Application Report

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# Closed-Loop Analysis of Load-Induced Amplifier Stability Issues Using $Z_{OUT}$

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## ABSTRACT

This application note discusses techniques using amplifier closed-loop output impedance ($Z_{OUT}$) to solve load-dependent stability issues.

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## 1 Introduction

Amplifier stability is load-dependent. An amplifier that is stable with a resistive load may ring or oscillate with a reactive load.

The traditional method for evaluating the impact of loading on the stability of a closed-loop amplifier involves analyzing Bode plots of the amplifier loaded loop-gain function. The load element interacts with the amplifier open-loop output impedance ($Z_O$) to alter the frequency response of the amplifier loop gain function, and the available stability margins. Analyzing these load-induced changes in loop gain requires open-loop conditions or *breaking* the amplifier feedback loop. Breaking the loop is not possible in the case of closed-loop amplifier devices such as current-sense amplifiers because the feedback loop is internal to the device and cannot be manipulated.

Nevertheless, there is a closed-loop amplifier property that is affected by changes to loop gain, and can be used for stability analysis without breaking the loop. This property is the amplifier closed-loop output impedance ($Z_{OUT}$), an increasingly common specification in the data sheets and SPICE macro-models of TI current sensing and precision amplifier products. This application report demonstrates through TINA-TI SPICE simulations a method of using $Z_{OUT}$ to analyze the stability of an amplifier load transient response under various load conditions.

# 2 Basic Properties of Electrical Impedance

## *2.1 Measuring Impedance*

Before delving into the details of closed-loop stability analysis, a review of a few basic concepts relating to electrical impedance is helpful. Impedance is the current-to-voltage transfer gain over frequency, of an electrical circuit or component. The stimulus is an ac small-signal current input of variable frequency, and the response is the resulting change in voltage at the test frequency. Impedance is then calculated by applying Ohm's law to the recorded changes in current and voltage.

Figure 1 depicts the impedance test circuit. The unknown impedance ($Z_{BLOCK}$) is excited by an ac small-signal current ($I_{LOAD}$) of frequency f, with the other terminal driven to a fixed dc voltage ($V_{DC}$). If $V_{DROP}$(f) and $I_{LOAD}$(f) are both measured at time instants $t_1$ and $t_2$, then by Ohm's law, the following two equations are derived:

$$V_{DROP}(f)(t_1) = V_{DC} + I_{LOAD}(f)(t_1) \times Z_{BLOCK}(f) \tag{1}$$

$$V_{DROP}(f)(t_2) = V_{DC} + I_{LOAD}(f)(t_2) \times Z_{BLOCK}(f) \tag{2}$$

Subtracting Equation 2 from Equation 1 and solving for $Z_{BLOCK}$(f) yields Equation 3.

$$Z_{BLOCK}(f) = \frac{V_{DROP}(f)(t_1) - V_{DROP}(f)(t_2)}{I_{LOAD}(f)(t_1) - I_{LOAD}(f)(t_2)} = \frac{\Delta V_{DROP}(f)}{\Delta I_{LOAD}(f)} \tag{3}$$

$\Delta V_{DROP}$(f) and $\Delta I_{LOAD}$(f) represent *changes* in $V_{DROP}$ and $I_{LOAD}$ between time instants $t_1$ and $t_2$. Therefore, there is no difference in the value of $\Delta V_{DROP}$ whether $V_{DROP}$ is measured with respect to $V_{DC}$ or with respect to GND.

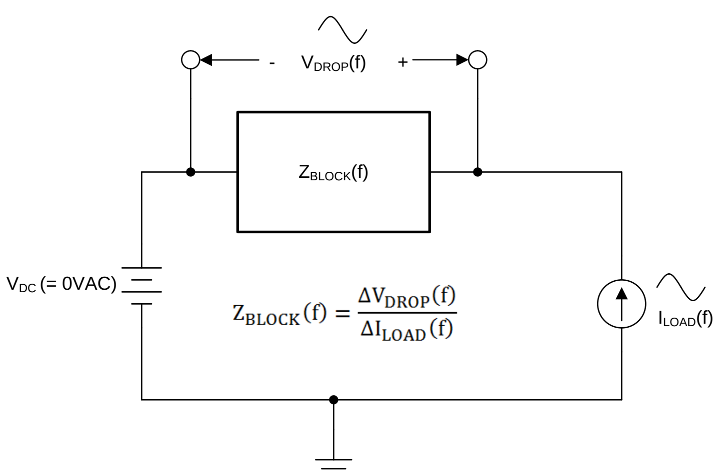


**Figure 1. Impedance Test Circuit**

Figure 2 shows the test circuit of Figure 1 configured for measuring the impedance of a capacitor ($C_1$). In this case, the response is measured directly relative to GND. The 1-GΩ resistor ($R_1$) is for simulation only and is required for dc convergence. Capacitor $C_1$ can be substituted with any two-terminal device to measure the impedance of the circuit.

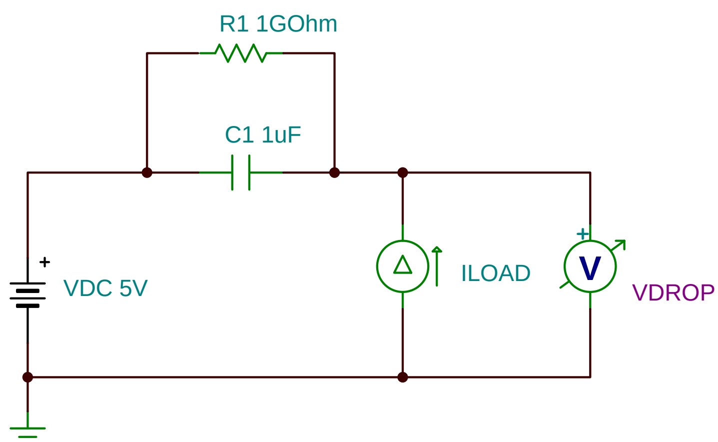


**Figure 2. Measuring $Z_{CAPACITOR}$**

## 2.2 Visualizing Impedance Using Bode Plots

Impedance, like any ac gain transfer function, is a complex function of frequency having magnitude and phase, and usually represented using Bode plots to simplify analysis. Figure 3 shows the Bode magnitude and phase plots of the ideal resistive (purely real) and reactive (purely imaginary) impedance elements.

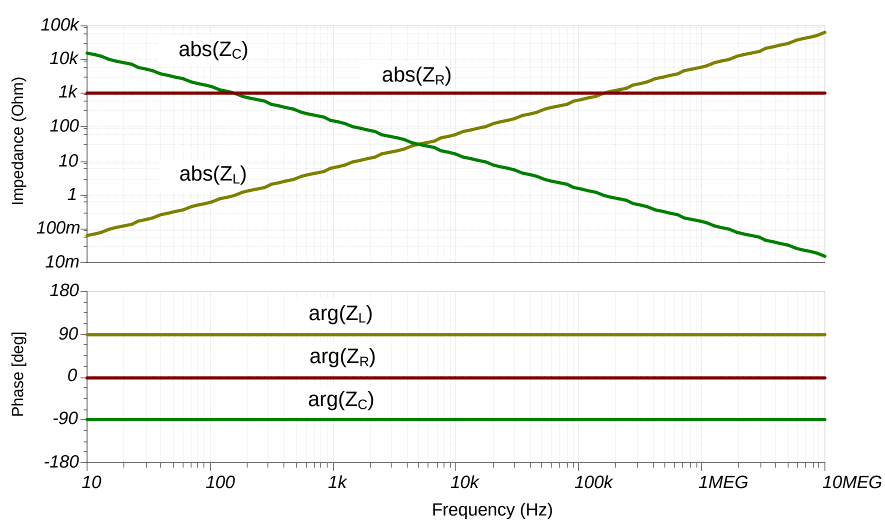


**Figure 3. Bode Plots of Ideal Resistive and Reactive Impedances**

Observe that the linear magnitude characteristic of the capacitor over frequency is the result of using logarithmic scales for the vertical and horizontal axes.

### 2.3 Poles of $Z_{EQ}$ Determine Load Transient Stability

Impedances combine in series or parallel configurations. The combination of resistive and reactive elements in a circuit produces poles and zeros in the Thévenin equivalent impedance function ($Z_{EQ}$). The locations of the poles of $Z_{EQ}$ in the complex s-plane determine the stability of the circuit response to load transients. For a detailed review of pole-zero analysis of system transfer functions, see *Understanding Poles and Zeros* .

For most circuits, a stable transient response is one that converges asymptotically to a finite, steady-state value without ringing. Based on Figure 4, the required transient response is obtained from a transfer function ($Z_{EQ}$) with poles that are not only all in the left half plane (LHP), but are also purely real (that is, lie on the negative real axis marked by the red line in Figure 4).

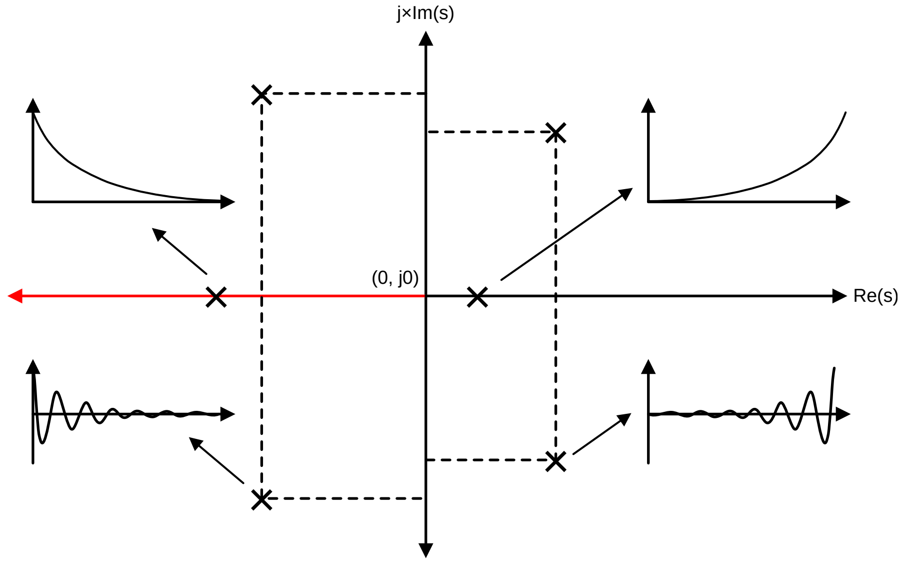


**Figure 4. Transient Response Based on Various Pole Locations in S-Plane**

### 2.4 Finding Poles and Zeros of $Z_{EQ}$

The poles and zeros of $Z_{EQ}$ can be found by solving for the roots of the denominator and numerator, respectively. For example, the $Z_{EQ}$ transfer function of the circuit in Figure 5 is given by Equation 4.

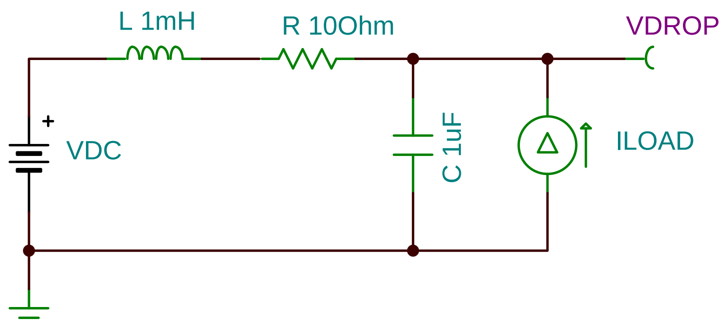


**Figure 5. Example Circuit With Both Series and Parallel Impedance Combinations**

$$Z_{EQ}(s) = (Z_L + Z_R) \| Z_C = \frac{R + sL}{1 + sRC + s^2 LC} \tag{4}$$

Table 1 summarizes the procedure to solve for the poles and zeros of $Z_{EQ}$.

**Table 1. Computation of Poles and Zeros of Equation 4**

| Poles of $Z_{EQ}$ | Zeros of $Z_{EQ}$ |
|---|---|
| $\text{Denominator}(Z_{EQ}(s)) = 0$ <br> $\Rightarrow 1 + sRC + s^2LC = 0$ <br> $\Rightarrow s_{p1,p2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ | $\text{Numerator}(Z_{EQ}(s)) = 0$ <br> $\Rightarrow R + sL = 0$ <br> $\Rightarrow s_{z1} = -\frac{R}{L}$ |
| Substituting Values of R, C and L From Figure 5: | |
| $s_{p1,p2} = (-5000 \pm j31225)\frac{rad}{s}$ <br> $f_{p1,p2} = \frac{\lvert Im(s_{p1,p2})\rvert}{2\pi} = 4.97 \ \text{kHz}$ | $s_{z1} = (-1000 + j0)\frac{rad}{s}$ <br> $f_{z1} = \frac{\lvert Re(s_{z1})\rvert}{2\pi} = 1.59 \ \text{kHz}$ |

The following observations can be made based on Table 1:

- $Z_{EQ}$ has a single real LHP zero (that is, has negative real part) corresponding to f = 1.59 kHz.
- $Z_{EQ}$ has a pair of LHP complex conjugate poles corresponding to f = 4.97 kHz, indicating an oscillatory transient response with 4.97 kHz frequency, as shown in Figure 6.

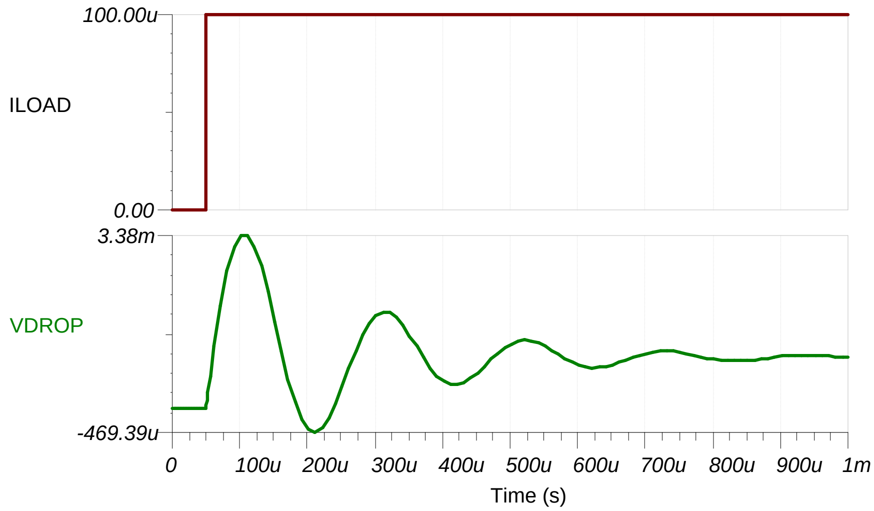


**Figure 6. Unstable Transient Response of Circuit in Figure 5**

In many cases, the poles and zeros of $Z_{EQ}$ can also be identified graphically using Bode plots. The general shape of the $Z_{EQ}$ magnitude plot can be obtained by superimposing the magnitude plots of the individual impedances and tracing the path of the *dominant* impedance at each frequency. Impedances with higher magnitude dominate in a series combination, and impedances with lower magnitude dominate in a parallel combination. Figure 7 shows an example.

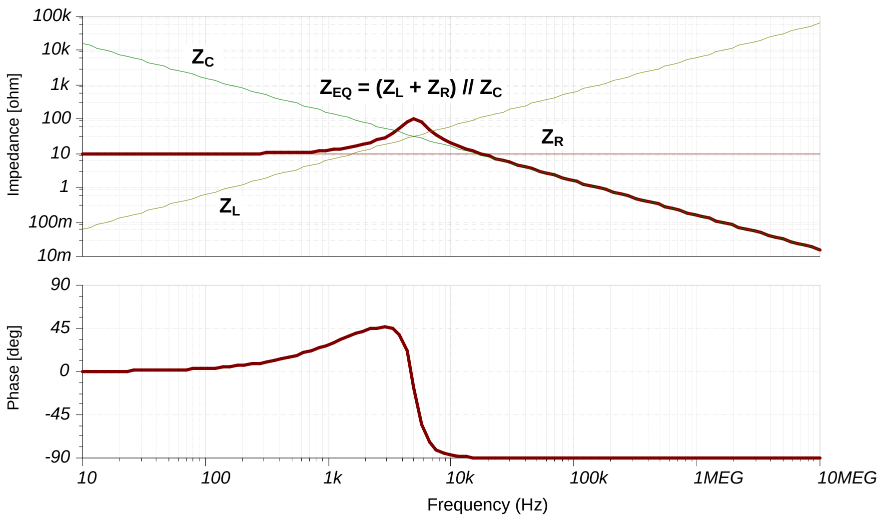


**Figure 7. Bode Plots of $Z_{EQ}$ for Circuit in Figure 5**

After the shape of the $Z_{EQ}$ magnitude plot has been constructed, standard rules (summarized in Table 2) can be applied to identify whether the break frequencies correspond to poles or zeros, as well as whether they are real or complex.

**Table 2. Pole and Zero Signatures on Bode Plots**

| Change in Slope of Magnitude Plot Around $f_{BREAK}$ | Change in Phase Around $f_{BREAK}$ | $f_{BREAK}$ Corresponds to |
|---|---|---|
| 20 dB/decade | 45°/decade over $0.1f_{BREAK} < f < 10f_{BREAK}$ | Single real LHP zero |
| N × (20 dB/decade) | N x (45°/decade) over $0.1f_{BREAK} < f < 10f_{BREAK}$ | N real repeated LHP zeros |
| –20 dB/decade | -45°/decade over $0.1f_{BREAK} < f < 10f_{BREAK}$ | Single Real LHP pole |
| N x (–20 dB/decade) | N x (-45°/decade) over $0.1f_{BREAK} < f < 10f_{BREAK}$ | N real repeated LHP poles |
| Peak | Sharp decrease | Pair of complex conjugate LHP poles |
| Notch | Sharp increase | Pair of complex conjugate LHP zeros |
| 20 dB/decade | -45°/decade over $0.1f_{BREAK} < f < 10f_{BREAK}$ | Single real RHP zero |
| –20 dB/decade | 45°/decade over $0.1f_{BREAK} < f < 10f_{BREAK}$ | Single real RHP pole |
| Peak | Sharp increase | Pair of complex conjugate RHP poles |
| Notch | Sharp decrease | Pair of complex conjugate RHP zeros |

Applying the rules specified in Table 2 to Figure 7, $Z_{EQ}$ has a real LHP zero around the $Z_R$ – $Z_L$ matching frequency (1.59 kHz), and a pair of complex conjugate poles around the $Z_L$ – $Z_C$ matching frequency (4.97 kHz). In this case, the complex conjugate poles physically represent resonance due to the $Z_L$ – $Z_C$ interaction when $I_{LOAD}$ stimulates the circuit.

## 2.5 Stabilizing the Load Transient Response

Ringing can be eliminated by making sure that $Z_{EQ}$ has no complex poles. Referring to the expression for $s_{p1,p2}$ in Table 1, the poles of $Z_{EQ}$ are real if the quantity under the square root is positive. Assuming the values of L and C are retained, and the value of R is varied, Equation 5 specifies the values of R that stabilize the circuit.

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} \geq 0$$

$$R \geq 2 \times \sqrt{\frac{L}{C}} = 63.25 \quad \Omega \tag{5}$$

Table 3 computes the effect of setting R = 2 × √(L/C) on the poles and zeros of $Z_{EQ}$.

**Table 3. $Z_{EQ}$ Poles and Zeros With R = 2 × √(L/C)**

| Poles of $Z_{EQ}$ | Zeros of $Z_{EQ}$ |
|---|---|
| $s_{p1,p2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\frac{1}{\sqrt{LC}}$ | $s_{z1} = -\frac{R}{L} = -\frac{2}{\sqrt{LC}} = 2 \times s_{p1,p2}$ |
| Substituting values of L, and C from Figure 5: | |
| $s_{p1,p2} = (-31622 \pm j0) \ \frac{rad}{s}$ <br> $f_{p1,p2} = \frac{\lvert Re(s_{p1,p2}) \rvert}{2\pi} = 5.03 \ \ kHz$ | $s_{z1} = (-63245 + j0) \ \frac{rad}{s}$ <br> $f_{z1} = \frac{\lvert Re(s_{z1}) \rvert}{2\pi} = 10.06 \ \ kHz$ |

$Z_{EQ}$ no longer contains complex poles, and Figure 8 confirms the circuit's stable load transient response.

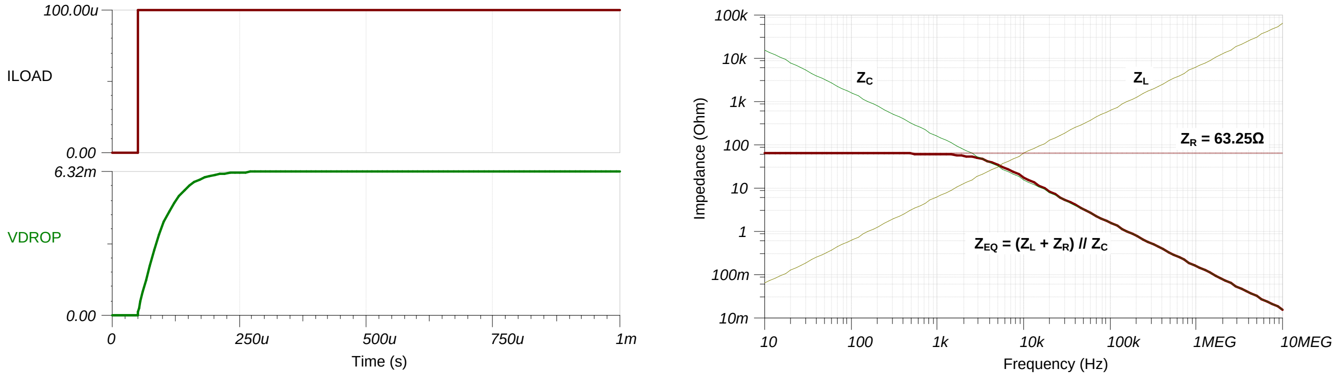


**Figure 8. RLC Circuit of Figure 5 is Stable With R = 2 × $\sqrt{(L/C)}$**

In most cases, Bode plots can also be used in an iterative fashion to improve stability. The goal is to eliminate any peaks in the $Z_{EQ}$ magnitude plot. Peaks caused by resonant LC interactions are eliminated by increasing the value of the resistance. Figure 9 illustrates the effect of iteratively increasing the value of R in the Figure 5 circuit on the magnitude peak and step response. The steady-state error is increasing because $I_{LOAD}$ is a step function and nonzero at steady-state, thus creating a bigger voltage drop as the series resistance is increased.

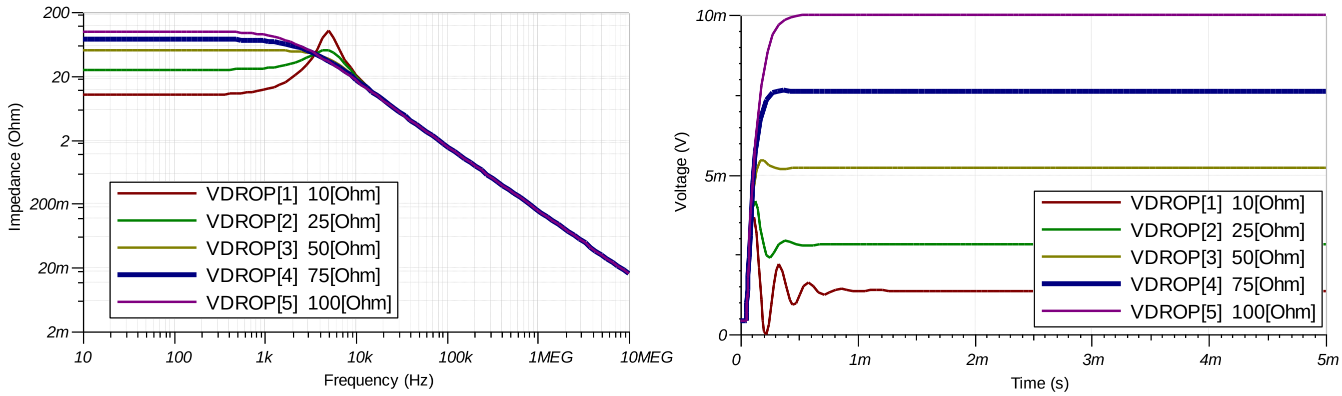


**Figure 9. Iterative Stability Improvement of Figure 5 Circuit**

A value of R = 75 Ω (blue traces in Figure 9) is suitable because this value produces the most desirable response in terms of gain peaking and settling speed. The value is also compliant with Equation 5.

## 3 Properties of Closed-Loop Amplifier Output Impedance ($Z_{OUT}$)

$Z_{OUT}$ is the frequency-dependent Thévenin equivalent impedance offered to a load by a closed-loop amplifier. $Z_{OUT}$ is measured using the test circuit depicted in Figure 10, and is derived by replacing $Z_{BLOCK}$ in Figure 1 with the closed-loop amplifier model, consisting of an op amp and a feedback network.

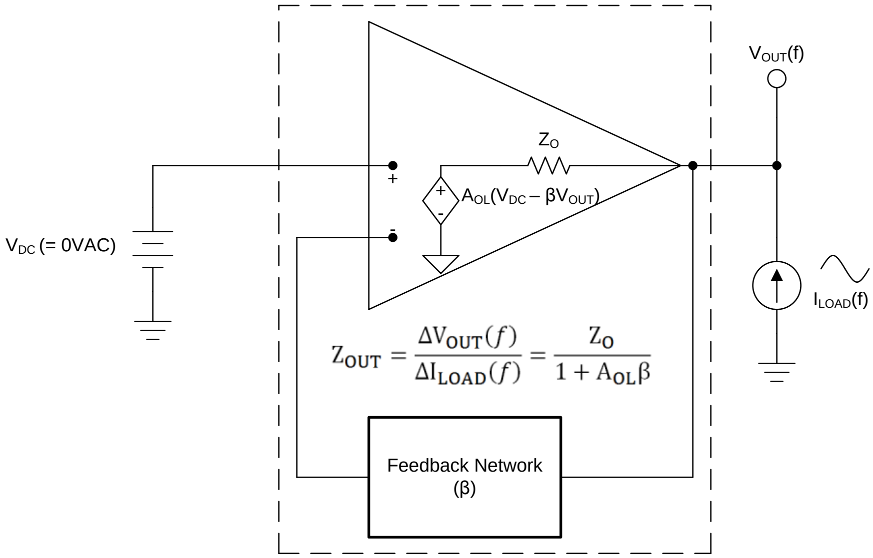


**Figure 10. $Z_{OUT}$ Test Circuit**

$A_{OL}$ is the amplifier open-loop voltage gain, and β is the voltage gain of the feedback network.

$Z_O$ is the op amp open-loop output impedance. Writing ac node equations (similar to Equation 3) based on Figure 10, and solving for the gain ($\Delta V_{OUT}$ / $\Delta I_{LOAD}$) produces Equation 6.

$$\Delta V_{OUT} = -A_{OL}\beta \times \Delta V_{OUT} + \Delta I_{LOAD} \times Z_O$$

$$\Rightarrow Z_{OUT}(f) = \frac{\Delta V_{OUT}(f)}{\Delta I_{LOAD}(f)} = \frac{Z_O}{1 + A_{OL}\beta} \tag{6}$$

The following observations can be made based on Equation 6.

- Any changes in amplifier stability affecting loop gain ($A_{OL}\beta$) also affect $Z_{OUT}$. Therefore, $Z_{OUT}$ can be used to assess amplifier stability.
- The frequency response of $Z_{OUT}$ can be derived from the amplifier $A_{OL}\beta$ and $Z_O$ transfer functions. Familiarity with commonly occurring $Z_{OUT}$ regions over frequency is important for analyzing amplifier load transient behavior

A simple amplifier model can be constructed to explore the effects of loop gain and $Z_O$ on $Z_{OUT}$. For ease of analysis, β is assumed to be constant so that loop gain is determined by $A_{OL}$ alone. $A_{OL}$ can be represented using the dominant pole approximation given by Equation 7.

$$A_{OL}(s) = \frac{A_{OL}(0) \times 2\pi f_{P1\_AOL}}{s + 2\pi f_{P1\_AOL}}$$

where

- $A_{OL}(0)$ is the amplifier dc open-loop gain.
- $s = -2\pi f_{P1\_AOL}$ is the dominant pole of the $A_{OL}$ transfer function.
- $(A_{OL}(0) \times 2\pi f_{P1\_AOL})$ is the amplifier gain-bandwidth product. (7)

Substituting Equation 7 into Equation 6 and simplifying yields Equation 8.

$$Z_{OUT} = \frac{Z_O}{1 + A_{OL}\beta} = \frac{Z_O}{1 + \frac{A_{OL}(0)\beta \times 2\pi f_{P1\_AOL}}{s + 2\pi f_{P1\_AOL}}}$$

$$Z_{OUT} = Z_O \times \frac{s + 2\pi f_{P1\_AOL}}{s + 2\pi f_{P1\_AOL} \times (1 + A_{OL}(0)\beta)} \quad (8)$$

Observe that $Z_{OUT}$ has:

- a real zero at $s_{Z\_ZOUT} = -2\pi f_{P1\_AOL}$ corresponding to $f_{Z\_ZOUT} = f_{P1\_AOL}$
- a real pole at $s_{P\_ZOUT} = -2\pi f_{P1\_AOL}(1 + A_{OL}(0)\beta$ corresponding to $f_{P\_ZOUT} = f_{P1\_AOL} \times (1 + A_{OL}(0)\beta)$
- $f_{Z\_ZOUT} < f_{P\_ZOUT}$

Additional poles or zeros may be present based on the characteristics of $Z_O$. As described in *Modeling the output impedance of an op amp for stability analysis* , $Z_O$ can be some combination of the fundamental impedances: resistive, capacitive or inductive. Therefore, Equation 8 can be examined under each of these three $Z_O$ conditions.

1. If $Z_O$ is purely resistive then $Z_{OUT}$ contains no additional poles or zeros and has the general frequency characteristics described in Table 4.

**Table 4. $Z_{OUT}$ for Resistive $Z_O$ and Single-Pole $A_{OL}$**

| $Z_{OUT}$ | Frequency Range |
|---|---|
| Resistive | $0 < f < f_{Z\_ZOUT}$ |
| Inductive | $f_{Z\_ZOUT} < f < f_{P\_ZOUT}$ |
| Resistive | $f > f_{P\_ZOUT}$ |

2. If $Z_O$ is purely capacitive ($Z_O$ = 1 / (sC)), then $Z_{OUT}$ contains an additional pole at f = 0, and has the general frequency characteristics described in Table 5.

**Table 5. $Z_{OUT}$ for Capacitive $Z_O$ and Single-Pole $A_{OL}$**

| $Z_{OUT}$ | Frequency Range |
|---|---|
| Capacitive | $0 < f < f_{Z\_ZOUT}$ |
| Resistive | $f_{Z\_ZOUT} < f < f_{P\_ZOUT}$ |
| Capacitive | $f > f_{P\_ZOUT}$ |

3. If $Z_O$ is purely inductive ($Z_O$ = sL), then $Z_{OUT}$ contains an additional zero at f = 0, and has the general frequency characteristics described in Table 6.

**Table 6. $Z_{OUT}$ for Inductive $Z_O$ and Single-Pole $A_{OL}$**

| $Z_{OUT}$ | Frequency Range |
|---|---|
| Inductive | $0 < f < f_{Z\_ZOUT}$ |
| Double Inductive | $f_{Z\_ZOUT} < f < f_{P\_ZOUT}$ |
| Inductive | $f > f_{P\_ZOUT}$ |

The term *double inductive* denotes a $Z_{OUT}$ region with a +40-dB/decade slope in magnitude. A double-inductive impedance can only occur in an active circuit using negative feedback, but modeling the impedance as a passive element in a combination is useful for stability analysis. On a linear frequency scale, the magnitude of $Z_{OUT}$ in the double-inductive region increases as $(\Delta f)^2$ for a $\Delta f$ change in frequency. Thus, the impedance transfer function of a double-inductive element ($L_D$) is $Z_{LD} = s^2 L_D$.

The effects of amplifier $Z_O$ and $A_{OL}$ on $Z_{OUT}$ can be validated through simulation. Figure 11 shows a single-pole op amp $A_{OL}$ model with a dc gain of 100,000 V/V (or 100 dB), and a single dominant pole at 15.9 Hz. $Z_O$ has been initialized to 0 Ω. The 1-GH inductor makes sure that the circuit is in closed-loop configuration for dc convergence, and in open-loop configuration for ac-gain measurement. The 1-GF capacitor keeps the inverting input of the op amp from floating when the feedback loop is broken.

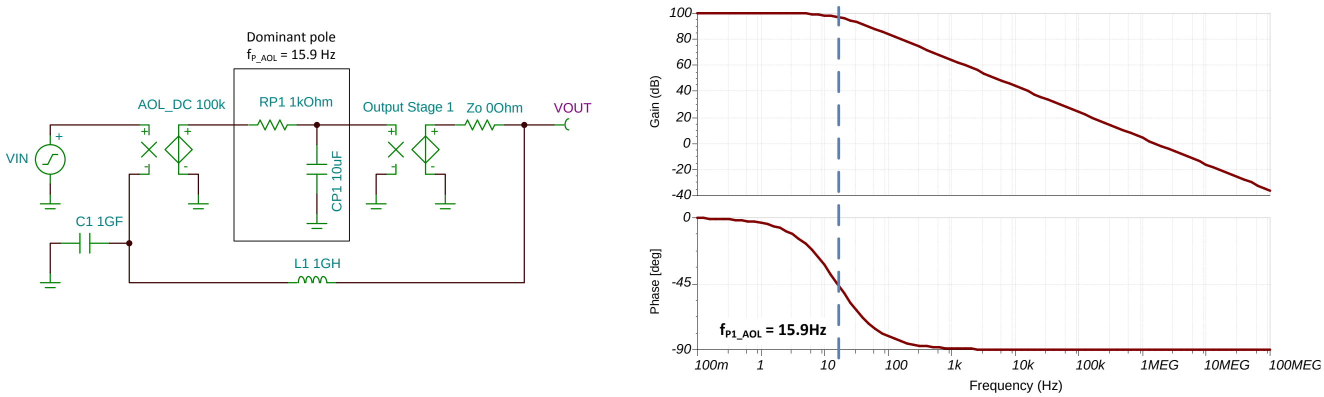


**Figure 11. Op Amp $A_{OL}$ Model With a Single Pole**

For measuring $Z_{OUT}$, the amplifier must be in closed-loop gain configuration with the appropriate $Z_O$ element, and excited with an ac current source according to Figure 10. Using β = 1 V/V for ease of analysis, the circuits of Figure 12 through Figure 14 depict $Z_{OUT}$ versus frequency for the three basic $Z_O$ types. $f_{Z\_ZOUT} = f_{P1\_AOL}$ = 15.9 Hz and $f_{P\_ZOUT} = f_{P1\_AOL}\,(1 + A_{OL}(0)\beta)$ = 1.59 MHz.

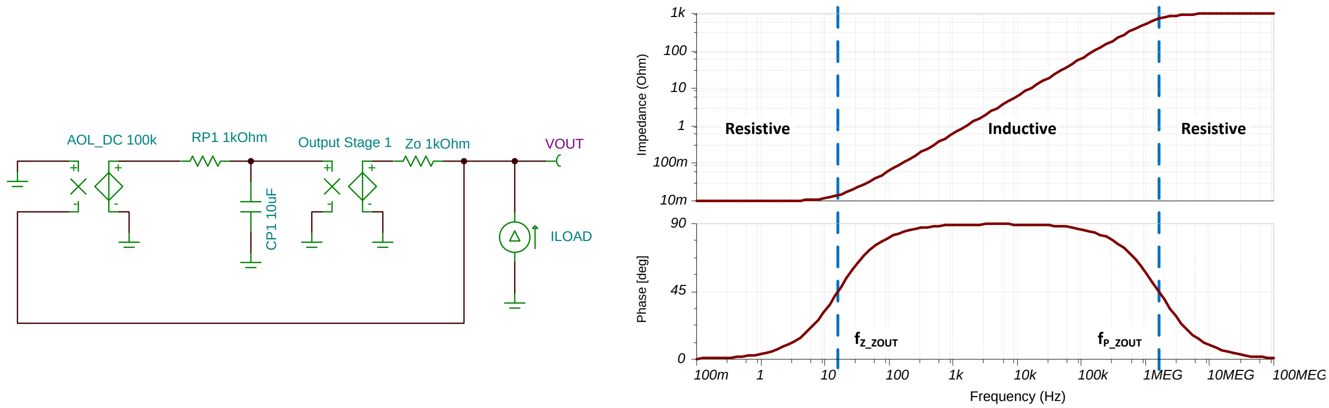


**Figure 12. $Z_{OUT}$ Simulation With Single-Pole $A_{OL}$ and Resistive $Z_O$**

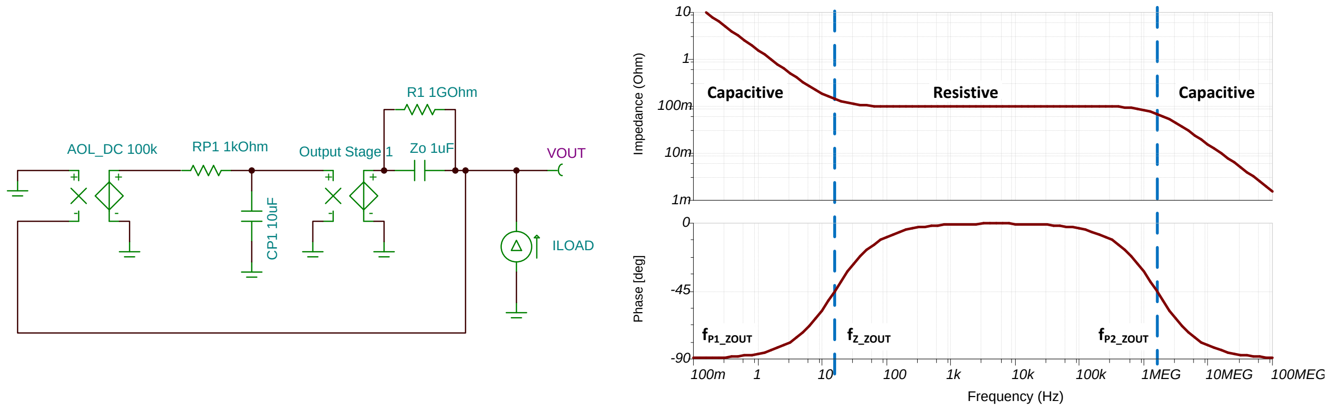


**Figure 13. $Z_{OUT}$ Simulation With Single-Pole $A_{OL}$ and Capacitive $Z_O$**

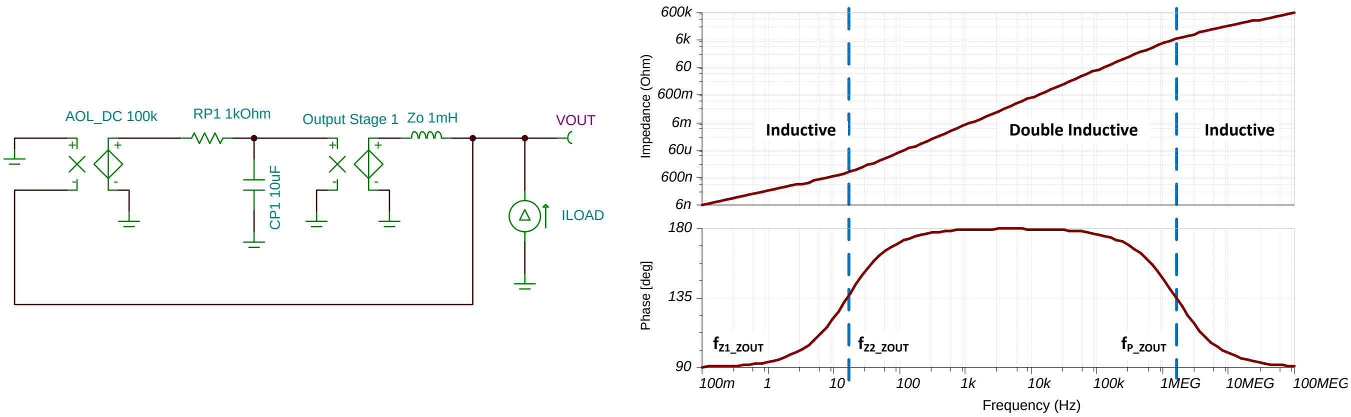


**Figure 14. $Z_{OUT}$ Simulation With Single-Pole $A_{OL}$ and Inductive $Z_O$**

## 4 Stability Analysis of a Loaded Amplifier Using $Z_{OUT}$ and $Z_{LOAD}$

The stability of a loaded amplifier can be evaluated by examining the Thévenin equivalent impedance ($Z_{EQ}$) for complex poles. $Z_{EQ}$ for a loaded amplifier is simply the parallel combination of $Z_{OUT}$ and $Z_{LOAD}$, as shown in Equation 9:

$$Z_{EQ} = Z_{OUT} \| Z_{LOAD}$$

where

- $Z_{OUT}$ can be resistive, capacitive, inductive or double inductive over frequency.
- $Z_{LOAD}$ can be resistive, inductive, or capacitive over frequency (9)

Therefore, the properties of $Z_{EQ}$ can be studied by considering the various combinations of $Z_{OUT}$ and $Z_{LOAD}$. Obviously, a resistive $Z_{OUT}$ requires no special consideration because a resistive $Z_{OUT}$ is unconditionally stable with any resistive or reactive $Z_{LOAD}$. From a stability perspective, only reactive $Z_{OUT}$ regions are relevant when combined with specific $Z_{LOAD}$ elements, as discussed in subsequent sections.

### 4.1 *Inductive $Z_{OUT}$ Driving a Capacitive $Z_{LOAD}$ or Capacitive $Z_{OUT}$ Driving an Inductive $Z_{LOAD}$*

This section analyzes stability issues caused by LC interactions between $Z_{OUT}$ and $Z_{LOAD}$. The analysis is the same regardless of which element is inductive and which is capacitive. In this case, however, the analysis focuses on the more common scenario where $Z_{OUT}$ is inductive and $Z_{LOAD}$ is capacitive. Both conditions produce complex conjugate poles in $Z_{EQ}$. The circuit shown in Figure 15 is a typical example.

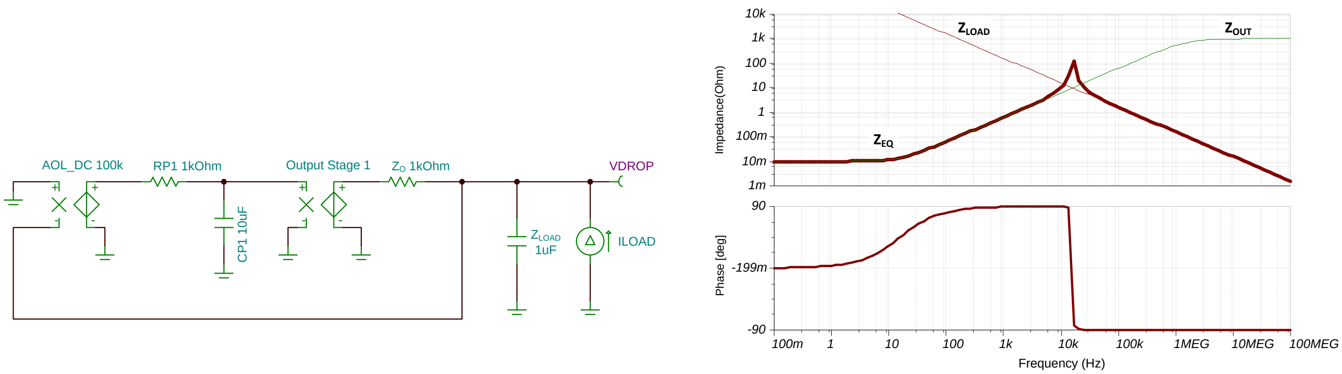


**Figure 15. Example of Circuit With Resonant LC Interaction Between $Z_{OUT}$ and $Z_{LOAD}$**

With the individual impedances superimposed, tracing the path of the lower impedance reveals a peak in the $Z_{EQ}$ magnitude plot around the $Z_{OUT}$ – $Z_{LOAD}$ matching frequency. The peak signifies complex conjugate poles and the oscillatory load transient response is shown in Figure 16.

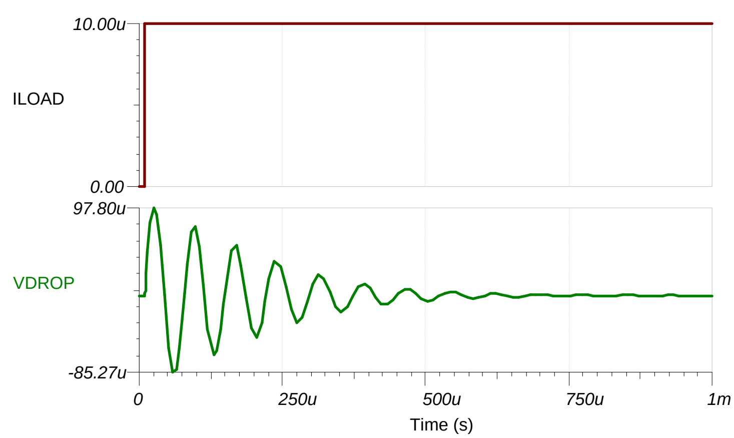


**Figure 16. Unstable Load Transient Response of Circuit in Figure 15**

As discussed in Section 2.5, an LC resonant peak in $Z_{EQ}$ can be eliminated by adding sufficiently large series resistance: $R \geq 2 \times \sqrt{(L/C)}$ (see Equation 5). While the circuit diagram shows the value of C (= $Z_{LOAD}$), the value of L must be calculated using the $Z_{OUT}$ magnitude plot by identifying a point in the inductive region. Observing that $|Z_{OUT}| \approx 50$ mΩ at f =75 Hz, the value of L can be calculated as $L = |Z_{OUT}| / (2\pi f) \approx 106$ µH. Thus, the required resistance value for stability is $R \geq 20.6$ Ω. Stabilized responses are shown in Figure 17 and Figure 18. Clearly, a value of R in the 20-Ω to 30-Ω range is suitable.

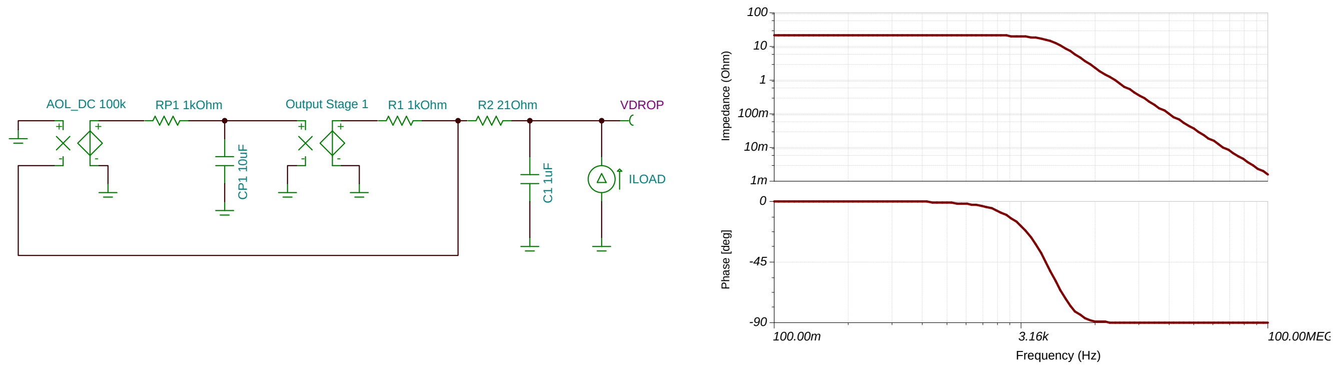


**Figure 17. Adding Series Resistance Eliminates LC Resonant Peak in $Z_{EQ}$**

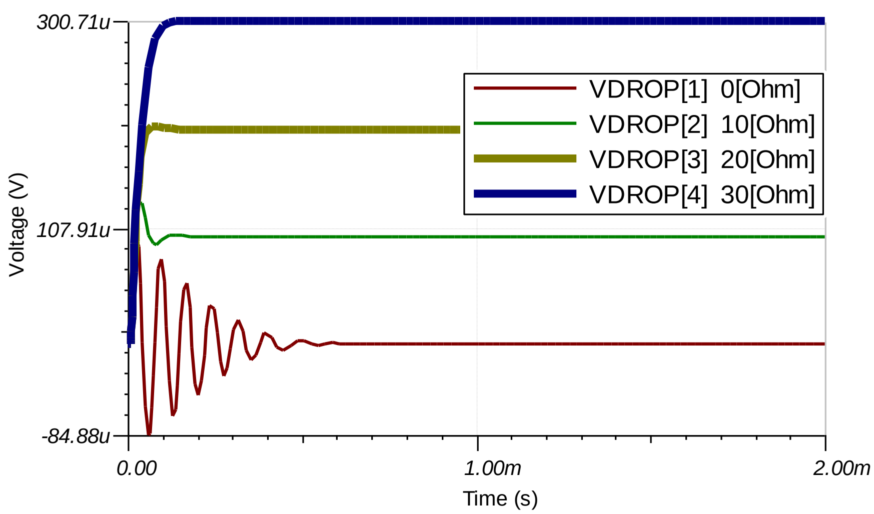


**Figure 18. Load Transient Response of Figure 17 Circuit is Stable for 20 Ω < R < 30 Ω**

The series resistance creates a steady-state dc error, and takes several milliseconds to settle, which may not be acceptable in some applications. A simple workaround is to move the resistor in series with the load capacitor, as shown in Figure 19.

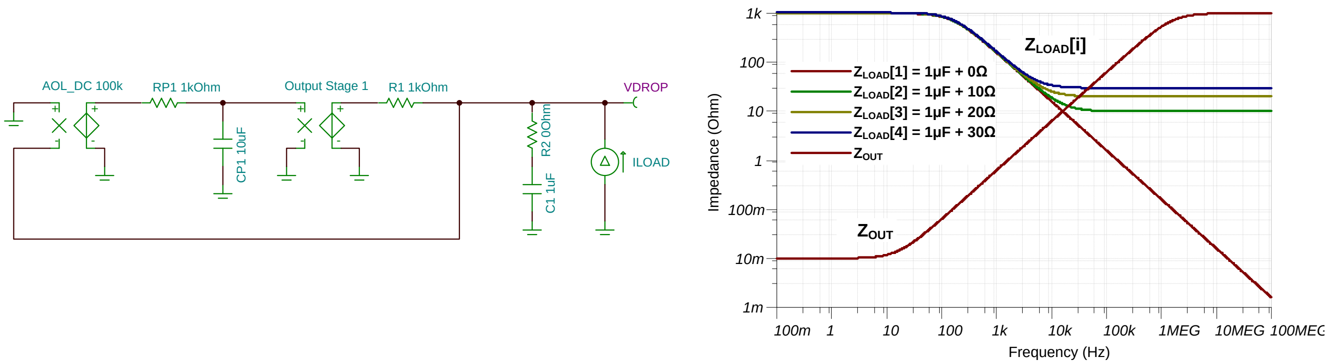


**Figure 19. Increasing ESR Also Eliminates Resonant LC Peak in $Z_{EQ}$**

There is still a stability improvement because, as shown in Figure 19, the added ESR makes $Z_{LOAD}$ more resistive around the $Z_{OUT}$ – $Z_{LOAD}$ matching frequency, thereby eliminating resonance. Figure 20 depicts the load transient response as ESR is increased.

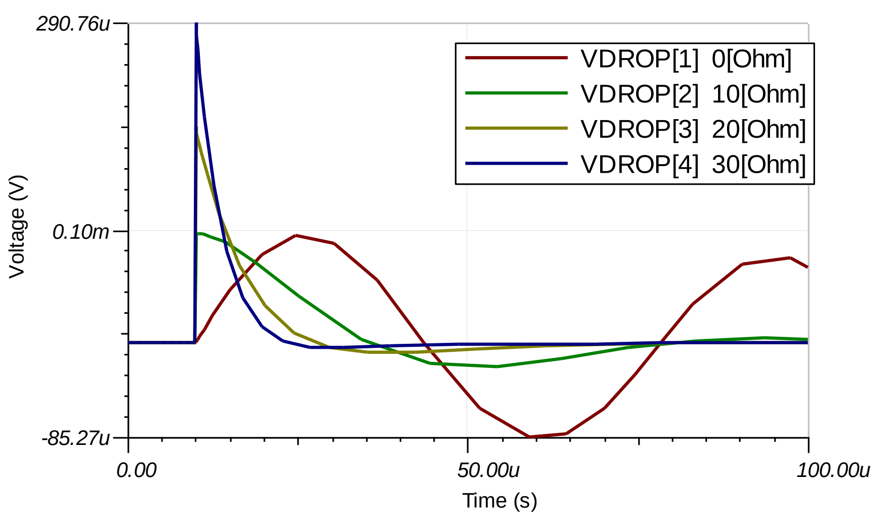


**Figure 20. Load Transient Response With Increasing $Z_{LOAD}$ ESR**

Observe that the response becomes increasingly stable, and the dc steady-state error as well as settling time become negligible. However, the peak overshoot becomes progressively higher as the capacitor initially sinks most of the load current (a positive step function, in this case), generating a bigger voltage drop across the ESR. This overshoot may be acceptable in applications where dc accuracy and fast settling are more important.

## 4.2 Double-Inductive $Z_{OUT}$ Driving Resistive $Z_{LOAD}$

Figure 21 shows the model of a circuit with a double-inductive $Z_{OUT}$ driving a resistive $Z_{LOAD}$.

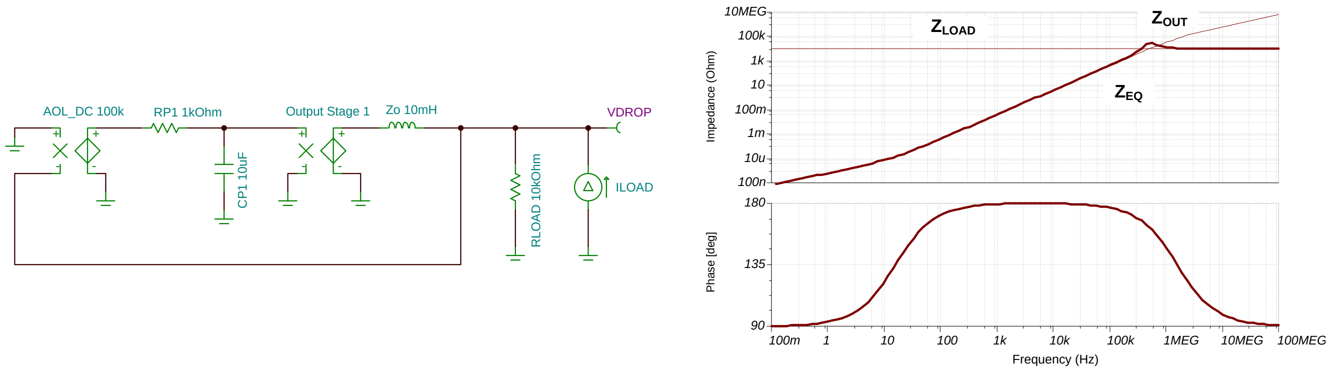


**Figure 21. Example Circuit Showing Double-Inductive $Z_{OUT}$ Driving Resistive $Z_{LOAD}$**

There is no LC resonance in this case; however, $Z_{EQ}$ has an unexpected peak around the $Z_{OUT} – Z_{LOAD}$ matching frequency, and the load step response is unstable, as shown in Figure 22.

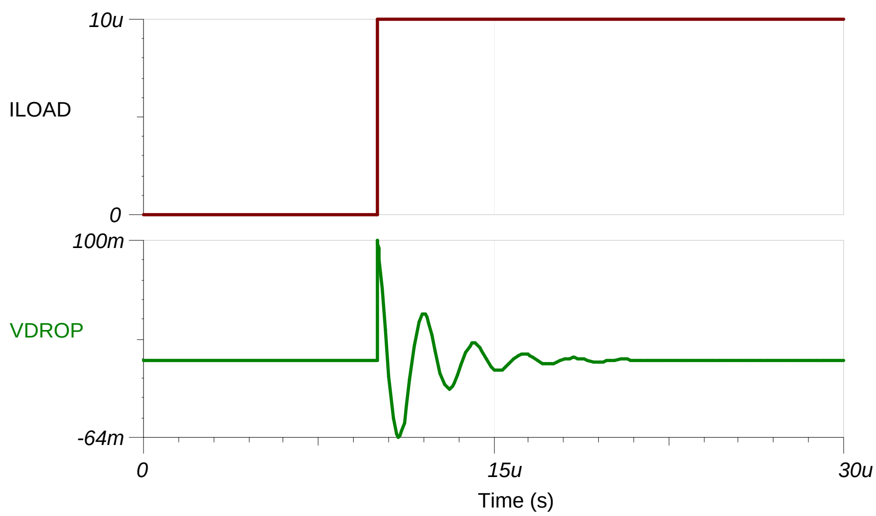


**Figure 22. Unstable Load Transient Response of Figure 21 Circuit**

For stability analysis in this case, the $Z_{EQ}$ transfer function must be solved algebraically.

Equation 10 expresses $Z_{EQ}$ as the combination of inductive ($L_1$, $L_2$) and double-inductive ($L_D$) portions of $Z_{OUT}$ with resistive $Z_{LOAD}$ = $R_{LOAD}$ = 10 kΩ. $L_2$ can be eliminated from the expression for $Z_{EQ}$ because $R_{LOAD}$ dominates $Z_{EQ}$ at high frequencies.

$$Z_{EQ} = Z_{OUT} \| Z_{LOAD} = ((sL_1 + s^2 L_D) \| sL_2) \| R_{LOAD} \approx (sL_1 + s^2 L_D) \| R_{LOAD} \tag{10}$$

$$\Rightarrow Z_{EQ}(s) = \frac{sR_{LOAD}(L_1 + sL_D)}{R_{LOAD} + s(L_1 + sL_D)}$$

$$\text{Poles} \quad \Rightarrow \text{Denominator}(Z_{EQ}(s)) = 0$$

$$\Rightarrow s^2 L_D + sL_1 + R_{LOAD} = 0$$

$$\therefore \quad s_{p1,p2} = -\frac{L_1}{2L_D} \pm \sqrt{\left(\frac{L_1}{2L_D}\right)^2 - \frac{R_{LOAD}}{L_D}} \tag{11}$$

Substituting the values of $L_1$ = 132 nH and $L_D = |Z_{OUT}|/(2\pi f)^2 \approx 1$ n units (from the $Z_{OUT}$ Bode plot) confirms that $Z_{EQ}$ has a pair of complex conjugate LHP poles near the $Z_{OUT} - Z_{LOAD}$ matching frequency of approximately 500 kHz.

$$s_{p1,p2} = (-130 \pm j3.13M)\ \frac{rad}{s}$$

$$\therefore\ f_{p1,p2} = \frac{|Im(s_{p1,p2})|}{2\pi} = 498\ \ kHz$$

Stabilizing the load transient response requires the quantity under the square root in Equation 11 to be positive

$$\left(\frac{L_1}{2L_D}\right)^2 - \frac{R_{LOAD}}{L_D} \geq 0$$

$$\Rightarrow L_1 \geq 2 \times \sqrt{R_{LOAD} \times L_D} = 6.4\ \ mH \qquad (12)$$

Equation 12 suggests that the complex poles can be eliminated by increasing the value of $L_1$, which is equivalent to adding series inductance to $Z_{OUT}$. According to Figure 23, the oscillatory response disappears when the value of the series inductance exceeds 6 mH. Figure 24 shows how adding series inductance eliminates the double-inductive characteristic of $Z_{EQ}$.

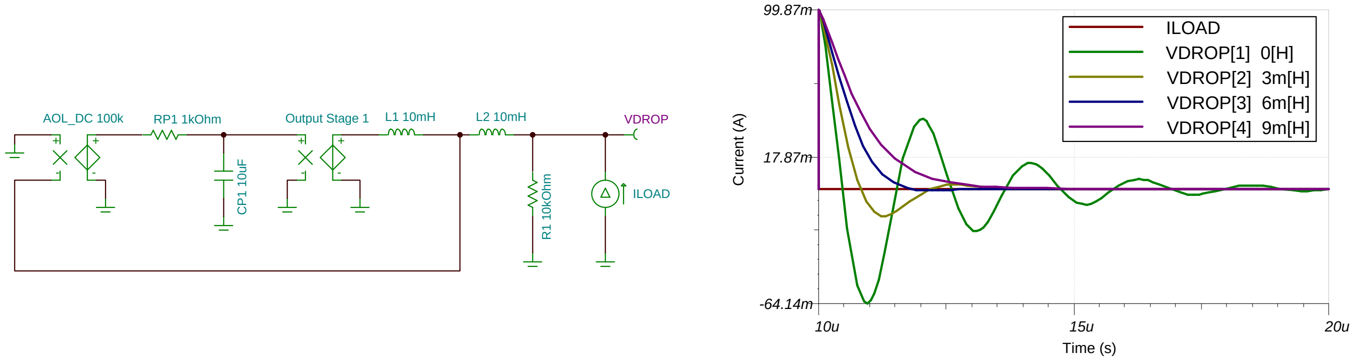


**Figure 23. Adding Series Inductance to $Z_{OUT}$ Stabilizes Load Step Response**

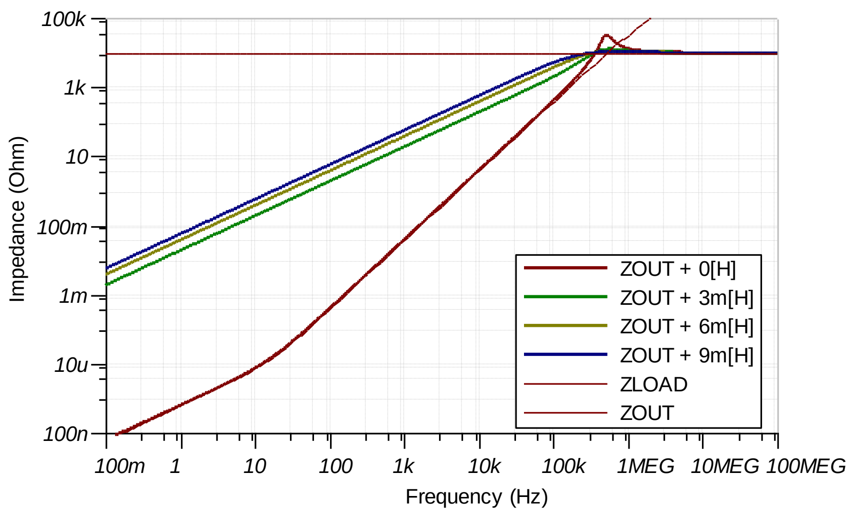


**Figure 24. Double-Inductive Region No Longer Dominates $Z_{OUT}$ With Higher ESL**

### 4.3 Double-Inductive $Z_{OUT}$ Driving Capacitive $Z_{LOAD}$

The $Z_{EQ}$ Bode plots for this configuration show signs of gross instability. According to the magnitude plot of Figure 25, there are three poles corresponding to the $Z_{OUT}$ – $Z_{LOAD}$ matching frequency, but phase lead increases sharply by 90° in just one octave around the matching frequency. Table 2 shows that this signature represents a pair of complex conjugate RHP poles that produce an exponentially increasing oscillatory response, as confirmed by Figure 26.

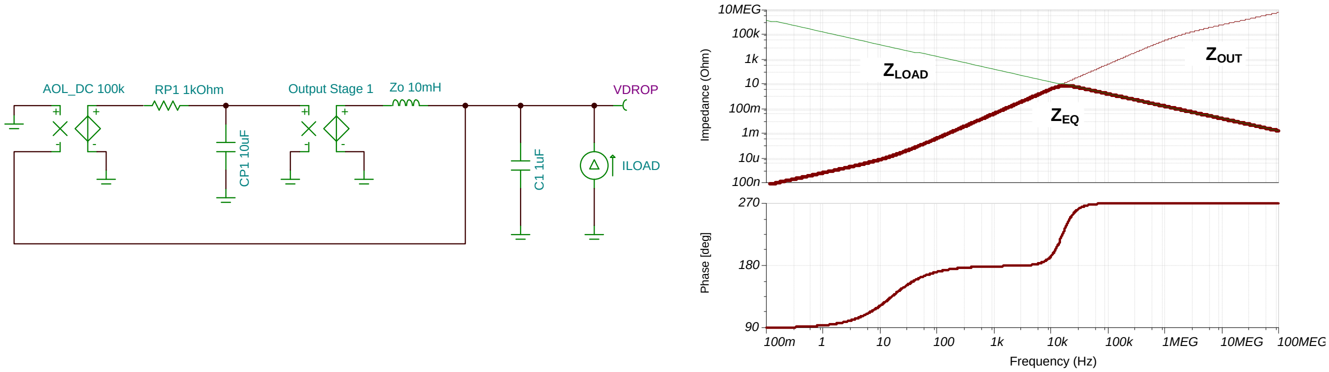


**Figure 25. Example Circuit With Double-Inductive $Z_{OUT}$ Driving Capacitive $Z_{LOAD}$**

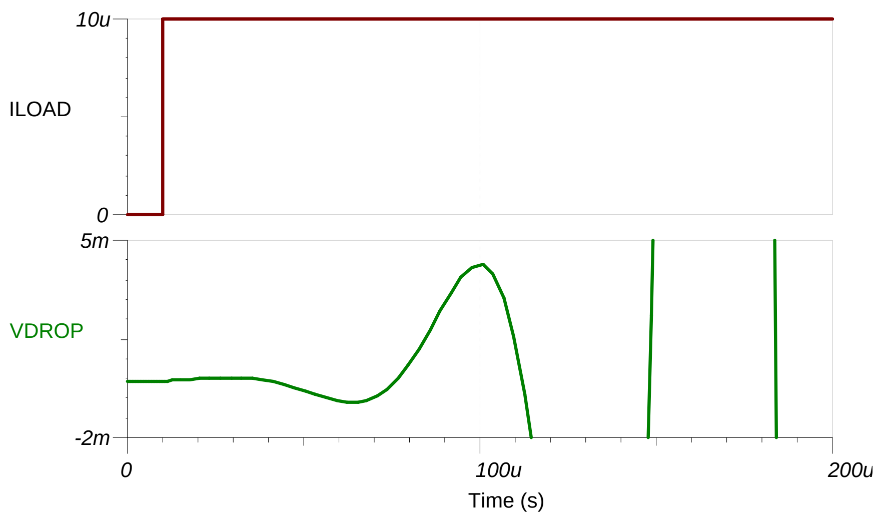


**Figure 26. Unstable Load Step Response of Figure 25 Circuit**

In this case, it is difficult to find the poles of $Z_{EQ}$ algebraically because this case involves solving a third-order polynomial equation, which is nontrivial. For reference, the $Z_{EQ}$ transfer function is given by Equation 13.

$$Z_{EQ} = Z_{OUT} \| Z_{LOAD} = \left( \left( sL_1 + s^2 L_D \right) \| sL_2 \right) \| \frac{1}{sC_{LOAD}} \approx \left( sL_1 + s^2 L_D \right) \| \frac{1}{sC_{LOAD}}$$

$$\Rightarrow Z_{EQ}(s) = \frac{s^2 L_D + sL_1}{s^3 L_D C_{LOAD} + s^2 L_1 C_{LOAD} + 1} \quad (13)$$

In lieu of an algebraic analysis, the strategies discussed in Section 4.1 and Section 4.2 can be applied. The basic idea is to add series inductance so that $Z_{OUT}$ transforms into being more inductive around the $Z_{OUT}$ – $Z_{LOAD}$ matching frequency. Consequently, the circuit simplifies to an LC resonant circuit that can then be stabilized relatively easily by adding series resistance.

From the Bode plot $|Z_{OUT}| \approx 10\ \Omega$ at the $Z_{OUT} - Z_{LOAD}$ matching frequency ($f_P$) of approximately 16 kHz. For $Z_{OUT}$ to become more inductive, the impedance of the series inductor at 16 kHz must be higher than 10 Ω. Use Equation 14 to select a suitable value.

$$L_1 > \frac{2 \times |Z_{OUT}(f_P)|}{2\pi f_P} \approx 200\ \ \mu H \tag{14}$$

The value of the series resistance required to eliminate the resonant peak appearing in Figure 27 can now be calculated using Equation 5: $R \geq 2 \times \sqrt{(L_1/C_{LOAD})} \approx 30\ \Omega$. Figure 28 and Figure 29 depict the Bode plot and transient response of the stabilized circuit, respectively.

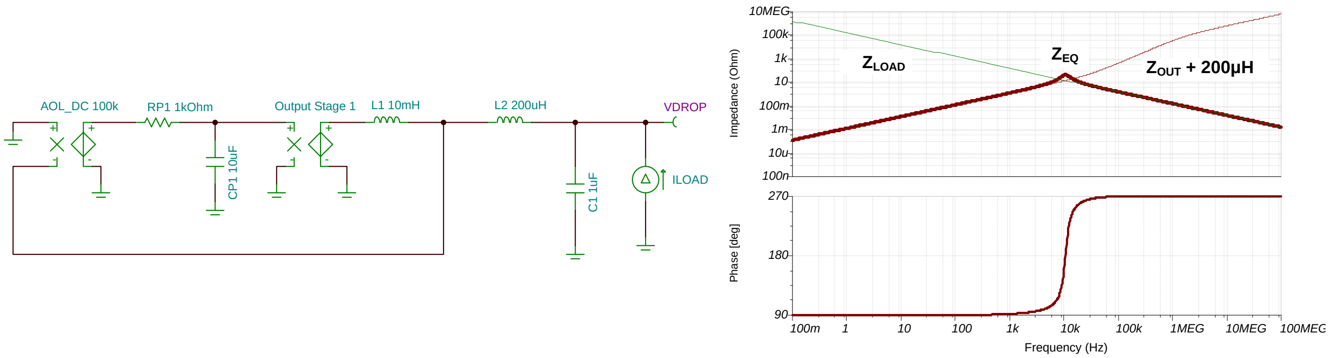


**Figure 27. Modified $Z_{OUT}$ is More Inductive Around $f_P$ = 16 kHz**

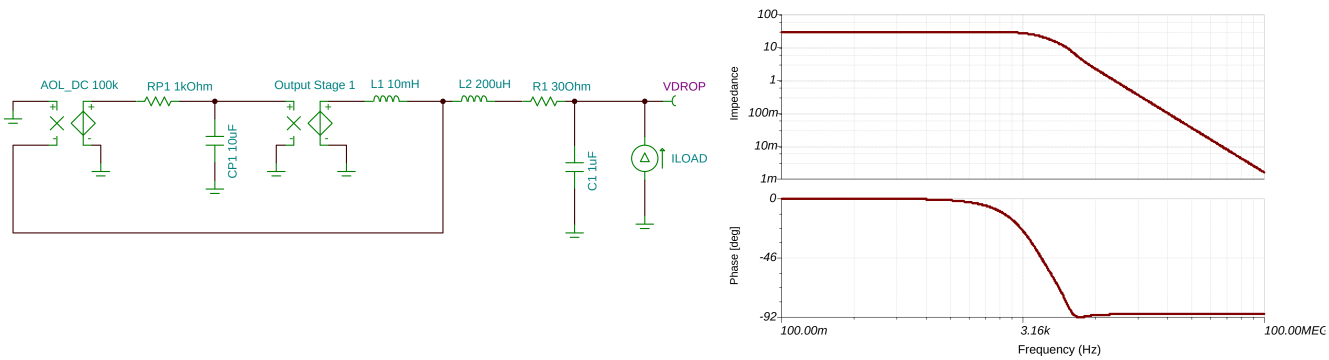


**Figure 28. Adding Series Resistance Eliminates Peak in Modified $Z_{EQ}$ Magnitude**

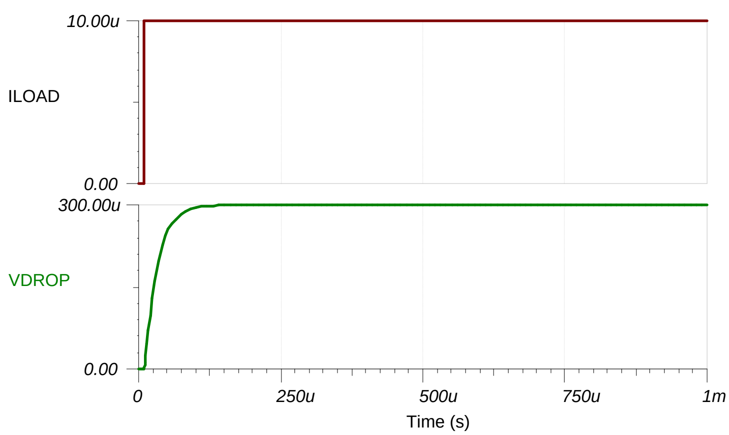


**Figure 29. Load Step Response of Stabilized Circuit**

Finally, the compensation strategies presented in this article must be considered alongside other system constraints such as cost, component availability, performance, and so on. For example, introducing a low-cost op amp buffer between the amplifier and load capacitor in the circuit shown in Figure 25 could be more cost-effective and enable faster settling across the load capacitor than using a 200-µH inductor for compensation. However, an op amp buffer would likely also consume more board area and supply current, and introduce additional errors in the signal path, thus requiring careful consideration against design objectives.

## 5 Conclusions

The closed-loop output impedance ($Z_{OUT}$) of an amplifier makes it possible to evaluate the stability of the amplifier load transient response under closed-loop conditions.

A stable load transient response is characterized by exponential settling to steady state without ringing. This requires a Thévenin equivalent impedance function that has purely real poles in the left half of the complex s-plane (LHP).

Simple amplifier simulation models constructed using the dominant pole $A_{OL}$ approximation provide useful insights into the frequency response of $Z_{OUT}$ and the various impedance profiles offered to a load.

Compensation strategies were developed using algebraic and geometric methods to overcome load transient stability issues for various amplifier $Z_{OUT}$ and load configurations. These strategies were validated through simulation.

## 6 References


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