Application Report

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Sensitivity Analysis for Power Supply Design

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ABSTRACT

In power supply design, tolerance in component values, and reference voltages and currents cause deviation to the desired system output. However, the deviation of the system output due to a variation of each input variation is different. The output is more sensitive to certain inputs than others. In this application note, sensitivity analysis is explored and a design example is provided.

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Sensitivity Analysis Overview

Sensitivity analysis is the study of how a certain system reacts to a variation of its inputs. It is an analysis technique for systematically changing variables in a model to determine the effects of such changes to its output.

Because of tolerance in component values, such as resistors, capacitors, inductors, and even reference voltage and current levels, sensitivity analysis is a useful technique in power supply design. It helps determine the effect of each input tolerance in the system and its effect on the overall performance of the model. The classical sensitivity function is defined as:

$$S_x^y = \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} \quad (1)$$

Thus

$$S_x^y = \frac{\partial y}{\partial x} \frac{x}{y} \quad (2)$$

x represents the value of the components such as resistors, capacitors, inductors, and even reference voltages and currents of the system. *y* is the output parameter of interest such as frequency, phase, or output voltage and current. The sensitivity result determines the per-unit change in *y* due to a given per-unit change in *x*.

Equation 2 can be simplified further for small changes as follow:

$$S_x^y = \frac{\Delta y / y}{\Delta x / x} \quad (3)$$

Sensitivity Analysis with the TPS3808G01

The TPS3808G01 is a microprocessor supervisor circuit. It uses the SENSE pin to sense if the system voltage drops below the preset threshold (V_{IT}). Figure 1 shows the SENSE pin for the TPS3808G01 part. The sense input provides a terminal at which any system voltage can be monitored by using a voltage divider as shown in Figure 2. The sense input pin is connected to a comparator that is referenced to 0.405 V.

Ideally, a comparator would have infinite input impedance that produces no current at the inputs. In practice; however, a real comparator has measurable input impedance and some degree of leakage current. These effects impact the accuracy of the trip point set by the resistive divider at the inputs, because this leakage current cannot be exactly determined and varies from device to device. For the TPS3808G01, it is called “Is” and its maximum and minimum values are provided in the data sheet as ±25 nA.

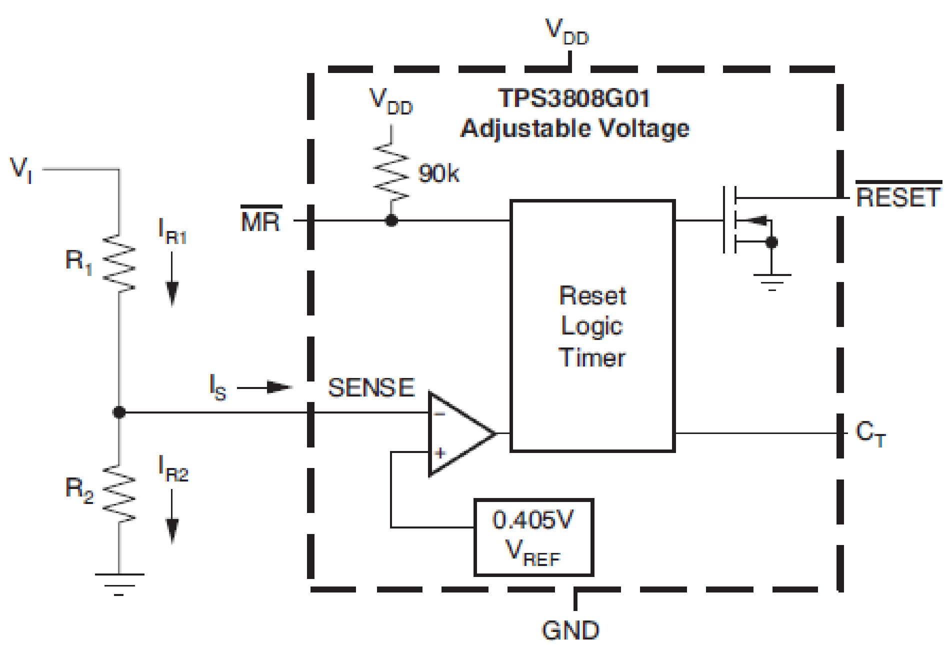

Figure 1. Functional Block Diagram of the TPS3808G01

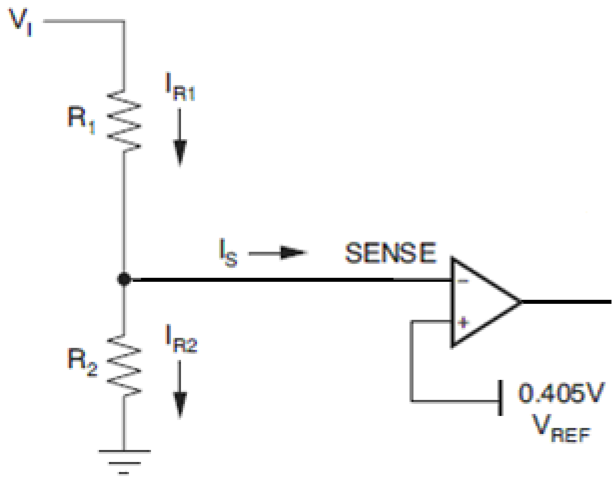

Figure 2. Sense pin for the TPS3808G01

$$V_I = \frac{V_S(R_1 + R_2)}{R_2} + I_S R_1 \tag{4}$$

$$V_S = V_{REF} \tag{5}$$

R1, R2, Is, and V_{REF} are variables and have tolerances. V_I is the target output voltage.

Sensitivity analysis determines how the tolerances on R1, R2, Is, and V_{REF} affects the output V_I.

Sensitivity of V_I with respect to R1:

$$S_{R1}^{VI} = \frac{\partial V_I}{\partial R_1}\frac{R_1}{V_I} \tag{6}$$

$$\frac{\partial V_I}{\partial R_1} = \frac{V_S}{R_2} + I_S \tag{7}$$

$$S_{R1}^{VI} = \frac{(V_S + I_S R_2) R_1}{V_S(R_1 + R_2) + I_S R_1 R_2} \tag{8}$$

Sensitivity of V_I with respect to R2:

$$S_{R2}^{VI} = \frac{\partial V_I}{\partial R_2}\frac{R_2}{V_I} \tag{9}$$

$$\frac{\partial V_I}{\partial R_2} = \frac{V_S R_2 - V_S (R_1 + R_2)}{R_2} \tag{10}$$

$$S_{R2}^{VI} = \frac{-V_S R_1}{V_S (R_1 + R_2) + I_S R_1 R_2} \tag{11}$$

Sensitivity of V_I with respect to I_S:

$$S_{Is}^{VI} = \frac{\partial V_I}{\partial I_S} \frac{I_S}{V_I} \tag{12}$$

$$\frac{\partial V_I}{\partial I_S} = I_S \tag{13}$$

$$S_{Is}^{VI} = \frac{I_S R_2 R_1}{V_S (R_1 + R_2) + I_S R_1 R_2} \tag{14}$$

Sensitivity of V_I with respect to V_{REF}:

$$S_{Vref}^{VI} = \frac{\partial V_{out}}{\partial V_{ref}} \frac{V_{ref}}{V_{out}} \tag{15}$$

$$\frac{\partial V_{out}}{\partial V_{ref}} = \frac{R_1 + R_2}{R_2} \tag{16}$$

$$S_{Vref}^{VI} = \frac{V_{ref} (R_1 + R_2)}{V_{ref} (R_1 + R_2) + I_S R_1 R_2} \tag{17}$$

Design example:

The threshold voltage is set by the ratio of the two resistors R1 and R2 in the divider. Multiple combinations of R1 and R2 can keep the ratio constant. However, to select the actual resistor values, there are tradeoffs to consider. With higher resistances, the leakage current at the comparator input can affect the threshold voltage accuracy. With lower resistances, the current through the divider is increased and this lowers the efficiency of the whole system. Especially for battery-powered applications, this current can be a significant drain on the battery life and the run time.

For selecting optimally-sized resistors, a detailed analysis has been done in the SLVA450 application report. In this application note, the following two equations were used to calculate R1 and R2:

$$R_2 = \frac{V_{IT}\left|\%Acc/100\right|}{\left|I_S\right|(\frac{V_{IT}}{V_{REF}} - 1)} \quad (18)$$

$$R_1 = R_2(\frac{V_{IT}}{V_{ref}} - 1) \quad (19)$$

V_{IT}: The threshold input voltage
Acc: Accuracy of the divider

In this design example, a 3.3 V input is used; anything below a 10% drop from that value is not desired (V_{IT}~ 2.97 V). For 1% desired accuracy (Acc). R2 and R1 are calculated to be 187KΩ and 1.18 MΩ respectively. The reference voltage (V_{REF}) and absolute leakage current (abs(I_S)) are provided in the data sheet of the TPS3808G01 as 0.405 V and 25 nA respectively. Using the standard values of R1 and R2, the lower threshold of the input voltage is calculated as follows:

$$V_I = (1 + \frac{R_1}{R_2})V_{ref} = (1 + \frac{1.18M\Omega}{187K\Omega})0.405 = 2.974V \quad (20)$$

$$S_{R1}^{VI} = \frac{(0.405 + 25\times10^{-9}\times187\times10^{3})1.18\times10^{6}}{0.405(1.18\times10^{6} + 187\times10^{3}) + 25\times10^{-9}\times1.18\times10^{6}\times187\times10^{3}} = 0.865 \quad (21)$$

$$S_{R2}^{VI} = \frac{-0.405\times1.18\times10^{6}}{0.405(1.18\times10^{6} + 187\times10^{3}) + 25\times10^{-9}\times1.18\times10^{6}\times187\times10^{3}} = -0.855 \quad (22)$$

$$S_{Is}^{VI} = \frac{25 * 10^{-9}\times1.18 * 10^{6}\times187\times10^{3}}{0.405\times(1.18\times10^{6} + 187\times10^{3}) + 25\times10^{-9}\times1.18\times10^{6}\times187\div10^{3}} = 9.865\times10^{-3} \quad (23)$$

$$S_{Vref}^{VI} = \frac{0.405(1.18\times10^{6} + 187\times10^{3})}{0.405(1.18\times10^{6} + 187\times10^{3}) + 25\times10^{-9}\times1.18\times10^{6}\times187\times10^{3}} = 0.99 \quad (24)$$

The following table provides the best values for R1 and R2 based on the most used VI and 10% voltage threshold V_{IT}. Then the sensitivity numbers are provided accordingly.

V_I (V)	V_{IT} (V)	Is (A)	V_S (V)	R1 (Ω)	R2 (Ω)	S_{R1}^{VI}	S_{R2}^{VI}	S_{Is}^{VI}	S_{Vref}^{VI}
6	5.40	2.50E-08	0.405	2.16E+06	1.75E+05	0.926	-0.916	9.90E-03	0.990
5	4.50	2.50E-08	0.405	1.80E+06	1.78E+05	0.911	-0.901	9.90E-03	0.990
3.3	2.97	2.50E-08	0.405	1.19E+06	1.88E+05	0.865	-0.855	9.90E-03	0.990
1.8	1.62	2.50E-08	0.405	6.48E+05	2.16E+05	0.752	-0.743	9.90E-03	0.990
1.2	1.08	2.50E-08	0.405	4.32E+05	2.59E+05	0.629	-0.619	9.90E-03	0.990
1	0.90	2.50E-08	0.405	3.60E+05	2.95E+05	0.554	-0.545	9.90E-03	0.990

Table 01: Different VI with 10% drop threshold voltages and the corresponding sensitivity calculations

The obtained results show that the V_I is not sensitive to the leakage current, less than 1%. However, it is sensitive to the reference voltage. Any change in V_{REF} affects V_I by 99%. For the resistors, R1 and R2, the sensitivity is changing as the resistor values changes. Both sensitivities, with respect to R1 and R2, are almost the same in value but in opposite signs.

Conclusion

Sensitivity analysis is a very useful method to determine how the tolerance of inputs can affect the overall performance of the system. Based on the outcome of this analysis, the analog power designer can choose his variables while keeping the tolerance of the output of his system within the desired range. This helps in the component selection during the power supply design process.

References

1. *TPS3808 Data sheet (SBVS050J)*
2. *Optimizing Resistor Dividers at a Comparator Input (SLVA450)*
3. *Ballast Resistors Allow Load Sharing Between Two Paralleled DC/DC Converters (SLVA250).*
4. *Effect of Resistor Tolerances on Power Supply Accuracy (SLVA423)*