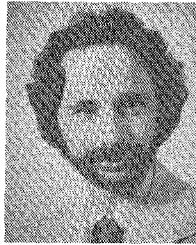
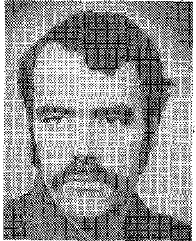


David P. Laude received the B.S. degree in electrical engineering from Florida Atlantic University, Boca Raton, in 1976, and the M.E. degree in 1977, also from Florida Atlantic University.

In April of 1977, he joined the Analog Products Group of Harris Semiconductor, Melbourne, FL, where he has been engaged in the design of precision monolithic voltage references, a precision operational amplifier, and subscriber line interface circuits.



James D. Beasom received the B.S. (with distinction) in engineering science from Pennsylvania State University, University Park, and the M.S. in electrical engineering from the University of Michigan, Ann Arbor.

He joined Harris Semiconductor in July 1966 as a Senior Engineer assigned to a process and device development group. While a member of that group, he studied thermal oxides and developed processes for the fabrication of n-channel and p-channel MOSFET's. Subsequently, he joined the Product Engineering Group as a Circuit Engineer working on the development of three custom, radiation-hardened digital circuits. Later, he transferred to the Device Group of the Engineering Department. His assignments in that group included linear p-n-p design; development of complementary linear device processes; development of processes for the fabrication of n-p-n's and FET's in a monolithic substrate; design of hardened TTL devices; and development of a number of linear and digital circuits employing these processes. Currently, he is the Manager of the Analog Technology Department. He directs and participates in the design and development of new linear devices and processes. His group is responsible for the initial device design and process development for all Harris Semiconductor linear circuits. It is also responsible for pilot wafer fabrication of these products to prove yields and reliability before they are released for production. He is currently developing processes for the fabrication of high-frequency and high-current complementary linear devices and the fabrication of complementary bipolars and FET's in a monolithic substrate. He is also developing a GHz frequency range MOS IC process. His patents include: an improved field effect transistor, a process for simultaneous fabrication of n-p-n's, p-n-p's, and MOS devices, a process for forming buried layers to reduce collector resistance in top contact transistors, and an improved semiconductor device (quasi-vertical transistor).

Accurate Analysis of Temperature Effects in I_C–V_{BE} Characteristics with Application to Bandgap Reference Sources

YANNIS P. TSIVIDIS, MEMBER, IEEE

Abstract—**The inaccuracy of the analyses commonly used for predicting the temperature behavior of the I_C-V_{BE} characteristics of transistors and the output of bandgap reference sources is pointed out. The problem is traced to a basic assumption implicit in such analyses, namely that the variation of the bandgap voltage of silicon with temperature is linear; this assumption is shown to be of poor accuracy. By taking into account the nonlinearity in this variation, new accurate formulas are derived. Both the previous analyses and the proposed analysis are compared to experiment; a valuable improvement is demonstrated. Equations which should prove of value in the design of bandgap reference sources and bipolar transistor temperature transducers are given. Higher order effects are discussed.**

I. Introduction

An accurate evaluation of the temperature effects in I_C-V_{BE} characteristics is needed in the design of bandgap reference sources and temperature transducers implemented in bipolar technology [1]-[8]. Bandgap reference sources are extensively used in A/D and D/A converters, voltage regulators, and measurement systems; temperature transducers find applications in instrumentation and measurement.

In the equations usually found in the literature for the temperature dependence of I_C-V_{BE} characteristics and the output of bandgap reference sources [2]-[4], [7], [9]-[11], two parameters appear: the extrapolated bandgap voltage of silicon to 0 K, and a process-dependent parameter denoted by η [3], [11] or a variety of other symbols. It is found that when the values expected from physics are used for these parameters, the equations do not agree with measurements [3], [7], [11]-[16]; for this reason, ficticious values are often used for these parameters in an attempt to make the equations fit the experimental data.

In this paper, it will be shown that the above discrepancies between theory and experiment are due to an assumption which, explicitly or implicitly, is made in the derivation of the equations used: that the variation of the bandgap voltage of silicon with temperature is linear. This assumption will be shown to be not justified for the level of accuracy commonly

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The author is with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA 02139, on leave from the Department of Electrical Engineering, Columbia University, New York, NY 10027.

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desired in the design of bandgap reference sources and temperature transducers. An accurate analysis will then be presented, in which the nonlinear variation of the bandgap voltage with temperature is taken into account. When such an analysis is used in evaluating a circuit, it becomes possible to clearly identify the part of the temperature variations attributable to nonidealities, such as resistor temperature coefficients, op amp offset voltage drift, etc; thus, the task of identifying, and reducing the effect of such nonidealities becomes easier. It is believed that the proposed analysis will lead to the design of bandgap reference sources and temperature transducers with improved performance.

II. Reasons for the Inaccuracy in Previous Analyses

In the equations used in previous analyses [2]-[4], [7], [9]-[11], the bandgap voltage of silicon enters solely through a related quantity, the *extrapolated bandgap voltage to 0 K*, usually denoted by V_{GO} (or V_{go}), which is defined as the extrapolated bandgap energy to 0 K divided by the electron charge. To see the reason for the inaccuracy in these analyses, one must consider the origin of V_{GO}. The collector current of an n-p-n transistor in the forward active region, neglecting the Early effect, is given by [17]:

$$I_C(T) = I_s(T) \exp\left(\frac{qV_{BE}}{kT}\right) \tag{1}$$

where T is the absolute temperature, I_C the collector current, V_{BE} the base-emitter voltage, q the electron charge, k the Boltzmann constant, and $I_s(T)$ is given by

$$I_s(T) = \frac{qAn_i^2(T)\,\overline{D}(T)}{N_B} \tag{2}$$

with A the base-emitter junction area, $n_i(T)$ the intrinsic carrier concentration, $\overline{D}(T)$ the "effective" minority carrier diffusion constant in the base, and N_B the Gummel number (total number of impurities per unit area in the base). For $n_i^2(T)$, the expression assumed in the above references (implicitly or explicitly) is of the form

$$\hat{n}_i^2(T) = BT^3 \exp\left(-\frac{qV_{GO}}{kT}\right) \tag{3}$$

where B is a constant; the hat above $n_i^2(T)$ is used to indicate that the expression is inaccurate, and to distinguish it from the more accurate expressions which will be given below; this practice will be followed throughout this paper.

To see why V_{GO} appears in (3), one should note that the exact expression for n_i^2 is actually of the form [17]

$$n_i^2(T) = ET^3 \exp\left[-\frac{qV_G(T)}{kT}\right] \tag{4}$$

where $V_G(T)$ is the *bandgap voltage at temperature T*; a plot of $V_G(T)$ is shown in Fig. 1. For many applications and in the usual operating ranges, this plot can be approximated by a straight line, also shown in Fig. 1 and denoted by $\hat{V}_G(T)$. If maximum accuracy is desired around a "reference" temperature T_r (often, but not necessarily, room temperature),[1] the approximating straight line should be taken tangent to the exact curve at $T = T_r$; therefore, $\hat{V}_G(T)$ will be given by

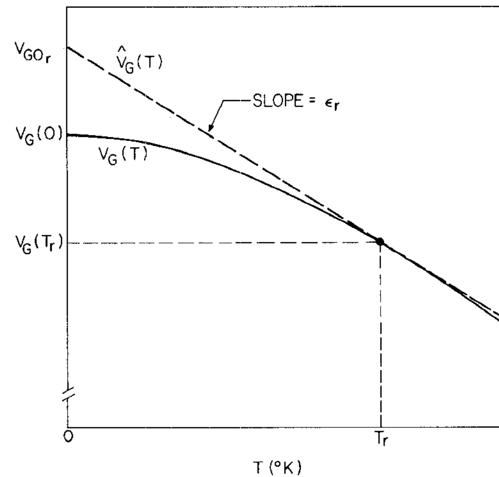

Fig. 1. Bandgap voltage versus absolute temperature and its first degree approximation (not to scale).

$$\hat{V}_G(T) = V_{GOr} + \epsilon_r T \tag{5}$$

where

$$\epsilon_r = \left(\frac{dV_G}{dT}\right)_{T=T_r} \tag{6}$$

$$V_{GOr} = V_G(T_r) - \epsilon_r T_r. \tag{7}$$

The quantity V_{GOr}, illustrated in Fig. 1, is what is commonly denoted by V_{GO}. We make two observations:

1) V_{GOr} is the extrapolated bandgap voltage to 0 K; hence, the subscript 0. Calling it "the bandgap voltage" or "the bandgap voltage at 0 K," as is sometimes done, can lead to confusion.

2) The value of V_{GOr} is not unique, but instead depends on the temperature T_r at which the straight line is tangent to the exact curve; hence, the subscript r in V_{GOr}.

From experimental data for $V_G(T)$ (to be discussed in Section VIII) it follows that the nonlinearity in its temperature variation is more severe at low temperatures, and therefore, it is at those temperatures that (5) is expected to be most inaccurate; this is illustrated in Fig. 1.

If now $\hat{V}_G(T)$ from (5) is substituted for $V_G(T)$ in (4), the approximate equation (3) is obtained, with $V_{GO} = V_{GOr}$ and $B \equiv E \exp(-q\epsilon_r/k)$. Thus, were the slope of $V_G(T)$ vs. T fixed, as (5) would imply, its value ϵ_r would be absorbed into a multiplicative *constant* in the expression for $n_i^2(T)$ in (3), and therefore, in the expression for $I_s(T)$ in (2). Hence, it would not affect the value of $I_s(T)/I_s(T_r)$, since it would cancel out in it; this ratio will be shown to appear directly in the expression for $V_{BE}(T)$. As will be seen, however, (5) is of poor accuracy; the error involved in it appears directly as an

[1] This temperature is often denoted by T_0; the subscript 0 is unfortunate, since it has already been used to denote "0 K" in V_{GO}, and obviously T_0 has nothing to do with 0 K.

error in the expression for the base-emitter voltage, and, consequently, as an error in the expression for the output of bandgap reference sources. As a result, large discrepancies between the temperature variation predicted for such outputs and that actually measured are common. In addition, this error masks the various nonidealities and makes it difficult to track them down.

III. An Accurate Evaluation of the Base-Emitter Voltage Temperature Dependence

Accurate equations will now be developed starting from (1). Let us consider two temperatures: an arbitrary temperature T and a specified reference temperature T_r. Applying (1) for each temperature, one can derive the following equation:

$$V_{BE}(T) = \left(\frac{T}{T_r}\right)\left\{V_{BE}(T_r) + \left(\frac{kT_r}{q}\right)\ln\left[\frac{I_s(T_r)}{I_s(T)}\cdot\frac{I_C(T)}{I_C(T_r)}\right]\right\} \tag{8}$$

This is the most general equation one can obtain before any approximations are made; it is accurate, to the extent that (1) is. Let us now use (2) in (8), with $n_i^2(T)$ as given by (4). In applying these equations at T and T_r, we will make the common approximation that the quantity E in (4) is a constant (this will be discussed further in Section VII). Also, we will define an "effective" mobility, $\bar{\mu}(T)$ for minority carriers in the base, by

$$\bar{\mu}(T) = \frac{q\bar{D}(T)}{kT}. \tag{9}$$

Using the above approximation and definition, (8) can be written as follows:

$$V_{BE}(T) = V_G(T) - \left(\frac{T}{T_r}\right)V_G(T_r) + \left(\frac{T}{T_r}\right)V_{BE}(T_r) + \frac{kT}{q}\ln\left[\left(\frac{T_r}{T}\right)^4\frac{\bar{\mu}(T_r)}{\bar{\mu}(T)}\frac{I_C(T)}{I_C(T_r)}\right]. \tag{10}$$

This equation is valid for any form of temperature dependence for $\bar{\mu}(T)$. If $\bar{\mu}(T)$ can be represented with sufficient accuracy by [3]:

$$\bar{\mu}(T) = CT^{-n} \tag{11}$$

with C and n appropriate constants, then (10) can be written as

$$V_{BE}(T) = V_G(T) - \left(\frac{T}{T_r}\right)V_G(T_r) + \left(\frac{T}{T_r}\right)V_{BE}(T_r) - \eta\left(\frac{kT}{q}\right)\ln\left(\frac{T}{T_r}\right) + \left(\frac{kT}{q}\right)\ln\left[\frac{I_C(T)}{I_C(T_r)}\right] \tag{12}$$

where $\eta \equiv 4 - n$. The assumption (11) will be discussed further in Section VII.

For the purposes of the following section, it will be instructive to express $V_{BE}(T)$ as the sum of a constant term, a term linear in T, and higher order terms. If a Taylor expansion at $T = T_r$ is performed on (12) and the terms are grouped together, we obtain

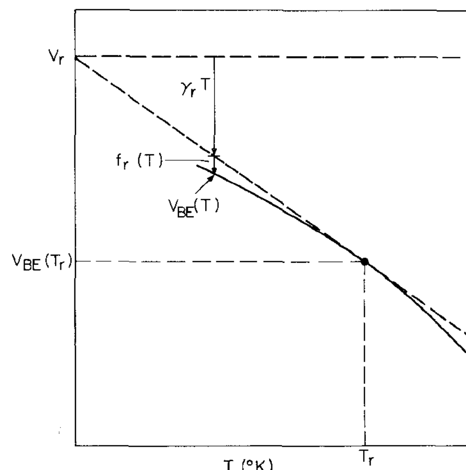

Fig. 2. Base-emitter voltage versus temperature and associated quantities (not to scale).

$$V_{BE}(T) = \underbrace{V_r}_{\text{constant term}} + \underbrace{\gamma_r T}_{\text{linear term}} + \underbrace{f_r(T)}_{\text{higher order terms}} \tag{13}$$

where the constant term is given by

$$V_r = V_{GOr} + (\eta - \theta_r)\left(\frac{kT_r}{q}\right) \tag{14}$$

with

$$\theta_r \equiv \left.\frac{\left(\frac{dI_C}{dT}\right)}{\left(\frac{I_C}{T}\right)}\right|_{T=T_r}. \tag{15}$$

The coefficient of the linear term in (13) is given by

$$\gamma_r = \left(\frac{dV_{BE}}{dT}\right)_{T=T_r} \tag{16a}$$

$$= -\left[\frac{V_r - V_{BE}(T_r)}{T_r}\right] \tag{16b}$$

and the group of higher order terms in (13) is given by

$$f_r(T) = V_G(T) - V_{GOr} + \left(\frac{T}{T_r}\right)[V_{GOr} - V_G(T_r)] - (\eta - \theta_r)\left(\frac{kT_r}{q}\right) + (\eta - \theta_r)\left(\frac{kT}{q}\right) - \eta\left(\frac{kT}{q}\right)\ln\left(\frac{T}{T_r}\right) + \left(\frac{kT}{q}\right)\ln\left[\frac{I_C(T)}{I_C(T_r)}\right]. \tag{17}$$

The geometrical interpretation of (13) and the quantities involved in it is shown in Fig. 2. It is clear that V_r, γ_r, θ_r and $f_r(T)$ depend on the choice of the reference temperature T_r.

The simplified form of the above equations for special forms of $I(T)$ (e.g., for I proportional to T), will be considered in Section V.

IV. Analysis of Bandgap Reference Sources

In bandgap reference sources a *correction voltage* $V_c(T)$ is added to $V_{BE}(T)$ in an attempt to cancel, or at least reduce, its temperature dependence. The output voltage of the reference source $V_{out}(T)$ is then

$$V_{out}(T) = V_{BE}(T) + V_c(T). \tag{18}$$

A. Correction Voltage PTAT

A common form of $V_c(T)$ is proportional to absolute temperature (abbreviated as PTAT). For such a voltage to cancel the linear term in (13), we must have

$$V_c(T) = -\gamma_r T \tag{19}$$

where γ_r can be found from (16b), and $V_c(T)$ can be generated by well-known circuit techniques [2]-[5]. $V_{out}(T)$ now becomes

$$\boxed{\begin{aligned} V_{out}(T) &= V_G(T) + \left(\frac{T}{T_r}\right)[V_{GOr} - V_G(T_r)] \\ &\quad - \eta\left(\frac{kT}{q}\right)\ln\left(\frac{T}{T_r}\right) + (\eta - \theta_r)\left(\frac{kT}{q}\right) \\ &\quad + \left(\frac{kT}{q}\right)\ln\left[\frac{I_C(T)}{I_C(T_r)}\right] && (20a) \\ &= V_r + f_r(T) && (20b) \end{aligned}}$$

where V_r and $f_r(T)$ are given by (14) and (17). Notice that, since (13) was obtained by a Taylor expansion at $T = T_r$, both $f_r(T)$ and its first derivative will be equal to 0 at $T = T_r$. Therefore, from (20b) we have

$$\boxed{\begin{aligned} &V_{out}(T_r) = V_r && (21) \\ &\left.\frac{dV_{out}}{dT}\right|_{T=T_r} = 0 && (22) \end{aligned}}\ .$$

The variation of V_{out} with temperature is defined by

$$\Delta V_{out}(T) \equiv V_{out}(T) - V_{out}(T_r). \tag{23}$$

From (20b), (21), and (23) we have

$$\boxed{\Delta V_{out}(T) = f_r(T)}\ . \tag{24}$$

B. Compensation of Higher Order Terms

A better reference source can be obtained if both the linear and the higher order terms in (13) are eliminated. This can be done in two ways; one of them is to use $V_c(T)$ to cancel all temperature-dependent terms. From (13) and (18), the required $V_c(T)$ is

$$V_c(T) = -\gamma_r T - f_r(T) \tag{25}$$

which gives

$$\boxed{\begin{aligned} &V_{out}(T) = V_r, \quad \text{all } T && (26) \\ &\frac{dV_{out}}{dT} = 0, \quad \text{all } T && (27) \\ &\Delta V_{out}(T) = 0, \quad \text{all } T && (28) \end{aligned}}\ .$$

Another way is to use $V_c(T)$ as given by (19), and to make $I_C(T)$ vary in such a manner that $f_r(T)$ is eliminated. The required form of $I_C(T)$ can be found by setting $f_r(T) = 0$ in (17). In principle, this would again result in (26)-(28). In practice, it is found that a satisfactory form for $I_C(T)$ is $I_C(T_r)(T/T_r)^b$ where b is an appropriate constant [7].

The accurate expressions given for $f_r(T)$ in this paper will hopefully lead to systematic circuit techniques for eliminating the higher order terms.

V. A Common Special Case

Let us assume that the collector current is proportional to some power of T [10]:

$$I_C(T) = FT^{\delta}. \tag{29}$$

Using this in (12), we obtain a simplified expression for $V_{BE}(T)$:

$$\boxed{\begin{aligned} V_{BE}(T) &= V_G(T) - \left(\frac{T}{T_r}\right)V_G(T_r) + \left(\frac{T}{T_r}\right)V_{BE}(T_r) \\ &\quad - (\eta - \delta)\left(\frac{kT}{q}\right)\ln\left(\frac{T}{T_r}\right) \end{aligned}}\ . \tag{30}$$

From (15), θ_r is now independent of temperature and is given by

$$\boxed{\theta_r = \delta}\ . \tag{31}$$

Equations (14) and (16b) are valid with θ_r given by (31). The expression for the higher order terms is, from (17)

$$\boxed{\begin{aligned} f_r(T) &= V_G(T) - V_{GOr} + \left(\frac{T}{T_r}\right)[V_{GOr} - V_G(T_r)] \\ &\quad - (\eta - \delta)\left(\frac{kT_r}{q}\right) - (\eta - \delta)\left(\frac{kT}{q}\right) \\ &\quad \cdot \left[\ln\left(\frac{T}{T_r}\right) - 1\right] \end{aligned}} \tag{32}$$

For most of the existing practical implementations, I_C is basically PTAT, in which case the above equations apply with $\delta = 1$.

VI. Analytical Expressions for the Bandgap Voltage

Measurements of the energy gap (qV_G) of several semiconductors versus temperature have been published by several authors. According to [18], an expression for $V_G(T)$ is

$$V_G(T) = V_G(0) - \frac{\alpha T^2}{T + \beta}. \tag{33}$$

For silicon the values for the constants in this equation are, according to [18], $\alpha = 7.021 \times 10^{-4}$ V/K and $\beta = 1108$ K. $V_G(0)$ is given in [18] as 1.1557 V, but as stated there should be corrected by the dissociation energy of the exciton, which at the time had not been accurately measured; using for the latter the value reported in [19], we obtain $V_G(0) = 1.1704$ V. In a later work [20], (33) has been compared to other results and it was suggested that the constants in it should instead have the values $\alpha = 4.73 \times 10^{-4}$ V/K, $\beta = 636$ K, and $V_G(0) = 1.170$ V. The uncertainties for these quantities are listed, respectively, as $\pm 0.25 \times 10^{-4}$ V/K, ± 50 K and ± 0.001 V. In addition to the disagreement between the values provided in these references, it should be noted that the values in both of them were chosen so as to match (33) to measurements taken from near 0 K to over 400 K. Notice that most of the nonlinearity of $V_G(T)$ is at low temperatures. As a result, the values chosen represent a compromise "best fit" from 0 K to 400 K, and are not really the values that one should choose when working with IC's, which usually operate between 200 K and 400 K.

In another reference [21], accurate measurements of $V_G(T)$ have been reported; these have been matched to within 0.2 mV by the following empirical relation:

$$\boxed{V_G(T) = a - bT - cT^2}. \tag{34a}$$

Unfortunately, the measurements in [21] were limited to below 300 K; according to [21], the constants in (34a) are

$$\boxed{\left.\begin{array}{l} a = 1.1785 \text{ V} \\ b = 9.025 \times 10^{-5} \text{ V/K} \\ c = 3.05 \times 10^{-7} \text{ V/(K)}^2 \end{array}\right\} \text{ for } 150 \text{ K} < T < 300 \text{ K}}. \tag{34b}$$

In lack of accurate measurements at higher temperatures and in view of the fact that most of the nonlinear variation of $V_G(T)$ occurs below 300 K anyway, we have extrapolated (34a) by a first degree polynomial for $T > 300$ K. A simple calculation using (34a) and (34b) gives, for such an approach

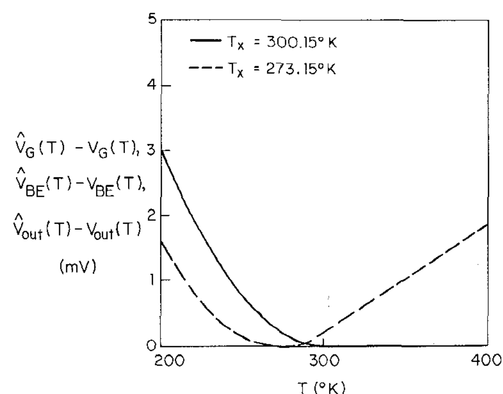

Fig. 3. Discrepancy between the approximate and the accurate expressions for the bandgap voltage, the base-emitter voltage, and the output of bandgap reference sources, for two choices of T_r.

$$\boxed{\left.\begin{array}{l} a = 1.20595 \text{ V} \\ b = 2.7325 \times 10^{-4} \text{V/k} \\ c = 0 \end{array}\right\} \text{ for } 300 \text{ K} \leqslant T < 400 \text{ K}} \tag{34c}$$

where all five decimal places in the value of a have been kept for the sake of continuity at $T = 300$ K, and do not imply a corresponding high accuracy. The linear variation of $V_G(T)$ for $\geqslant 300$ K, implied by (34a) and (34c), appears to be a reasonable assumption in view of experimental evidence [22]. The upper limit of validity of (34c) has been specified as 400 K in lack of experimental data for $T > 400$ K.

Thus, for temperatures above room temperature we consider (5) to be satisfactory; by comparing (5) to (34a) and (34c), it is obvious that, for $T_r \geqslant 300$ K, $V_{GOr} = a = 1.20595$ V, and $\epsilon_r = -b = -2.7325 \times 10^{-4}$ V/K. The value of V_{GOr}, thus obtained, is not far from the commonly assumed value of 1.205 V; the latter value has probably been obtained by extrapolating the high temperature part of the $V_G(T)$ curve. As seen from Fig. 1, the value of V_{GOr} for smaller values of T_r will be smaller, e.g., using (7) and (34) it is found to be 1.2013 V at 0°C (273.15 K).

It should be noted that although (34a), (34b) matches the measurements shown in [21] to within 0.2 mV, the uncertainty in the measurements themselves is given as about 1 mV. This uncertainty is due to the uncertainties in the energies of the exciton and the *TO* phonon; if these energies are assumed to have a negligible temperature variation [23], then the 1 mV uncertainty in $V_G(T)$ is all due to an uncertainty of 1 mV in the value of a only in (34), and therefore, the temperature *variation* predicted by that equation can be assumed to be very accurate, at least for 150 K $< T <$ 300 K.

The discrepancy between the approximate expression, $\hat{V}_G(T)$, given by (5), and the accurate expression (34), is shown in Fig. 3. The situation for two different choices of T_r is shown. As will be mentioned in Section VIII, the same curves show the corresponding discrepancy for $V_{BE}(T)$ and $V_{out}(T)$.

VII. Discussion of the Approximations Made and Higher Order Effects

The equations given in Section V are sufficient as long as the bias current is proportional to some power of T over the whole range of interest. However, this is not always the case; for example, the temperature dependence of the resistors [4] used to set the current can result in an $I_C(T)$ which cannot be adequately described by (29). In such a case, the more general formulation of (12)-(17) should be used. To consider the validity of *these* equations now, the following comments are in order.

1) $\bar{\mu}$ is an "effective" mobility of the *minority* carriers in the base; however, the bulk of the existing data on mobility is for *majority* carriers instead. We can make here a common assumption, namely that the temperature dependence of the minority carrier mobility is the same as that observed when the same carriers are in a material of the opposite polarity and the same doping concentration [24].

2) Even if the above assumption is made and (11) is used, the constants entering this equation depend on the impurity concentration [17]. For a standard pnp device, it is reasonable to assume that the concentration is constant throughout the base; however, for a standard process npn device the impurity concentration varies widely with depth, and therefore, so do the constants in the mobility expression. Now, $\bar{\mu}(T)$ is a single *effective* mobility, which includes the effect of all individual mobilities at various depths in the base (this follows from (9) and the derivation of (2) [17]). It does not necessarily follow, then, that what is true for the mobility at a specific impurity concentration is true for the effective mobility.

3) Even if $\bar{\mu}(T)$ can be represented by (11), the temperature range over which this equation is accurate is open to question. The CT^{-n} type temperature dependence is usually specified as "around room temperature."

In view of the above comments, the validity of (11) is not unquestionable. If there is reason to believe that it is not accurate, and $\bar{\mu}(T)$ is approximated by a different function, then the more general equation, (10), should be used

Let us now consider the intrinsic carrier concentration. The quantity E in (4) is proportional to the third power of the average density of states effective mass [17]; this, in turn, is a function of temperature [20], and therefore E is not really a constant. Expressions for $n_i^2(T)$ more complex than (4) do exist, but a significant improvement over (4) is not evident (see the summary on n_i in [20]).

For some fabrication processes, deviation from the basic relation (1) is observed; instead, devices made by these processes are represented by

$$I_C(T) = I_s(T) \exp\left(\frac{V_{BE}}{lkT}\right) \tag{35}$$

where l is slightly greater than 1, can vary with both current and temperature, and from device to device over the same wafer. In such cases, the accuracy of the proposed equations is likely to decrease.

Problems can also develop due to the parasitic resistances in series with the collector, base, and emitter. For example, let us assume that only the collector series parasitic resistance R_C is significant, and that it is linear. The intrinsic collector-emitter voltage will be

$$V_{C'E} = V_{CE} - R_C I_C. \tag{36}$$

This can become a problem, especially if V_{CE} is low, since then the slope of the I_C-$I_{C'E}$ curves can be significant; e.g., consider the case of a diode-connected device with I_C being PTAT. With increasing temperature, V_{CE} decreases and I_C increases; both of these effects are in the direction of decreasing $V_{C'E}$. Thus, the V_{BE} needed to support a given I_C can be higher than what would normally be required, and the variation of V_{BE} with temperature can be smaller than predicted. Parasitic resistances can be contributing factors to the nonideality associated with (35).

Finally, in practical implementations problems observed include the variation of the parameter η from device to device on the same wafer, as well as the effect of strain on the I_C-V_{BE} relation.

VIII. Experimental Results and Comparison to Proposed and Previous Analyses

To compare the analysis in this paper to previous analyses, let $V_G(T)$, $V_{BE}(T)$, and $V_{out}(T)$ denote the values of the bandgap voltage, the base-emitter voltage, and the reference source output voltage as given by the equations proposed here; let $\hat{V}_G(T)$, $\hat{V}_{BE}(T)$, and $\hat{V}_{out}(T)$ by the corresponding quantities derived if (5) is assumed [2]-[4], [7], [9]-[11].

Equation (5) represents an expansion at $T = T_r$; as such, it preserves correctly the value of both the bandgap voltage and its first derivative with respect to T *at* $T = T_r$. Because of this, the values of $V_{BE}(T)$ and $V_{out}(T)$ and their first derivatives *at* $T = T_r$ would also be preserved *if* there were a way to determine the other parameters in the equations, e.g., $(\eta - \delta)$ in (30). However, even in such a case, the temperature variation predicted for these voltages would be in error; from the analysis in this paper, it is easily shown that the error in all three quantities is equal:

$$\boxed{\begin{aligned}\hat{V}_{out}(T) - V_{out}(T) &= \hat{V}_{BE}(T) - V_{BE}(T)\\ &= \hat{V}_G(T) - V_G(T)\end{aligned}} \tag{37}$$

These errors, using (5) and (34), have been plotted versus T in Fig. 3, for $T_r = 273.15$ K and $T_r = 300.15$ K, corresponding to 0°C and 27°C. The errors shown in Fig. 3 are comparable to the temperature variation ΔV_{out} in (23), as predicted by the previous analyses, and are positive; thus, the previous analyses predict temperature variations ΔV_{out}, which can be much smaller than those predicted by the proposed analysis. This is illustrated in Fig. 4.

A comparison to measurements has been made using experimental data on npn transistors made using five different processes, labeled I through V. The data for process I were provided by R. C. Dobkin of National Semiconductor, Inc., and

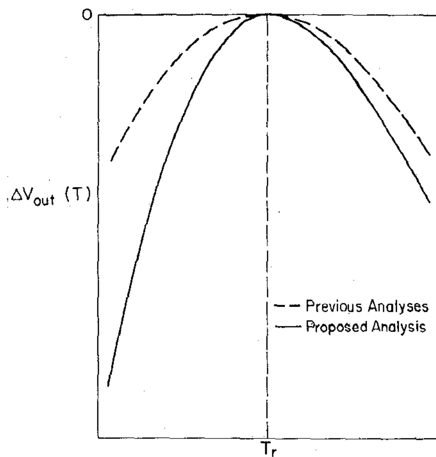

Fig. 4. Form of bandgap reference output voltage variation (23) for the proposed and the previous analyses.

Fig. 5. Discrepancy between theory and experiment; the constants in the equations have been evaluated as explained in the text. Results for five different processes are shown. Circles indicate proposed theory; crosses indicate previous theory. For processes II, III, and IV, $l \neq 1$ (see text).

are for I_C proportional to T (PTAT); those for processes II, III, and IV were provided by M. Timko of Analog Devices, Inc., and are also for I_C PTAT. The data for process V were taken from [7], and are for a constant I_C. The measurements have been compared to the proposed equation (30), and to the equations used elsewhere [2]-[4], [7], [9]-11]; the latter can be written as follows:

$$\hat{V}_{BE}(T) = V_{GOr}\left(1 - \frac{T}{T_r}\right) + \left(\frac{T}{T_r}\right) V_{BE}(T_r) - (\eta - \delta)\left(\frac{kT}{q}\right)\ln\left(\frac{T}{T_r}\right). \tag{38}$$

For each equation and each set of data, $V_{BE}(T_r)$ as given from the measurements has been used. To the best of the author's knowledge, there is no way to accurately calculate the value of η (and hence, that of $\eta - \delta$) from process data; hence, for both the proposed equation (30) and the previous equation (38), $(\eta - \delta)$ has been calculated by writing these equations for T equal to the lowest measurement temperature, with V_{BE} as measured, and solving for $(\eta - \delta)$. Using the value found by this procedure, $V_{BE}(T)$ has been evaluated for each measurement temperature using both equations. Notice that, because of the way $(\eta - \delta)$ was calculated, both equations will agree with experiment at the lowest temperature measurement; however, *no* attempt has been made to adjust the parameters to achieve a best overall curve fitting.

A comparison is shown in Fig. 5. For each process, the deviation of the theoretical equation from the measurements is shown for the proposed analysis [circles, (30)] and the previous analysis [crosses, (38)]. The advantage of (30) is apparent.

The accuracy of the proposed analysis for temperatures below 300 K, where the accurate constants in (34b) can be used, is within the accuracy specified for (34a) in [21], i.e., 0.2 mV; actually, it is must better than that for most of the data.

At higher temperature, although the advantage of (30) is still clear, the accuracy of that equation decreases for processes II, III, and IV; this is because these processes clearly exhibit the nonideality represented by (35); in fact, the parameter l was observed to vary significantly with both current and temperature. Thus, by assuming (30) was valid and by calculating $(\eta - \delta)$ at the lowest temperature, what we have really done is effectively calculate $l(\eta - \delta)$ at that point; this value is not accurate at other temperatures. Finally, the temperature for these three processes was measured with an accuracy of about 0.1°C; this corresponds to an error in V_{BE} of about 0.2 mV.

The values found for η by means of the procedure explained above using the accurate equation (30), were 2.330, 2.303, 2.286, 2.316, and 2.405, respectively, for processes I, II, III, IV, and V. Attempts to match the approximate equation (38) by curve-fitting the experimental data result instead in fictitious, large values for η [7], [11]-[14], [16]; this is because in curve-fitting (38) a large value of η is needed so that the temperature variation of the last term in that equation can increase, in order to make up for the missing terms corresponding to the higher order terms in $V_G(T)$. Using the resulting high values of η, then, the only way that the approximate analysis can lead to the correct value for the output voltage at room temperature is to use a fictitious, low value for V_{GOr}, as can be seen from (14) and (21). Thus, for example, for process V the fictitious values of $\eta = 3.429$ and $V_{GOr} = 1.1761$ V must be used to make (38) match the measurements [7], instead of the correct values $\eta = 2.405$ and $V_{GOr} = 1.20595$ V used in conjunction with the accurate equation (30).

No attempt will be made to compare theory and experiment for the output of complete bandgap reference source circuits, because in practical implementations various *circuit* nonidealities must be taken into account; this author does not have access to such detailed information. However, it is obvious that if, e.g., a circuit implements (18) and (19) "exactly," then the agreement between theory and experiment for $V_{out}(T)$ will be as good as that for $V_{BE}(T)$, which has been investigated in detail above.

IX. Conclusions

It has been shown in this paper that the main cause of disagreement between measurements and commonly used equations for $V_{BE}(T)$ is due to the assumption that the variation of the bandgap voltage with temperature is linear. The error in this assumption appears directly as an error in predicting V_{BE}, as well as an error in predicting the output of bandgap reference sources.

Equations have been provided which take into account the nonlinearity in the variation of the bandgap voltage with temperature. In lack of accurate bandgap voltage measurements at high temperatures, a linear extrapolation of an existing low-temperature equation has been proposed. Very good agreement between the proposed equations and measurements has been demonstrated; the errors are of the order of the uncertainty in predicting the bandgap voltage variation, and the order of accuracy of the measurements, which both represent state of the art.

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Yannis P. Tsividis (S'71-M'70), for a photograph and biography, see this issue, p. 928.

Correspondence

Resistor Termination in D/A and A/D Converters

JEN-YEN HUANG

Abstract—**A new termination technique for diffused resistors is developed. This termination provides low contact resistance and little misalignment error. Data indicate that the termination is accurate to at least 10 bit. This makes possible the realization of D/A or A/D converters exceeding 10-bit performance while staying with low cost standard diffusion process.**

I. Introduction

Monolithic A/D converters using $2^N R$ resistance ladders have gained great popularity and have enjoyed tremendous commercial success in the past few years [1], [2].

One major reason for their success is the easy adaptability to standard digital MOS process. The end result is a low cost, self-contained and very easy to use A/D converter.

The ladder consists of 2^N resistors connected in series and an external reference voltage is applied directly across it. Each of the $2^N - 1$ connection points forms a voltage tap which, when selected, is directed to the output of a D/A converter or the input of a comparator in an A/D converter. This set of $2^N - 1$ voltages is obviously the most important factor in determining the accuracy of the converter, therefore, a set of 2^N well matched resistors must be built.

The cheapest way to build IC resistors is through diffusion and the matching among diffused resistors depends heavily on mask making tolerances and on process variations, for example, resistance gradient, body proximity effect, voltage coefficient, limitations of photolithography process, etc. Nevertheless, diffused resistors built on a standard MOS process designed for high volume digital circuits are more than adequate for an 8-bit converter. This is best shown by the popular ADC0816 8-bit A/D converter [1], in which a 256-R resistor string is diffused into the silicon through a standard process.

One straightforward way of building this resistor string is to drop a contact on each end of 256 individual resistors and then connect them in series with metallization. This, however, introduces 512 new variables due to contact resistance variations.

Contact resistance from aluminum to diffusion has been studied extensively [3]-[5]. For a typical 256 n^+ resistor string, contact resistance contributes approximately 25 percent of the total ladder resistance, and its variation across the ladder can become a significant error source.

To alleviate this problem one can use a long diffused string as shown in Fig. 1(a). There is no contact in series with the string except the one at each end (where they are connected to external pins). Thus, 512 variables are reduced to only two. But on these two remaining contacts, we have a new variable to consider in addition to the contact resistance variations, which is contact misalignment with respect to diffusion.

In Fig. 1(a), the spacings, $d1$ and $d2$, vary from lot to lot due to contact misalignment. This directly affects the resistance values of the first and last resistors, R_1 and R_{2N} [Fig. 1(b)]. Variation in R_1 contributes to zero-offset error (voltage error on tap 1) and that in R_{2N} to full-scale error (voltage error on tap $2^N - 1$). Typically, a 13 percent voltage error can occur on tap 1 from contact misalignment alone. In order to minimize the effects from these error sources, a better termination is needed to reduce the contact resistance and to eliminate the contact misalignment problem.

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The author is with National Semiconductor Corporation, Santa Clara CA 95051.

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