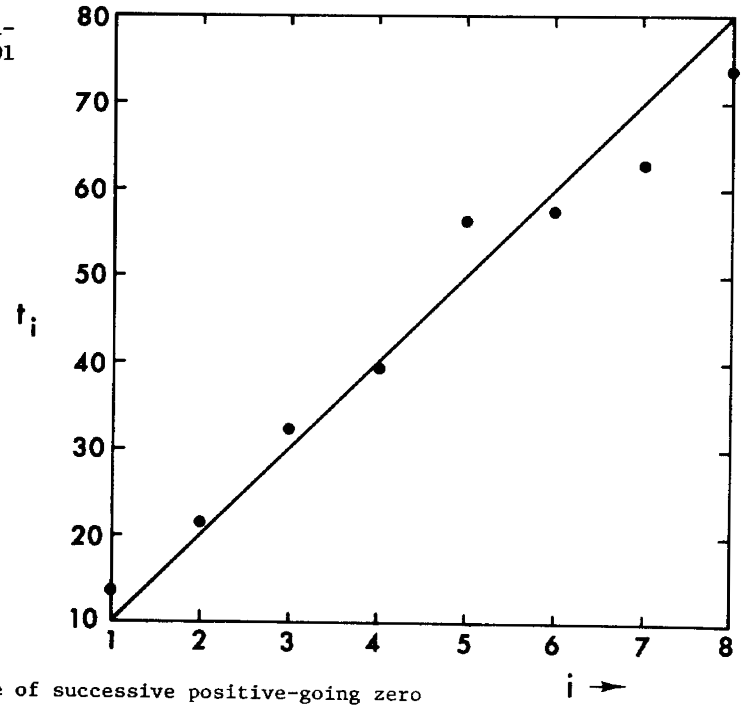

Figure 1: Plot of accumulated time of successive positive-going zero crossings of a simulated test signal. The clock frequency is 10 times the test signal frequency, and the test signal has white phase noise with standard deviation $\delta\phi = 0.6\pi$.

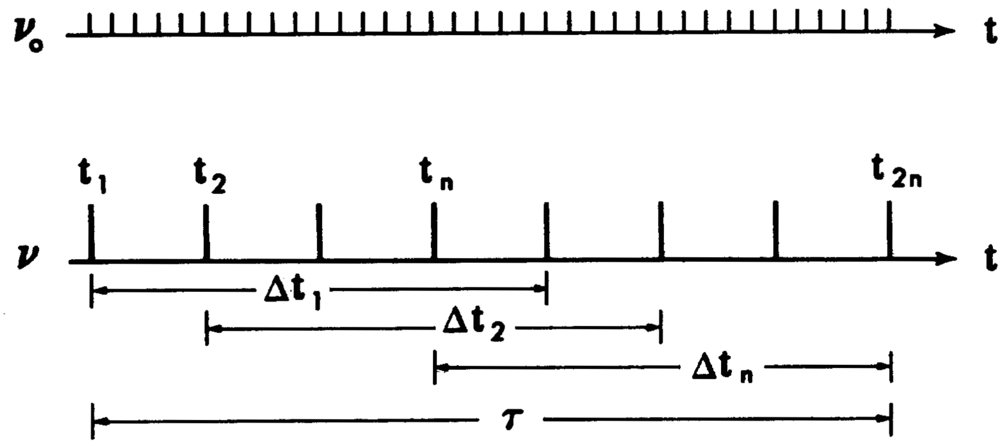

Figure 2: Characterization of frequency meter algorithm. The time scale is furnished by a counter counting the clock frequency ν_o. The total measurement interval $\tau \equiv (2n-1)/\nu$ is subdivided into n overlapping intervals $\{\Delta t_i\}$.

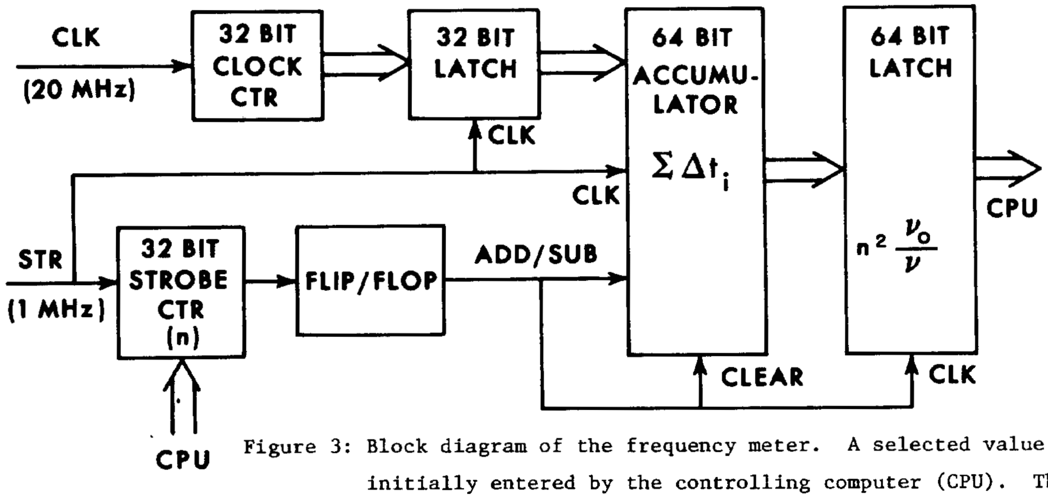

Figure 3: Block diagram of the frequency meter. A selected value for n is initially entered by the controlling computer (CPU). The value of the accumulator at the end of the measurement cycle is read by the computer and converted into a measurement of the strobe (STR) frequency using the known value of the clock (CLK) frequency.

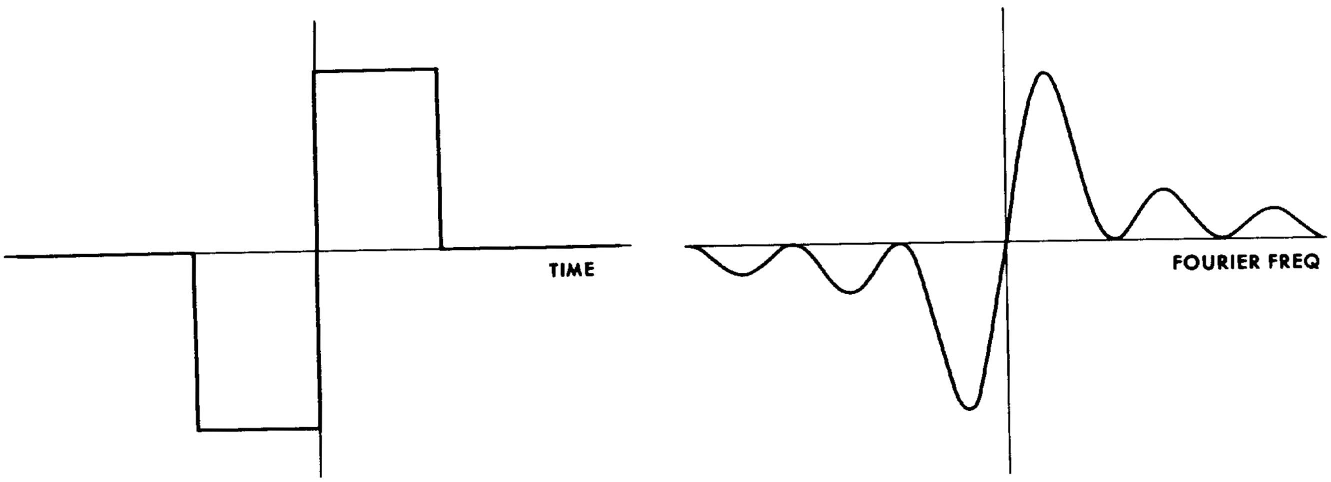

Figure 4: Transform pair for the frequency meter. The impulse response (a) is a single cycle of a square wave, implying a frequency response (b) of $\sin(\omega)^2/\omega$.

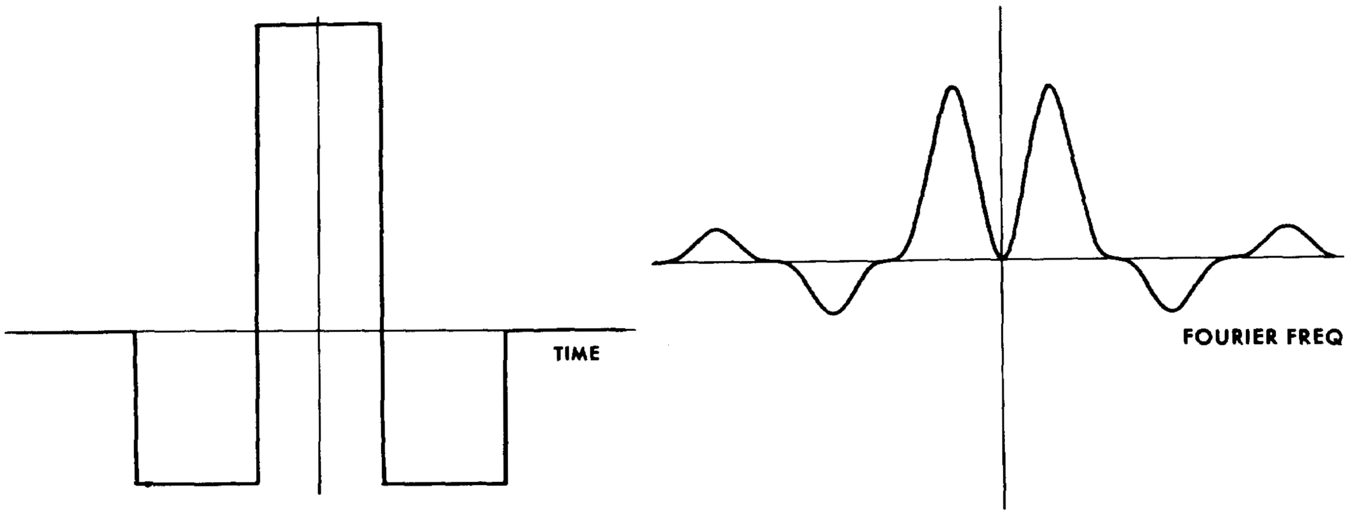

Figure 5: Transform pair for a frequency difference (variance) filter. This filter distinguishes between white phase and flicker phase noise[5]. Impulse response (a) and frequency response (b).

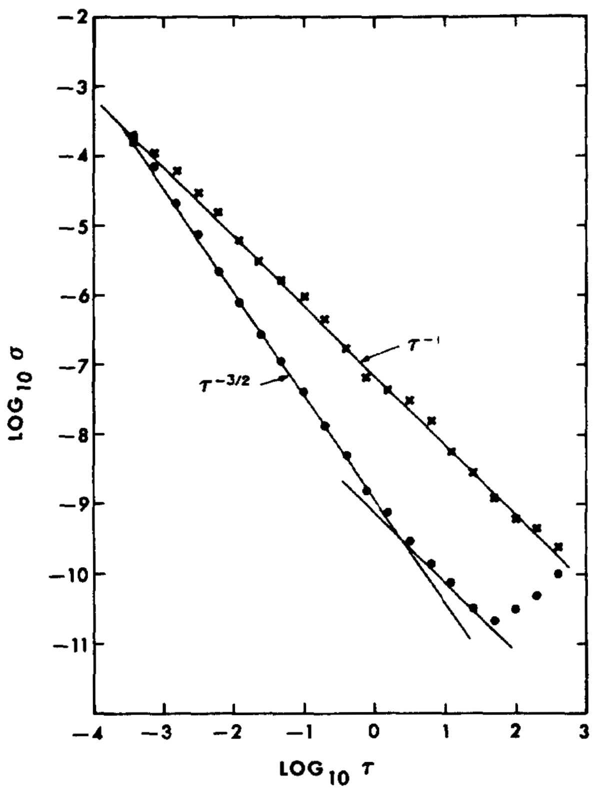

Figure 6: Frequency stability of a synthesized 21 kHz signal compared with its internal reference at 10 MHz. Sigma, the square root of the conventional Allan variance (crosses) decreases as τ^{-1} for white or flicker phase noise. Sigma for the frequency meter (dots) decreases as $\tau^{-1.5}$ for white phase noise and as τ^{-1} for flicker phase noise[5]. The increase in sigma for $\tau > \sim 100$ seconds is due to a frequency noise peak with a period ∿one hour (see Figure 7). Instrumental dead time ∿0.1 seconds.

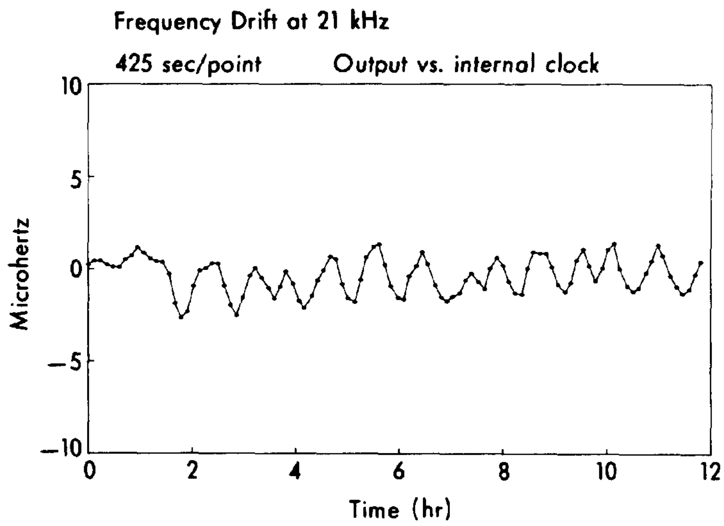

Figure 7: Frequency drift of a synthesized 21 kHz signal compared with its internal reference at 10 MHz. Source of the drift is unknown.